LECTURES ON POLITICAL ECONOMY

By

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INTRODUCTION

Johan Gustaf Knut Wicksell, the author of these lectures, is an economist of outstanding achievement whose work has not yet received in English-speaking countries the attention it deserves. In Scandinavia where he taught, and in Central Europe and Italy where he has long been read, his influence has already been extensive and important. But, in other parts, even at the time of his death in 1926, he was probably less known than any other economist of commensurate rank. In recent years, however, largely as a result of the writings of Professor Hayek and Mr. J. M. Keynes, his theories concerning the rate of interest and the price level have become more widely known and his reputation is on the increase. It is safe to say that as the main body of his work becomes available this process is likely to continue.

Wicksell was born in 1851. He was thus nine years younger than Marshall, three years younger than Pareto, and the exact contemporary of Böhm-Bawerk and Wieser. His interest in Economics developed comparatively late: his first important work, *Über Wert, Kapital und Rente,* was not published until 1893. He graduated in philosophy and mathematics, and it was not until after taking his second degree in 1885 that he turned his attention seriously to the subject which became his life-work. After ten years' further study in France, Germany, Austria, and England he took his doctorate in economics. In 1900 he was appointed assistant professor of Political Economy at Lund. From 1904 to 1916 he held the chair in the same university. He died in 1926.

[1] In preparing this introduction I have been greatly helped by articles dealing with Wicksell and his work by Professors Ohlin and Somarin, which appeared in the *Economic Journal,* vol. xxxvi, p. 503 seq., and the *Zeitschrift für Nationalökonomie,* Bd. ii, S. 221 seq., respectively. A succinct and well-documented account of Wicksell's work on the theory of Money and Capital and its influence on certain contemporary writers is to be found in an as yet unpublished thesis submitted by Mr. Solomon Adler to the University of London for the degree of M.Sc. (Econ.) in 1932, and a useful discussion of parts of this theory is to be found in Kirchmann, *Studien zur Grenzproduktivitätstheorie des Kapitalzinses.*

Wicksell's central contributions to theoretical economics are all outlined, if not fully developed, in three books, all in German, which appeared in rapid succession at the commencement of his career in the nineties: *Über Wert, Kapital und Rente*, which appeared in 1893 [1]; *Finanztheoretische Untersuchungen*, which appeared in 1896; and *Geldzins und Güterpreise*, which appeared in 1898. In the first he developed an outline solution of the main problems of the pure theory of value and distribution. In the second he applied certain elements in this solution to the special problems of the theory of public finance and the incidence of taxation. In the third he developed his now celebrated theory concerning the relationship between the money rate of interest and the general level of prices. His *Vorlesungen über Nationalökonomie*, of which the present volumes are a translation, were published first in Sweden in two parts, General Theory, and Money and Credit, in 1901 and 1906 respectively, and contain, with much new material, a systematic restatement of the main theorems of the first and the third of these earlier treatises.

It would be a great mistake, however, to regard Wicksell's work as an economist as limited to these four major publications. He published much on the population problem, played an active part in the discussion of public affairs in Sweden, and throughout his career was a regular contributor to the scientific journals in Sweden and elsewhere. The files of the *Ekonomisk Tidskrift* are full of lengthy articles by Wicksell, tantalizingly inaccessible to those of us who have not the good fortune to possess a sufficient knowledge of Swedish.[2] The German periodicals contain a number of contributions, and the *Economic Journal* and the *Quarterly Journal of Economics*, once at least, each secured an important article from his pen.[3] Few economists

[1] Some of the matter included in this book had been published in *Conrad's Jahrbücher* in the preceding year.

[2] Some of these contributions are now available in one or other of the world languages. The article on Professor Bowley's Mathematical Economics, with its discussion of the theory of Bilateral Monopoly, appears in the *Archiv für Sozialwissenschaft*, Bd. 58, pp. 252–281. Professor Hayek has included a celebrated article on Prices and the Exchanges in his *Beiträge zur Geldtheorie*, and two others on Dr. Gustav Åkermann's *Realkapital und Kapitalzins* and Prof. Cassel's "Theory of Social Economy" appear in English as appendices to the present volume. But an English translation of a comprehensive selection of these papers is still urgently to be desired.

[3] A short list of Wicksell's principal contributions to foreign periodicals is given by Professor Ohlin, op. cit., p. 512.

of his generation were more productive or—if those articles which are accessible in one or other of the world languages are any criterion—maintained so consistently high a level.

It is not easy in a few paragraphs to give a just view of the place in the history of modern economic theory of Wicksell's main achievements. As we have seen, he was the contemporary of men like Böhm-Bawerk and Pareto, whose work falls naturally under the headings appropriate to the so-called Schools—the School of Vienna, the School of Lausanne, the School of Marshall. But Wicksell fits into no such classification. No economist of similar rank has been more open to outside influences. But the influences were not all from one quarter. From the outset of his work in the nineties, he stands apart from the disputes of the Schools, deriving equally from the good elements in each of them—a pioneer of a generation which stands beyond these early factions and can perceive both the common denominator and the particular contribution in their respective systems. There is no economist whose work more strongly exemplifies both the element of continuity and the element of progress in the central tradition of theoretical Economics. Few have known better the works of the English classics or used them to greater advantage. To those brought up in the English tradition of post-classical Ricardian criticism his lucid reformulations of their doctrines must come as something of a revelation. But his debt to the later schools is no less evident. In the broad outlines of his value theory, the Austrian influence is strong; and in his capital theory the influence of Böhm-Bawerk is obvious. But the whole is set in a framework which derives essentially from Walras, and the detail owes not a little to Wicksteed and to Edgeworth. In short, in spite of his dates, Wicksell is of the present generation.

In all this, of course, he bears a strong resemblance to Edgeworth, our own great eclectic. There are indeed many elements in common in their work. Many of the problems which interested them were the same—distribution, public finance, the theory of monopoly—and they both brought to their solution that essential seriousness characteristic of those who are conscious of working with the instruments of an established scientific technique. But there was this important difference. Whereas Edgeworth's eclecticism showed itself mainly in the analysis

of particular problems, Wicksell's showed itself even more strongly in a tendency to synthesis. His particular investigations are important. But even more important are his reconstructions of general theory. He had the feeling for broad effects, the capacity for wide abstraction of the great system-makers. But being a scientist and not a mere system-maker, the system he constructed was not specifically his own but the system common to the best work of the past hundred years of economic theory.

In this respect, perhaps, he is more to be compared with Marshall, and more than one critic has made the comparison.[1] But here, too, there are important differences. There can be little doubt that in general knowledge of the details of economic relationships in the modern world, Marshall was greatly Wicksell's superior, as indeed he was the superior of most others of his generation. But as a systematizer of pure theory he had the defects of his qualities. The peculiar blend of realistic knowledge and theoretical insight which enabled him to present with such ingenuity the world as he saw it, was not necessarily conducive to clear presentation of abstract theoretical issues. He was so anxious to explain the reality he knew, to make his theory appear *plausible,* that he was apt to be impatient with refinements which, though useless for this purpose, might be fruitful in other connections. Moreover, as Mr. Keynes has pointed out, he lacked that æsthetic feeling for order and proportion which is essential to a theoretical synthesis on the grandest scale. It was just here that Wicksell excelled. There is no work in the whole range of modern economic literature which presents a clearer general view of the main significance and interrelations of the central propositions of economic analysis than these lectures. The arrangement is exemplary. The successive propositions are presented in a setting which emphasizes both their implications and—what is just as important—their limitations: and the whole is built up in such a way that at each successive point in the argument attention is always focused upon the new elements in the problem, the rest having been satisfactorily disposed of at an earlier stage. In this no doubt Wicksell learnt much from Walras. But no one would contend that the exposition of the *Éléments d'Économie Politique Pure,* littered

[1] See, e.g., Schumpeter, "Knut Wicksell," *Archiv für Sozialwissenschaft,* Bd. 58, pp. 238–257.

up as it is with so much superfluous and somewhat crude mathematics, is a model of expository clarity.

In certain respects, the closest comparison is with Wicksteed. For Wicksteed had the architechtonic instinct, and he, too, had derived both from Lausanne and Vienna. He had not, however, Wicksell's feeling for the English classics, and the development of his thought was on different lines. Strongly influenced by Pareto's modifications of utility theory, in later years he became more and more interested in the philosophical and methodological implications of the general theory of value. Wicksell, on the other hand, who was a bit old fashioned on pure utility theory, turned his attention more and more to the development of that part of the Jevonian-Böhm-Bawerkian theory of capital, which, just because he rejected the classical writers so completely, in certain respects Wicksteed failed to comprehend [1]; and as time went on his interests became more technical and practical. But the two supplement each other in admirable fashion. The subjective side of modern theory is at its best in Wicksteed, the objective in Wicksell; a combination of the two covers much of the essential ground.[2] I am not clear that Wicksteed was acquainted with Wicksell.[3] But there is ample evidence that Wicksell knew Wicksteed's work and appreciated it long before much was thought of it in England.

Any enumeration of Wicksell's more outstanding contributions to the detail of Economic Science must commence, if it is to do justice to his own wishes, with his contributions to the theory of population. It was the reproach that his knowledge of the economics of the population problem was insufficient, which first directed his attention to scientific economics; and thoughout his life, the population problem in all its aspects retained the

[1] In this connection a comparison between Wicksteed's article on Jevons' "Theory of Political Economy" (*Works*, vol. ii, pp. 734–754) and the sections on Capital Theory in *Über Wert, Kapital und Rente* is very instructive.

[2] But not all. I should be very sorry to be thought to lend any countenance to the view, now apparently gaining ground in somewhat unexpected quarters, that in undergraduate teaching or in advanced studies we are yet in a position to dispense with the most thorough study of Marshall's *Principles*. It would be a sad thing if the uncritical acceptance of this great work, which so long tended to stiffle the development of other lines of thought in this country, were to be succeeded by an equally uncritical rejection of all the wisdom and the path-breaking intuitions that it contains.

[3] He must have been aware of *Über Wert, Kapital und Rente*, for it was reviewed together with his own *Co-ordination of the Laws of Distribution* in the *Economic Journal* for June, 1894.

strongest hold on his interest and emotions, so much so indeed that in 1909 he incurred the penalty of a short term of imprisonment on account of strong utterances on certain of its non-economic aspects—a period which he devoted to the preparation of a short book on this subject signed defiantly " Ystad Prison ". In the statistical field, he did much important work on the mechanics of population increase, and, in the field of economic theory, he was one of the first systematically to develop the concept of an optimum population. Whether it is so easy at any time to assign a specific magnitude to this elusive concept as Wicksell himself supposed, whether indeed we really yet know enough about the application of the laws of returns in this connection to be in a position to describe it in a way which is theoretically satisfactory, are questions on which differences of opinion between reasonable men may yet legitimately arise. But the emphatic pronouncements in the introduction to the *Lectures* on the place of population theory in a systematic treatment of economic problems are a sufficient indication of the importance Wicksell himself attached to this part of his work.

To the broad outlines of the theory of value Wicksell added little that was completely original. But he fused the main teachings of Walras and the early Austrians with great ingenuity and expository power, giving to the philosophical insight and profoundity of Menger and his followers, the superior precision and elegance of the mathematical formulation. Seldom have the complications involved in the transition from pure utility theory to the theory of exchange and price been stated with greater clarity and exactitude. To more recent developments of the theory of value he was not very sympathetic, probably on account of the very strong utilitarian bias in his general view of the subject. The student of the theory of public finance, however, should not miss his discussion of the principle of justice in taxation.[1]

In the theory of production Wicksell displays much greater originality. His statement of the marginal productivity theory is one of the most satisfactory available. As Dr. Hicks has shown,[2] the exposition in the *Lectures*, with its express condition

[1] *Finanztheoretische Untersuchungen*, p. 176 *seq.* Wicksell's views in this spect have been developed with great ingenuity by his pupil, Professor E. Lindahl, in his *Die Gerechtigkeit der Besteuerung*.

[2] *Theory of Wages*, p. 233.

that the various firms concerned must be at a stage at which futher economies of large scale production are absent, is immune from the strictures which have been passed by Pareto, Edgeworth and others on the version which is to be found in Wicksteed's *Co-ordination of the Laws of Distribution.* In this he may have been indebted to Walras. But in the light of the discussion of the theory of distribution in *Uber Wert, Kapital und Rente*, Wicksell must himself be looked upon as one of the founders of the marginal productivity theory.

Most conspicuous, however, in the sphere of the theory of production is Wicksell's contribution to that part which deals with problems of capital and interest. Here his eclecticism rises to the point of pure genius. By a judicious selection from the best elements in earlier theories he achieved a reformulation of this part of the theory of production from which, it is safe to say, all future work in this field which aspires to be taken seriously must commence. It is worth examining the nature of this achievement in rather more detail.

The part played in the classical system by the ingredients of a substantially correct theory of capital and interest is by no means so negligible as post-classical criticism has often assumed. On the one hand in the wage fund theory, on the other in the Ricardian modifications of the labour theory of value, particularly in the letters to McCulloch, there exist the rudiments of a theory in many essential respects not dissimilar from that which is to be found in Jevons, Bohm-Bawerk and Wicksell. In a series of brilliant reconstructions in the *Finanztheoretische Untersuchungen* and elsewhere, Wicksell himself indicated the significance of certain aspects of the classical doctrines in this respect. More recently Mr. Edelberg has shown [1] how, if one is willing to give Ricardo the benefit of the doubt in one or two connections, a whole theory of capital and interest on Wicksellian lines can be reconstructed from actual Ricardian material. In any case it cannot be said that important theories of capital and interest played a negligible part in the classical system. Indeed, if a choice had to be made between the classical theories and those modern systems which ignore the Jevonian-Böhm-Bawerkian reconstruction and reject the classical elements,

[1] "The Ricardian Theory of Profits," *Economica*, February, 1933, pp. 51–74.

there is much to be said for the view that the classical theories would be much less likely to mislead.

But the classical system as a whole was very vulnerable. It was open to general attack on its theory of value. It was everywhere deficient on points of formulation. And these particular theories of capital and interest were liable to attack, not merely for their obvious deficiencies in this respect, but also for political reasons. As time went on, the wage fund doctrine in particular, instead of being reformulated in those minor respects in which it was defective, became the target of continuous and completely hostile criticism, some of it justified in points of detail, but most of it analytically erroneous and totally beside the point. Nothing could be more superficial—for instance—than the criticisms put forward by writers such as Walker and J. B. Clark of the incontrovertible proposition that wages are paid out of capital. But for political reasons the classical theories of capital were unpopular and men jumped at any pretext for rejecting them. The result was that, particularly in English circles, much of the Economics of the fifty years after 1870 was what Wicksell calls a *Kapitallose Wirtschaftstheorie* —an economic theory of *a*capitalistic production. Considerations of capital theory proper, save of a more or less terminological nature, simply disappear from the picture. Professor Taussig's *Wages and Capital* was a gallant attempt to stem the tide—which incidentally carried through most of the modifications necessary to make the classical theory logically acceptable and completely disposed of the ridiculous myth that it had originated in selfishness and reaction. But it was in vain. When, after the war, Mr. Dennis Robertson and Mr. J. M. Keynes turned their attention to problems of fluctuation which involved similar considerations, the tradition of a theory of capital had so completely disappeared in English Political Economy that they had to start completely from the beginning. Nor was the position any better in certain continental circles. The work of Pareto, valuable as it is in other respects, adds little to knowledge in this connection. It would perhaps be putting it too strongly to say that there is no capital in his equations of economic equilibrium. But it would certainly be correct to say that there is no *time*. Now time is the essence of capital theory.

There was another stream of thought, however, in which the theorems of the classical economists were by no means altogether abandoned. In spite of his antipathy for Mill and his celebrated denunciation of his "four fundamental propositions on capital" —"all wrong," as he said, Jevons had taken over into his capital theory important classical elements. And in Böhm-Bawerk's "Positive Theory of Capital" something very like the classical wage fund theory, shorn of its obvious defects of formulation, makes its appearance. But Jevons' chapter on capital was only an outline; and, for various reasons, the influence of Böhm-Bawerk was not altogether fortunate. In his critical work, he was undoubtedly unjust to many of his predecessors. This, where it did not create repulsion, created the impression of a much greater lack of continuity than actually existed. And in his positive solution, which in most important respects was substantially correct, the emphasis and arrangement was such as to make understanding of the main elements much more difficult than need have been the case. The sections dealing with the element of time discount are admirably clear and have made a permanent mark on the discussion of the subject elsewhere. But the sections relating to the "third ground" for the existence of interest—the "technical superiority of present goods"—are developed in a mode which definitely invites criticism. What, as Wicksell points out, is really the central and fundamentally unassailable core of the Böhm-Bawerkian theory—the discussion of the influence of the varying productivity of productive processes of different lengths on prices, the use of the subsistence fund, and the formation of the rate of interest—only appears as a sort of practical application of these more disputable propositions at the very end of the book. It is clear that many of Böhm's readers never reach that last section. The result has been that in those parts where the oral tradition of Böhm-Bawerk's seminar was not influential, it came to be thought that the theory of the relation of time discount to interest was Böhm-Bawerk's chief contribution. The propositions relating to the "third ground" were held to have been disposed of by the criticisms of Professors Fetter and Fisher; and the most valuable element in the solution, therefore, what is really a marginal productivity theory of interest, properly stated in regard to the time element, tended to escape attention.

But not with Wicksell. For Wicksell the productivity side of the question was obviously at once the more important and the more deserving of further elucidation. Steeped as he was in the literature of the classical system, he had no difficulty in detecting the underlying continuity between Böhm-Bawerk's theory of the subsistence fund and the classical wage fund theory, and with his mathematical insight he divined, in spite of all Bohm-Bawerk's disclaimers, the substantial identity between the general marginal productivity analysis and the propositions relating to the varying productivity of different investment periods. He was thus able to present an account of equilibrium of capitalistic production which combined all the best features of these apparently divergent theories, and, by invoking the methods of Walrasian analysis, he was able to present it in a much more general setting than was the case with either Jevons or Böhm-Bawerk. It is true that this theory itself is not complete. It was fully developed in the *Lectures* only for the case of circulating capital. And although later on, in his review of Dr. Åkerman's book (printed below as Appendix 2) Wicksell developed a solution for the case of capital of varying degrees of durability, it is obvious that this is one of the fields of pure analysis in which most yet remains to be done. But the fundamental ideas of his theory—the place of the varying productivity of variations in the investment period, the idea of interest as the difference between the marginal productivity of direct and indirect uses of factors of production—these are notions which are not likely to be superseded and which are fundamental as a basis for future work.

I come finally to what is probably the best known of Wicksell's contributions—his celebrated theory concerning the relations between money and natural rates of interest and movements in the general level of prices. This is probably Wicksell's most original contribution. The main propositions are certainly not new. As Professor Hayek has shown [1] there is a very considerable body of passages in the classical literature, in which, in one form or another, they make their appearance. But, apart from one isolated passage in Ricardo, which Wicksell

[1] *Prices and Production*, chapter i, *passim*. "A Note on the Development of the Doctrine of 'Forced Saving,'" *Quarterly Journal of Economics*, vol. xlvii, pp. 123–133.

says explicitly was only brought to his notice after the publication of his own theory, these passages are not in the most conspicuous or most easily accessible works, and there seems little reason to question that, in so far as any idea implicit in the fundamental notions of Economics can be so described, his main idea was original.

Its influence has been far reaching. It is clear that in Wicksell's own treatment, in certain respects—not unimportant in regard to practical applications—it is not correctly developed. It can be shown that the proposition that the money rate of interest which keeps prices stable is also the rate which clears the market of voluntarily accumulated capital, breaks down when the conditions of capital supply are either progressive or retrogressive.[1] It is clear that it stands in much need of refinement before it can be applied to the interpretation of actual conditions—still more as a guide to practice. The notion of a single rate, either natural or monetary, needs to be replaced by the idea of a *structure* of rates; and the interrelations of these rates, and their relation, not merely to the stream of saving, but also to the risk factor, need much more study. But when all is said by way of qualification, it remains true that the discovery, or rather the rediscovery, of the general relationship involved is one of the greatest single steps forward in monetary economics since the proper elaboration of the quantity theory. It is the key, not only to the more complex problems of fluctuations of monetary value, but also to much that is central in the general theory of capital and the theory of business cycles. Monetary theory and capital theory alike are at an *impasse* when the theory of money is limited to the simple quantity theory and the theory of capital is divorced from the theory of the money market. The value of money is said to depend on the quantity of money and the velocity of circulation, the rate of interest on the marginal productivity of extensions of the investment period, and the rate of time discount. The relations between the supply of capital and the supply of money, between the money rate of interest and the rates of real accumulation and investment, not to mention the relations between

[1] See Hayek, *Monetary Theory and the Trade Cycle*, chapter v, and *Prices and Production*, chapter i; also G. Myrdal, "Der Gleichgewichtsbegriff als Instrument der Geldtheoretischen Analyse," in *Beiträge zur Geldtheorie*, ed. Hayek.

relative prices at various stages of production and the rate of borrowing of the entrepreneurs—all these problems, whose solution is essential to any comprehensive theory of economic change, remain unexplained until this fundamental conjunction has been effected. No doubt in this field it has been left for others to develop the implications of the broad principles which Wicksell laid down and even now much work still remains to be done. But the main credit of rediscovering these principles and bringing them once more into the centre of discussion must rest permanently with the author of these lectures.

The present translation is based upon the third edition, published in Sweden after the death of the author under the editorship of Professor Somarin. The two volumes into which it is divided, which deal with general theory and money and credit respectively, are to be published successively and will be sold separately. There have been added, as Appendices to Volume I, two of Wicksell's longer articles, one which adds to the capital theory of Book II further elucidations of the problem of durable capital not provided in the text, and another, which, in the form of a lengthy critique of Professor Cassel's *Theory of Social Economy*, underlines various details of Wicksell's general outlook. The inclusion of this latter must not be thought to imply any special endorsement by the editor of all the various criticisms it contains; there are, indeed, several not unimportant points, notably those relating to the measurability of utility, where Professor Cassel still seems to me to have the better of the argument. But it is always good to know exactly where important authorities differ, and it was thought that anything which should elucidate the relationship of the theoretical systems of the two most famous Scandinavian economists of our time would therefore be helpful.

Wicksell's aim in preparing the *Lectures* was to provide a work which would not only enlighten the professional economist but would also serve as a textbook for students. It is with this end in view that the present edition has been prepared. It is not perhaps suited as an introduction for very young students who have no preliminary acquaintance with economics or any of the natural sciences. For such, some such work as

Volume I of Wicksteed's *Commonsense of Political Economy* is to be preferred. But for more advanced students (i.e. students in the first year of preparation for the final examination, as distinct from students preparing for the intermediate) and for readers of maturity it is admirably fitted for use as a general textbook. I know no single work better suited to the needs of any natural scientist who wishes to get a general view of what theoretical economics is about, and to what extent it is scientifically respectable. In parts the exposition is mathematical. But here, as in the original, the more advanced sections and the sections involving calculus have been printed in smaller type and may be omitted on first reading. The main argument throughout is accessible to those who have no mathematical competence.

The task of editing the translation of a technical work of this sort is always somewhat arduous, and I am indebted to many friends at the London School of Economics who have lent assistance. The final version of the text owes much to Dr. J. R. Hicks, who generously gave much time to the checking and correction of the manuscript. In addition to providing the translation of the Appendices, Mr. Solomon Adler gave valuable assistance and advice concerning the rendering of technicalities, and Mr. E. S. Tucker has borne the main burden of the laborious task of seeing the book through the press.

LIONEL ROBBINS.

FROM THE AUTHOR'S PREFACE TO THE SECOND EDITION

The first edition of this book was a very limited one, for I did not wish to deprive myself of the opportunity of publishing a new edition and of availing myself of the improvements which experience and expert criticism might suggest. Unfortunately, very little criticism, either public or private, has reached me; but during the ten years or more in which I have been teaching I have naturally discovered various defects, which in this edition I have endeavoured to correct. By omitting the chapter on the theory of population, which was published a couple of years ago in a revised form as a "Verdandi" publication, it has been possible, without increasing the size of the work, to find space for certain additions, which, I hope, will increase its value and its usefulness. Thus the presentation of the theory of rent and the problem of distribution in a non-capitalistic economy has been expanded and, in connection with the theory of interest, some pages have been devoted to a résumé and criticism of Böhm-Bawerk's theory in its original form. Similarly, I have given a detailed alternative explanation [1] of the origin of interest and of the solution of the problem of distribution under capitalistic production, in which I assume that the whole of the available supply of current labour and land is either invested in production at once, at the same time, or possibly at different moments of time; after which, the products mature spontaneously under the influence of free natural forces—as for instance in the laying down of wine for consumption, etc. Interest then appears in its purest form as the "marginal productivity of waiting" (or of time), and the problem, in all its phases, is easily susceptible of exact treatment in a mathematical form, without it being

[1] This expression is perhaps not entirely suitable, since, as will easily be seen, the essence of the argument is in both cases the same. It is therefore also possible that I ought to have endeavoured to combine sections II, 2, C and D in a single uniform presentation. I have found myself unable, however, for various reasons, to do this. As they now stand, these two collateral presentations may materially support and explain each other.

necessary to have recourse to calculation with so-called simple interest, as in Böhm-Bawerk's well-known exposition.

Finally, the original brief discussion of the phenomena of the accumulation of capital has been expanded, and now includes an examination of Professor Cassel's interesting contributions to the still very meagre literature of this subject.

As will appear from what has been said, the present edition has a more "mathematical" character than its predecessor. In every case, however, I have prefaced the mathematical analysis by an elementary treatment with definite—though usually arbitrary—figures. The passages in smaller type can, for the most part, be read and understood without any special knowledge of mathematics, and for the remainder, as I have said in the text, the standard reached nowadays in secondary schools should suffice.

Opinions may differ as to the value of this method. For my own part, I am convinced that a constant and logical argument from simple assumptions conveys more real knowledge than variegated but superficial talk upon everything under the sun: national character, racial differences, will to power, class interests, etc. Again, as regards the controversy concerning the so-called historical and theoretical treatment of economics (of which the latter must of necessity be more or less mathematical), this is a matter which can, in my opinion, be settled only by a *division of labour*. We must be deeply grateful to those persons who, by the discovery and investigation of documents relating to economic history—matters treated in a very step-motherly fashion by earlier historians—have succeeded in illuminating the present by the light of the past, and in showing to us some links on a chain of development of which we ourselves and our environment constitute another link. But, on the other hand, if economics is some day to become a real science and guide to practical business it must inevitably advance to certain positive results and principles of universal application. It will not do to treat questions relating to economic policy, to trade and industry, and especially to population, as if they were metaphysical speculations in which each person can adopt the point of view which appeals most to his temperament—and still more frequently, perhaps, to his private interests. We are here concerned with substantial quantities, measurable magnitudes,

a and *b*, plus and minus. To secure an explanation of their relations which would be convincing to every thinking and unprejudiced person cannot be said to be outside the scope of economic inquiry, but must, on the contrary, be its ultimate goal.

I am, of course, far from regarding the following arguments, which are for the most part hypothetical, as an adequate foundation for a practical treatment of economic questions, though I have little doubt that they constitute a necessary preliminary—and, at the same time, provide a useful exercise for those concerned with such problems. In more than one case it may appear that a direct application of our principles to actual politico-economic problems would be quite natural. In such cases we must certainly be on our guard against over-hasty generalizations from results achieved by way of abstract deductions; and, unfortunately, the mathematical method affords no absolute guarantee against false deductions. But, in any case, that method has a great advantage over the merely descriptive method, in that errors committed cannot long be concealed, and false opinions cannot be defended long after they have been shown to be wrong.

KNUT WICKSELL.

LUND.
March, 1911.

LECTURES ON POLITICAL ECONOMY

GENERAL THEORY

INTRODUCTION

THE NATURE OF ECONOMICS: DIVISION OF THE SUBJECT

It is not easy to give a satisfactory definition of the term "political economy".[1] The conception itself is, indeed, somewhat vague—a natural state of affairs in the infancy of a science. Literally, the name indicates national housekeeping or the theory of national housekeeping. Yet, at any rate nowadays, a nation has no common housekeeping, but every individual manages his own affairs. The State itself constitutes a management of some affairs in common and the same is true of the local units; the housekeeping of those units is dealt with by the science of public finance, which, though it must be regarded as a part (and an important part) of political economy, is by no means the whole. In modern times, moreover, it has become customary to treat public finance as a distinct science.

The name political economy arose during the so-called "mercantile" age, when it was regarded as a duty of the State itself to exercise an extensive influence over the affairs of individuals, so that the latter enjoyed only a very restricted liberty, under the guidance and control of the State. At that time, therefore, it was appropriate to speak of political economy, a term which adequately represented the conception which underlay it. Its appropriateness diminished with the advent of the physiocratic ideas and the victory of the conception of unrestricted liberty and free trade, especially as the main thesis of the latter was that the State should interfere as little as possible in economic affairs and leave the individual, except in certain well-defined cases, free to attend to his own business.

[1] [Swedish *Nationalekonomi*: German *Nationalökonomie*.]

Thus, according to this view, the fundamental principle of political economy was that its subject matter, the national household, did not exist.

In our day, it is true, there has been a reaction against this ultra-liberal principle, but nevertheless it is still in reality the individualistic, purely private, system which predominates. For this reason many modern writers have desired to reject the qualifying adjective "political" or "national" and to speak merely of economics, or have invented entirely new names, such as "*plutology*" or "*catallactics*". But in the absence of a better name we may perhaps retain the old one,[1] provided that we are careful not to import into it the conception of a national unity in the economic field which does not exist in reality. In accordance with the modern outlook, the subject matter of political economy is becoming more and more the doctrine of economic phenomena, in their interrelations, seen *as a whole*; i.e. in so far as they uniformly affect whole classes of the community, or a whole people, or the totality of all peoples (what the Germans call *Weltwirtschaft*). By an *economic* phenomenon or activity is meant every systematic endeavour to satisfy a material need, or, more precisely, one which seeks with the available means to achieve the greatest possible result, or a given result with the least possible means. (The familiar expression, "to obtain the greatest possible results with the smallest possible means," is illogical and should therefore be avoided.)

In many cases such an activity, though directed to the advantage of an individual, at the same time promotes, or is at least not inimical to, the general good. He who works and produces only for his own gain also confers benefits on others —indirectly, by means of *exchange*; the improvement of the soil and of technical plant in general, which is effected by the present generation, possibly only in its own interests, will, nevertheless, be of benefit to the coming generation. In such cases individual and national economic interests coincide. But it is equally common, or even more common, for one economic interest to conflict with another; circumstances or activities which benefit one branch of industry, one class of society, or one generation, are often more or less injurious to another.

[1] [i.e. *Nationalekonomi* (Swedish) or *Nationalökonomie* (German).]

Examples of this kind are familiar to everybody; the most important is surely the distribution of property, in so far as possession of land or an exceptional monopoly of any kind necessarily excludes others from that land or that monopoly. Private and national economic interests then no longer coincide, and the question arises which is to be followed; in other words, which of two conflicting interests is to be preferred as contributing most to the general good. To answer this question is the practical and social duty of political economy, and it might be said that the definition of political economy as a practical science is the theory of the manner of satisfying human needs which gives the greatest possible satisfaction to society as a whole,[1] having regard to future generations as well as to the present. The existing individualistic organization of society, in so far as it is socially justified, must then be regarded as a means to the attainment of that end.

The solution of this problem is frequently very difficult and the result is, of course, always dependent not only on technical economic considerations, but also on the degree of our sympathies; that is to say, on our understanding of the interests and demands of others. When we say that a thing is beneficial or injurious from the point of view of political economy, this manner of speaking is based on an ethical or philosophical postulate; that is to say, on certain conceptions concerning the natural right of men to live and enjoy the good things of life. We either consider all to have the same rights and reckon each individual member of society as a unit, or else, for one reason or another, we recognize a difference between them, though in that case the reasons must be clearly stated if we are to regard our view as scientifically established.

As we all know, opinions on this question have changed greatly in the course of time. In earlier times, only the free, and afterwards only the propertied, classes were regarded as members of society in the true sense; slaves and those without property were regarded in much the same way as domestic animals in our day—merely as a means and not as an end. Aristotle's well-known saying that shuttles and the plectron of the lyre would have to move of themselves before slavery could cease,

[1] Here, too, one should avoid the very common, but fundamentally meaningless, expression "the greatest happiness of the greatest number".

is evidence of this view, though we need not go back so far in time to encounter similar opinions. Among eighteenth century Swedish writers on economics, mentioned by Arnberg in his *Frihetstidens politiska ekonomi* ("The Political Economy of the Age of Freedom"),[1] we repeatedly find remarks which show that the conception, so repellent to our minds, of a workman as a mere beast of burden was, as recently as two centuries ago, still general and deep-rooted. Indeed, it may be regarded in some degree as one of the merits of economic science that in this respect it has produced a revolution in public opinion. As soon as we begin seriously to regard economic phenomena *as a whole* and to seek for the conditions of the welfare of the whole, consideration for the interests of the proletariat must emerge; and from thence to the proclamation of *equal* rights for all is only a short step.

The very concept of political economy, therefore, or the existence of a science with such a name, implies, strictly speaking, a thoroughly revolutionary programme. It is not surprising that the concept is vague, for that often happens with a revolutionary programme. Indeed, many practical and theoretical problems remain to be solved before the goal of economic or social development can be said to be clearly understood. Something can still be said in favour of the older point of view, but in any case it should be said straightforwardly and without prevarication. If, for example, we regard the working classes as beings of a lower type, or if, without going so far as this, we regard them as not yet being ready for a full share in the product of society, then we should say so clearly and base our further reasoning upon that opinion. There is only one thing which is unworthy of science—to conceal or pervert the truth; that is to say, in this case, to represent the position as if those classes had already received all they could reasonably wish or expect, or to rely upon unfounded, optimistic beliefs that economic developments in themselves tend to the greatest possible satisfaction of all. This latter mistake was made especially by the so-called "harmony" economists in the middle of the last century—the American, Carey, and the otherwise admirable Frenchman, Bastiat—both of whom in

[1] Cf. also G. Schauman, *Studier i frihetstidens nationalekonomiska litteratur*, Helsingfors, 1910.

their own countries and in ours have had, and still have, many disciples.

The division of the subject which first suggests itself is into "theoretical" and "practical" political economy—economics in the narrow sense and national economic policy. Owing to the decisive difference which it makes to our handling of economic problems whether we assume the existence of private property and freedom of contract in anything like their present forms, or whether we do not, it might be more appropriate to subdivide the practical portion into two parts: one being an application of the theory *founded on existing conditions*, and the other a *critical examination* of the *foundation* itself.

The former of these would be, at the same time, a link between the latter and the theoretical portion. On the one hand, it amplifies the theoretical abstractions by a closer consideration of reality, whilst, on the other hand, the practical problems which emerge as soon as we approach reality can find their ultimate solution only in a criticism of the foundations of the whole economic life of society.

We thus arrive at the following division of our subject:—

(1) *A theoretical part* (pure, general, or theoretical economics), comprising a statement of economic laws or the connection between economic phenomena, in which, in order to discover or demonstrate these laws, we must necessarily proceed from certain *simplifying assumptions.*

(2) *A practical part* (applied economics, particular problems of the consumption, distribution and production of goods), comprising the application of these laws to various fields of activity in the *concrete economic life of society.*

(3) *A social part* (social economics or economic policy), comprising an investigation into the question how these economic laws and practical precepts should properly be applied in order to obtain *the greatest possible social gain*, and what changes in the existing economic and legal structure of society are necessary to this end.

In the *first* of these main parts there are certain subdivisions. First and foremost comes the theory of *human wants*, quantitative and qualitative, i.e. the general theory of *consumption*, which, since it is the purpose of all economic activity, should logically be placed first, even though in actual life it comes last in point

of time. As regards such needs, or consumption, the *quantitative* point of view emerges first, and in this respect the number of consumers is of decisive importance. Thus, in our first subsection, we naturally treat of the theory of *population*, its composition and changes. Man is, indeed, not only a consumer; he is also a producer. Yet he is, both phylogenetically and ontogenetically, both in racial and individual development, a consumer long before he is a producer. In the theory of production, moreover, man is only one of the productive factors; in the theory of consumption he and his purposes constitute the whole. Generally speaking, and even apart from the above division of the subject, it will be found that the theory of population, which can never be omitted from a complete treatise on political economy, can never find a suitable place in the system unless it forms an introduction to the whole. In actual fact, it is impossible to consider economic problems profitably, whether they are of a practical or theoretical kind, unless we constantly keep population and its changes in view. On the other hand, it would appear that certain problems of population are of such a complicated nature that they cannot be solved without a thorough knowledge of every part of the theory of economic structure. Thus we return to these problems at practically every point in a thorough economic investigation, and their solution may be regarded as its chief result.

We next turn to the *qualitative* side of human needs: to their extent and intensity, relative importance, etc., and the comparative importance which we accordingly attribute to the means of satisfying these needs. The development of this inquiry will lead us to the theory of *value* and to the associated general theory of *exchange*. On the other hand, exchange as it appears in reality in modern society, and the regulation of exchange by society which may be considered desirable, belong respectively to the second and third main sections of our subject.

The next subdivision is the general theory of *production* and of the factors of production: land (or nature), labour, and capital, their part in production and their relative shares in the distribution of the product—rent, wages, and interest—all examined on certain simplifying assumptions, such as universal free competition or competition limited in a certain manner. It is already clear that the theory of production cannot be

separated from the theory of *distribution*, though it should be noted that this applies only to distribution as it actually takes place under the individualistic economic system, or, more correctly, as it would take place on our simplifying assumptions. The social problem of distribution, on the other hand, which belongs to the third main division, is fundamentally different from this ; it embraces, among other things, the question, not yet raised at this stage, of property rights in the various factors of production.

In these two subdivisions we shall treat the subject mainly from the static point of view, i.e. we shall assume, in principle, a society which retains unchanged from year to year the same population, the same area of territory and the same amount of capital, and remains on the same level of technical achievement. By way of transition to a more dynamic point of view, which can only be successfully presented in combination with the practical part of our subject, we shall briefly treat the problem of saving or *accumulation of capital*—which is equivalent to production without corresponding consumption—as well as its negative counterpart, capital consumption.

Finally, we include in the general or theoretical part of our work the theory of the medium of exchange, money as well as organized credit, which subjects are clearly connected and partly coincide. Many monetary questions, it is true, have their proper place in the special or applied section of our subject, but to avoid unnecessary length we shall treat most of them together, more especially since the actual technique of money is of much greater interest to pure economic theory than the technical details of production or trade.

We thus obtain the following five subdivisions of Part I of our work :—

(i) The theory of population.[1]

(ii) The theory of value and exchange.

(iii) The theory of production and distribution.

(iv) The theory of capital, all of which are treated in Volume I, and

(v) The theory of money and credit, which is the subject of the second volume of the theoretical part of our work.

As I shall probably not be in a position to publish either

[1] [For reasons explained in the author's preface, this section was omitted in the second Swedish edition and is not included in the present translation.]

of the two other main parts, it is unnecessary to recount how I have conceived their content or how I have treated them in my lectures. I need only add that the third main division (or Social Economics) would include, as its last section, a theory of public finance—which is usually treated nowadays as a separate science, as a study of particular financial legislation—though in essence it undoubtedly constitutes a part, growing more important and extensive every day, of political economy.

This division of the subject accords in the main with that used by Walras in his *Éléments d'économie politique pure*, though it is not always based upon the same reasons. Formerly, following the example of J. B. Say and J. S. Mill, it was usual to divide economics into the theories of production, distribution, exchange, and consumption—a chronological order, as it were, according to which it was supposed that commodities must first be *produced*, then *distributed* between the persons participating in the production (workers, landlords, capitalists, etc.) and then *exchanged*, in so far as they were unable to avail themselves of their share in kind, and finally *consumed*. But this easy division of the subject is far from logical. Production and distribution cannot, as we have already pointed out, be understood except in combination, and the concept of value and exchange underlies both, a fact which has led to incessant anticipations and circumlocutions unfortunate from an expository point of view. And, again, there was not much left to say about consumption when everything else had been treated; so that the whole of this section was completely ignored by Mill. Yet, if this is allowed to happen, one loses sight of the fact that that which directs—or, more correctly, *ought* to direct—all economic activity is human needs. Thus the theory of *wants* or *value* should undoubtedly be placed first; and this is often done nowadays, even by writers of textbooks who, like Professor C. Gide, otherwise preserve the old division of the subject. On the other hand, it can hardly be right to postpone discussion of value, as Philippovich does, and only to treat of it in connection with the theory of commercial practice. The theory of value in its modern form has, as we shall see, been more or less responsible for the transformation of every branch of political economy and should, in combination with the theory of *population*, constitute the foundation of the whole edifice.

Another consequence of this traditional division of the subject has been that, within the various main divisions, theoretical, practical, and social problems have been treated together. At an earlier stage in the development of the science, this might be defensible—and there is no doubt that it helped to give to the works of Mill, as previously to those of Adam Smith (whose division of the subject is somewhat different), a high degree of literary charm. But in proportion as science develops and becomes specialized, a different method becomes necessary and, by adopting it, it becomes easier to escape the criticism, advanced so often and with so much justice against the older economists, that the range of validity of their conclusions was not always clearly established.

It is a more especial disadvantage of the traditional division of the subject that the theory of money came to be treated as a mere episode in the theory of exchange, without regard to its great theoretical and practical importance in every branch of economics. This is probably the real reason why, despite the voluminous writings on the *technical* aspects of money and credit, no complete theory of money and its functions has ever been advanced, and why it remains one of the least explored fields in the theory of political economy.

Passing over to pure or theoretical economics (with which the present volume will be solely concerned) we should point out that the exposition in the whole of this section must of necessity be abstract and schematic; the results will be correspondingly hypothetical, that is to say, they can only claim validity under our simplifying assumptions. Whether, and to what extent, they will accord with reality will evidently depend on two circumstances: first and foremost, whether our assumptions are themselves founded on reality, i.e. contain at least some elements of reality—which we must always demand, for otherwise all reasoning about them would be sterile. We can, for example, safely assume that men are actuated by selfish motives, because that is always, at least to a very large extent, true. But we can no more assume that they are filled with a desire to injure each other than that they are purely altruistic. Further, the conditions from which we abstract must be relatively unessential, at least as regards the question under discussion: when we are considering certain economic

PART I

THE THEORY OF VALUE

BIBLIOGRAPHY.—The three works which, appearing almost simultaneously but quite independently, put forward for the first time the main features of the modern theory of value are Carl Menger's *Grundsätze der Volkswirtschaftslehre*[1] (published after his death in a new and enlarged edition), Stanley Jevons' *Theory of Political Economy*, and Léon Walras' *Éléments d'économie politique pure* (both of which appeared in several editions). The simplest, and perhaps fullest, presentation of the theory, from Menger's point of view, and without the use of mathematical symbols, is given by Böhm-Bawerk in his famous essay *Grundzüge der Theorie des wirtschaftlichen Güterwerts*[1] (*Conrads Jahrbücher*, vol. xiii (1886)). An adaptation of this, in which some portions of interest have been omitted, is to be found in the same author's *Positive Theorie des Kapitals*. Among the many works in which the theory was subsequently developed may be mentioned Marshall's *Principles of Economics*, published in many editions; Wicksteed, *The Common Sense of Political Economy*; Pierson, *Principles of Economics*; Pareto, *Cours d'économie politique* and *Manuel d'économie politique* (1909); my own work, *Über Wert, Kapital und Rente*[1]; and, in Swedish, Johan Leffler's essays in *Ekonomiska Samhällslifvet*, vol. i, pp. 4–37 and 48–80. Although supplemented and corrected by the modern theories of value, the writings of the classical economists on value and price have by no means lost their importance. The well-known works of Adam Smith, Ricardo, and John Stuart Mill still provide, in this field, a number of instructive investigations and observations. A kind of reaction in the direction of the earlier point of view, though more apparent than real, is to be seen in G. Cassel's *Theoretische Sozialökonomie* (1918, 4th ed., 1927), also published in English (1923 and 1932).

[1] [These works are reprinted in the Series of Scarce Tracts, published by the London School of Economics.]

In this part we have first to examine the qualitative aspect of human needs and the differing significance which we attach to the available means, material, or otherwise, of satisfying those needs. In modern communities this significance finds its most striking and objective expression in the *exchange value* or *price* of the various objects, goods or personal services.

The theory of value and price has an importance which is not limited to systems where there is highly developed division of labour, with money and credit and more or less free competition. Even in a self-contained economy (e.g. in the administration of national or communal finance), indeed in every individual productive enterprise or consumption unit, valuation constantly takes place. And we find exchange, too, when that is understood in the wider sense of the term, i.e. a choice between the various uses of the same means of production or finished commodity; or between various means of achieving the same end. This would still be true if free competition ceased to exist, and gave way to some form of collectivism. Hence the theory of value is of fundamental and universal importance in economics.

Modern investigations in the theory of value have led to the setting up of a principle—or rather to the generalization and establishment of a principle already known and applied—called the *marginal principle*, whose application extends far beyond the actual province of the exchange of goods into the fields of production, distribution, and capital. In other words, it governs every part of political economy.

This so-called marginal principle is, in reality, only an adaptation of the fundamental idea from which higher mathematics and mathematical physics have developed; namely, the idea of regarding given magnitudes as variable (as a rule *continuously* variable) quantities, and of regarding their rates of change as new quantities (the Newtonian fluxions, the differential co-efficients of Leibniz). It was, therefore, very natural that the refined terminology and symbols of the infinitesimal calculus should be applied to the modern theory of value. Yet, in the nature of things, it is only the fundamentals of the calculus that can be used, so that no more of it need be known than is taught in schools.

There is ample reason, therefore, for inserting at this stage in our exposition a thorough examination of the theory of value,

though only in general outline and from a theoretical point of view. The realistic study of value or prices presupposes, in the first place, a knowledge of the theory of money and credit, the treatment of which is postponed to the second volume; and, in the second place, an investigation into trade and marketing—which belongs to a special division of economics.

For reasons of space we must omit many of the details and abstruse borderline cases, in which the theory of value abounds, and refer the reader to other more exhaustive accounts, especially to Böhm-Bawerk's essay in *Conrad's Jahrbücher*, mentioned in the bibliography, and to the works of Marshall, Wicksteed, and others.

1. *Exchange Value and its Causes. Earlier Explanations*

The means of satisfying our needs we call *utilities* or *commodities*—this last signifying utilities of a material kind. Immaterial utilities are called *personal services*, and these may include services rendered to oneself; for example, a walk, or gymnastic exercises. Even rest and sleep are such personal services and are just as important to the individual as those performed by someone else. By *goods* we mean objects, many identical units of which are available and which are the object of trade.[1]

The word "utility" is related to *useful*, a term which has many meanings: a thing may be useful in contrast to another which is merely pleasant, i.e. which has a lesser and more transitory use. More important, however, is the fact that most things may have either beneficial or injurious ulterior effects; the latter may even predominate, but, being more remote, they may be disregarded. Since, however, economic theory primarily describes and explains human economic activity as it is, and not as it should be, we must naturally include among utilities those objects which, from a philosophic point of view, might be considered harmful (e.g. many stimulants) so long as they are objects of widespread production and consumption. The Italian, Pareto, in his *Cours d'économie politique*, suggested that instead of the word "utility" we should use "ophélimité"

[1] [There follows, in the original, a paragraph which discusses questions of terminology, which are of no interest to English readers.]

(from the Greek ὠφέλιμος—useful). But this seems unnecessary, because there does not appear to have been any serious ambiguity or misunderstanding in economic science concerning the various meanings of the terms "use" or "utility".

Unfortunately, the same cannot be said of the closely related concept of *value*. Economists have disputed for over a century —and are still disputing—about its correct meaning, or rather about the relation between its different meanings. Happily, the dispute has now lost most of its acerbity and seems on the point of being abandoned. The definition of *exchange value* or *price* offers no great difficulty and gives rise to no special ambiguity. By *exchange value* we mean the ratio in which goods, commodities or services are exchanged for other goods, commodities or services, i.e. the quantity or number of units of every other kind of goods which may be exchanged for a given quantity, or a given unit, of the first-mentioned good. Thus, strictly speaking, a commodity has as many exchange values as there are other goods, commodities, and services for which it can be exchanged; in this way, the conception becomes indefinite. If, however, in exchange for a unit of one commodity, one obtains, or must be satisfied with, a smaller amount of *all* other goods, then we can reasonably say that the exchange value of the first-named commodity has fallen. We are accustomed in practice to use this expression as soon as a rise or fall has occurred in the exchange value of a commodity in relation to the majority of other more important commodities, even if its exchange value in relation to one or more less important commodities has moved in an opposite direction.

The word *price* is sometimes used with exactly the same meaning as *exchange value*; but most commonly the price of a good (and often its exchange value too) is supposed to be measured in the general standard of values or prices for all goods, which is called "money". From the various values of goods in terms of money, their money prices—or, if we so prefer, their money values—we can directly deduce, by division, their relative exchange values. The problem of the theory of value is to explain why one commodity has, either permanently or temporarily, one price and another commodity (or service) quite a different one.

At first sight it might appear that this valuation must be due to differences of utility—so that exchange value and usefulness would be one and the same thing—or at least proportional to each other. And, in fact, it frequently is the case that exchange value stands in a more or less direct relation to usefulness. This is always true wherever two utilities can replace one another and where both, even though more or less effectively, can satisfy the same need. If, for example, we look at our commonest fuels: beech, birch, pine wood, etc., it might be argued that their varying prices or exchange values in the market depend almost exclusively on their fuel value—on the amount of heat obtainable from a given volume or weight of each. Conditions are somewhat different with coal. In comparison with an equal weight of wood, coal has great thermal efficiency, but the various inconveniences and discomforts connected with the use of coal as fuel for a long time hindered its use for that purpose, so that it had little exchange value. And its exchange value is still low as compared with wood. The same is probably true of lignite, peat, etc. Conditions similar to those prevailing in regard to the above-mentioned three kinds of wood also prevail between the various animal foodstuffs, such as pork, beef, mutton, veal; between the vegetable foodstuffs, such as wheat, rye, oats, and potatoes, and to some extent also between textiles—silk, wool, linen, and cotton, etc. But, as these examples show, the relation between usefulness and exchange value is not, even under this assumption, quite evident and clear. In many cases it does not appear to exist at all. Where, on the other hand, two commodities cannot replace each other in consumption, but either wholly or in part satisfy different needs, it becomes a question whether their relative utilities can be measured or compared by any common standard. Experience also proves that the prices of two commodities often vary in quite different degrees (and their relative exchange values thus change) without there being any corresponding change in their physical properties.

At the very beginning of the history of economic science, attention was directed to this distinction.[1] One of the best-known passages in Adam Smith is that in which he explains that the

[1] Indeed, much earlier, Aristotle brought out this very difference between κτῆσις (acquisition) and χρῆσις (usefulness).

word "value" has two meanings, so that at one time it expresses the usefulness of an object (or what he calls its *value in use*), and at another its purchasing power over other utilities (i.e. its *exchange value*). Adam Smith also pointed out that those things which have the greatest value in use often have little or no exchange value—for example, water; and, on the other hand, the things which have the greatest exchange value frequently have little or no value in use, e.g. diamonds. But he stopped at this point. He speaks afterwards only of *exchange value* and never returns to the concept of *value in use*. And at this point science stood still, one may say, for almost a hundred years without it being noticed that Adam Smith's statement was really a striking paradox and involved a problem which necessarily demanded a solution. There were plenty of commentaries and disquisitions on this statement in the subsequent literature of political economy, but practically no *criticism*, no examination of its obvious contradiction. In what follows, we shall endeavour to make such an examination. But, before doing so, we must say something of the consequences which this uncritical reception of Adam Smith's statement occasioned to political economy.

Since, as was assumed, utilities and exchange values did not always coincide, but frequently diverged, exchange value must either depend upon something entirely different from utility, or upon utility and something else as well. The latter explanation was generally accepted (though the Socialists, with Karl Marx at their head, advocated the former). The result was the concept of relative *scarcity*: in order to have exchange value an object must, it was said, necessarily be useful, but, in addition, it must exist in *limited quantities*. If the supply is unlimited in proportion to the need for it (air, water, and the so-called *free* goods in general—in contradistinction to *economic* goods, which do not exist in unlimited quantities and with which we are, for that reason, economical), then the exchange value falls, in spite of the great utility, to zero. On the other hand, great scarcity can impart a high exchange value to objects of little usefulness (though some usefulness must always be present), e.g. rare stamps, animals, plants, precious stones, etc. With a slight modification, this point of view developed into the well-known proposition that if utility creates and regulates the

demand for a thing, its scarcity or the difficulty of producing it regulates and controls its *supply*. Its price is, therefore, determined, as we are accustomed to say, by the relation between demand and supply. With a given supply, a large demand leads to higher prices, and a small demand to lower prices. And *vice versa*, if the demand is fixed and the supply varies. If utility, and with it demand, falls to zero, or if it becomes *negative* (so that people wish to get rid of the commodity), then, of course, the price or exchange value will also be zero or negative—people will pay to get rid of it (e.g. rubbish, slag, and formerly even sawdust, etc.). Yet the same can also happen, it was said, to useful objects if the supply becomes superabundant —e.g. water in floods or cloudbursts, air when it comes in too large quantities or too rapidly. Dwelling-houses, after all, are principally designed to keep out an excess of air and water. Again, if a relatively large demand encounters a small supply, the exchange value may become very great, as, for example, in the case of the demand for gold and jewels, which, even ignoring the use of gold as a medium of exchange, are not without use—even if only of a limited kind. They are, therefore, eagerly sought for, but they can only be procured in small amounts.

All this is, doubtless, in the main perfectly correct and even obvious. But it is not the purpose of science to describe the obvious in elaborate terms. If we examine the matter a little more closely, the principle of the determination of value by supply and demand does not, in reality, throw much light on the real nature of the phenomena under discussion. It is obvious, for example, that only so-called *effective* demand influences prices. The demand of persons who are not in a position to pay the price asked for any particular commodity evidently has not the slightest influence on price, however great that demand may be. It may be compared to the longing glances of the numerous, though impecunious, persons who gaze at the precious objects in a jeweller's shop window. But the effective demand—in other words, the quantity of the goods that can be bought at the prevailing price—is, on the average, neither great nor small in relation to the supply, but is in fact *exactly the same*. Indeed, it is only on this condition that the market can be in a state of equilibrium. If the demand

is greater than the supply the price will rise ; if it is less the price will fall—but it cannot continue to rise or fall for ever. Since, therefore, supply and demand are equal where there is economic equilibrium and a stable price, whether that price be high or low, we must further ask : Why does the demand for and the supply of *this particular* commodity achieve equilibrium at *one particular* price, and that of another commodity at a totally different price ? The classical analysis of exchange values gives no direct answer to this question, though this drawback was felt by the classical economists themselves.

It may be pointed out that, in Adam Smith, the expression "effective demand" has a somewhat different meaning. It means the demand of those persons who are willing to pay the "natural price", i.e. the costs of production and transport ; if supply in the particular case were accidentally greater or less than *this* demand, then the price would fall below or rise above the "natural price".[1]

F. J. Neumann, in his essay on "Value" in Schönberg's *Handbuch*, entirely rejects the concept of supply and demand (offer and demand) whenever these are regarded as merely quantities. That, in his view, is extremely one-sided. On the contrary, in his view, supply and demand represent a whole complex of qualities : extensity, intensity, purchasing power on the part of those who demand, etc. ; for which reason it is absurd to say that demand is as great as, greater or less than, offer or supply. The obvious reply to all this is that the circumstances enumerated by Neumann doubtless affect the magnitude both of supply and of demand, and the total result must be that, when a certain price is quoted in the market, a certain definite quantity of goods of this kind will be offered and an equally definite quantity will be demanded. For my part, I cannot see the one-sidedness of such a view.

Without entirely abandoning the formula of supply and demand, to which they always resorted in case of need, attempts were made by the classical school to provide a more definite explanation of the exchange value of at least one group of commodities (in practice the most important), i.e. those which, as it was usual to say, could be produced in unlimited quantities.

[1] [The paragraph which follows this in the original has been omitted. It discusses the distinction between *utbud* and *tillgång* (*offer* and *supply*), which has no counterpart in English economics.]

The explanation related to their *cost of production* or eventually, according to a subsequent variation of terminology, to their *cost of reproduction.* If a commodity is not, generally speaking, an object of production in the ordinary sense (as, for example, certain natural products), or cannot be produced or reproduced (pictures by old masters), or if, finally, its manufacture is the result of a natural or legal monopoly, then we must still content ourselves with the thesis that the price is determined by supply and demand. For the majority of goods, on the other hand, which can in practice be reproduced in unlimited quantities under free competition, costs of production would, as has been said, determine the average or "natural" price, about which the market price always oscillates.

It is quite evident that, under free competition, the price of a commodity cannot be either above or below its cost of production if this includes everything required for bringing the commodity to market, including a "reasonable" (i.e. customary) compensation to the last seller for his labour and trouble. If it were otherwise, the commodity would either not be manufactured, or it would be manufactured in such large quantities that the price would necessarily fall owing to the increase in supply. But if this is to be a valid *explanation* of reciprocal (relative) exchange values, then the costs of production must evidently be something definite, something arising from independent (absolute) causes; they must not be dependent on the exchange values themselves. Herein lies the weakness of the classical theory of value. If we analyse more closely the conception of costs of production, we shall find that the latter resolve themselves into a reward or compensation for the use of the various factors of production, usually divided into the three main categories of land, labour, and capital. If, for example, the manufacture of two quantities, *a* and *b*, of two different goods requires the same amount of the same kind of labour, the employment of the same quantity of land of the same quality and the same quantity of capital for the same period of time, then we can say without fear of contradiction that both quantities of goods will be sold in the market at the same price. That is, after all, nothing more than saying that all labour of the same kind, all land of the same quality, and all capital employed for the same period of time will receive the

same reward, which is a natural and necessary consequence of free competition. If, on the other hand, as is nearly always the case, the production of these commodities requires land, labour, and capital in *different proportions*, e.g. more land, but less labour and capital, for *a* than *b*, then some means must be found for reducing the quantities of these various factors of production employed to a common measure, though, of course, no direct means of doing this is available. In order to express them in common units, we have to refer to the *remuneration* they demand, i.e. the relative magnitude of wages, rent, and interest. These, however, are not given, and the determination of them constitutes a problem of the same kind as our original problem, and one which can only be solved in connection with it.

The method adopted by economists of the classical school (particularly Ricardo) to escape from this dilemma shows considerable ingenuity; but as has been seen already from our consideration of the connection between the market price and the costs of production of a commodity, and as we shall show in further detail later, the attempt was foredoomed to failure. In the first place, they attempted to simplify the problem as much as possible. The various kinds of labour, such as skilled and unskilled, might, they thought, be reduced to a common standard in so far as labour of a higher quality was regarded as representing an extra number of working days, corresponding to the higher wages paid for it, and to the time which the workman had *previously* spent on his technical education. As regards capital, they found its chief rôle in production to lie in advancing wages or the necessities of life to labourers and providing necessary tools and raw materials. They assumed in consequence that capital (or the capitalists) in all branches of production would receive approximately the same share or percentage of the exchange value of the product (profits of capital). Ricardo expressly admitted that this rule was subject to important exceptions in consequence of the unequal proportions of fixed and circulating capital in the various branches of production. Finally, they thought that land could be disregarded and that rent could therefore be excluded from costs of production. They only regarded labour and capital employed at the *margin of production* as contributing to costs—either on marginal land, the least fertile (which is

superabundant and, therefore, pays no rent) or, in more intensive cultivation, on land which is already employed—where an addition to output can pay no extra rent for similar reasons. In this way, the factors of production governing exchange value were reduced practically to one only—labour. According to Ricardo, the exchange values of various goods should stand in more or less direct relation to the quantities of labour required to produce them under the most unfavourable conditions which are necessary for their production, i.e. on the margin of production. So great was the satisfaction felt with this result, which is formally so brilliant, that J. S. Mill in the introduction to his theory of value declared the classical theory of value to be "complete", so that there remained nothing for him, or for subsequent writers, to add.

Ricardo makes another simplifying assumption, which must be borne in mind in reading his works, if we are not to misunderstand them. He assumes that *gold*, the measure of value and prices, is always produced with the *same* labour costs, and also that profits on capital employed in the production of gold constitute the same percentage of wages or of the total product as in any other branch. From this he is led to the conclusion that the *amount* of labour employed in the production of a certain unit of goods directly expresses the number of ounces or grammes of gold for which this unit of goods is habitually exchanged in the market; in other words its *price* measured in gold. On this assumption, on the other hand, the general level of *wages* can never have the least effect on prices, as in that case they would also affect the price of gold (in money, i.e. reckoned in gold), which is an obvious contradiction. A rise in wages (money wages) can, moreover, according to Ricardo, take place only in combination with a corresponding fall in the profits of capital, where commodity prices remain unchanged; a change in commodity prices, again, necessarily presupposes that the amount of labour employed in their production has—owing to new inventions or to increased difficulties in production—become greater or less than previously.

By these various simplifying assumptions Ricardo greatly facilitated his analysis. In his work, the structure of economic theory appears, for the first time, as a coherent, logical system. But his conclusions thereby frequently assume an abstract and even unreal character. In this respect, he compares unfavourably with Adam Smith.

Even if we admit all these generalizations and simplifications for what they are worth, we are still faced with the fundamental error of the classical theory of value. Their margin of production is not a fixed limit, given *a priori*, but is variable and itself depends, among other things, upon the actual exchange value of the goods in question and, to that extent, upon what it has to explain.

Thus, for example, there are certain manufactured goods (especially articles of clay) for which the raw materials exist already mixed in nature in practically unlimited quantities, so that, for them, there is no margin of production: they can be produced with unchanged labour costs (per unit of goods) in any desired quantity. In the case of other commodities, on the other hand—particularly the means of subsistence—in any given state of technique, increased labour costs per unit are necessary if they are to be produced in larger quantities than before. If, therefore, any economic unit must itself provide for the production of these two kinds of goods, their relative exchange value or price will clearly depend, to a high degree, on the relative magnitude of the demands for them; for the extension of the margin of production and the costs of production at that margin for the latter commodity are only thereby determined.

Let us take another example. Suppose that an economic unit (a district or a whole country) is compelled by natural circumstances to restrict its production to two staple articles only, say corn and linen, the prices of which we will suppose, for the moment, to be determined by the world market. If the price of linen goods is relatively high, the community will devote itself principally to their manufacture and will cultivate corn only in proportion to its domestic needs; if, on the other hand, the price of corn is relatively high, then it will expand its production of corn and restrict its manufacture of linen to the minimum. Since, however, the production of linen requires little land in proportion to the labour employed, it is clear that, in the former case, when linen is the chief manufacture, the demand for land will be small, and agriculture will be restricted to the best land or will become less intensive. In both cases the result will be that the labour employed in the production of raw materials will become, even on the margin of production, inconsiderable. And, since this labour in the case of corn constitutes the whole, and in the case of linen only a minor part, of the necessary labour, the portion of labour

employed per unit of linen will be great in *relation* to that employed in the production of a unit of corn. On the other hand, if the production of corn, owing to changed price conditions, becomes predominant, the production of the raw material must be extended to inferior land, or else the cultivation of the better land must become more intensive. Whichever happens, the result will be that the amount of labour which is employed on the inferior land (or, in general, on the margin of production) in the production of the raw material will be very great. From this it will follow further that the total labour employed under the most unfavourable circumstances in the production of one unit of corn will be relatively great in relation to the labour employed in the production of one unit of linen. As illustrations we may mention the economic conditions in Northern Russia, Ireland, and, to some extent, certain Swedish provinces, at the time when the increasing cheapness of cotton goods began to oust the native linen products of those countries.

A third, and very important, example is the exchange value or purchasing power of gold itself in terms of goods, which—as even Adam Smith realized, though Ricardo purposely ignored it—is by no means constant, but depends on the labour costs in the mines on the margin of production. Naturally, however, this margin is itself variable. It expands when commodity prices are low and the purchasing power of gold is high, but it shrinks in the contrary case ; so that production is restricted to the richer mines or river beds, and the maximum labour employed in the production of a given quantity of gold becomes less.

In such cases, Ricardo's thesis that the exchange value of the product is proportionate to the quantity of labour required for its production at the margin is verified—if in each case, as we have done, we do not take into consideration the varying proportions of capital employed. Yet obviously, under such circumstances, it is not the costs of production which govern the exchange values. That, indeed, would be impossible if, as is assumed in the above example, the latter are fixed and determined beforehand by the world market. On the contrary, it is the exchange value of the goods which governs their costs of production—i.e. which determines how much labour shall be employed in the production of one unit of corn and in one unit of linen goods. Again, if we look at the matter more generally and observe either an isolated economic unit or the whole of the world's production and exchange, then it is clear that costs

of production and exchange values cannot stand in the simple relation of cause and effect which Ricardo supposed. As we shall see later, *they are mutually conditioned* like the various elements in a single economic system in equilibrium. But, in that case, it is also clear that reference to costs of production, even under the simplest imaginable assumptions, is impossible as a theoretical explanation of the exchange value of goods, however useful it may often be as a practical rule.

No doubt, the classical economists failed to realize this because, in the case of one of the most important groups of commodities, the means of subsistence, they regarded demand, or consumption (and therefore also the extension of the margin of production), as given by the size of the *population*. Statistics have not confirmed this: largely owing to indirect methods of use, the demand for and consumption of corn and other foodstuffs is almost as elastic and variable as that of other goods.

There is this further point. It happens in many cases, even where a commodity is manufactured under competitive conditions, that its costs of production *cannot be separated* or imputed because its production proceeds simultaneously and in combination with that of other goods, e.g. where one commodity is a by-product in the manufacture of another. Such cases, which have been given by Marshall the technical name "joint supply", are mentioned also by Mill in his chapter, "Some peculiar cases of value," [1] but, as the chapter heading indicates, Mill regarded them as exceptions to the rule. In reality (as Jevons remarked) they occupy a large, perhaps the largest, part of the field of production. We shall return to this subject in greater detail, but it may be pointed out here that all branches of agriculture fall within the category of joint supply: the cultivation of cereals and livestock, no less than that of textile materials and other commercial crops, are mutually determined in any well-ordered system of agriculture. Here the only question which arises is whether the total selling value of the products will cover the total costs of production, for the separate costs cannot be imputed. When, for example, before the introduction of corn duties in Sweden, some agriculturists maintained that the growing of rye at the low prices prevailing "did not pay", they nevertheless continued to grow it and proved by so doing

[1] [*Principles*, book iii, chap. xvi.]

that this crop constituted a necessary element in an agricultural system which must have paid as a whole, or else it would have been abandoned.

Here also, it would be possible, by an artifice resembling that of Ricardo for the elimination of rent from the costs of production, to impute the costs of various goods by supposing that one or other of them entered in varying degrees into the total output—which is in fact in full correspondence with actual conditions. Thus, for example, a breeder of sheep produces, at one and the same time, wool and mutton, but he can, as required, specialize on one breed of sheep or another, the wool-producing or the mutton-producing, and in that way obtain either more meat and less wool or *vice versa*. The possibility of transporting fresh meat in refrigerating chambers from Australia or the Argentine to Europe in fact compelled European sheep farmers to abandon the merino breed, with its fine wool, in favour of breeds yielding more meat. This, in its turn, gave rise to a crisis in the European clothing industry towards the end of the nineteenth century.

In the same way, in the manufacture of coal gas, coke is obtained, if desired, as a by-product. But here, too, the proportion between the two products is neither given nor determinate, for some coal yields more gas and less coke, and *vice versa*. If coking is the principal objective, as at iron works, more attention will be paid to the latter kind of coal, and *vice versa* if the production of gas is the more important. In this way, we obtain a kind of margin of production in which an increased production of one of the commodities corresponds to a definite increase in the costs of production. But even here it will appear that the costs of production are by no means pre-determined; they may vary in a high degree with the variations in the relative prices of the goods. In other words, the relation between costs of production and exchange values is, in this case also, not one of cause and effect, but of interdependence.

In reality, the classical theory of value did not give general satisfaction. The celebrated Proudhon included, though on somewhat confused grounds, the theory of value among his *Contradictions économiques*, and Bastiat, his opponent, introduces the chapter on value in his work, *Harmonies économiques*, with the significant words "Dissertation ennui: dissertation sur la valeur, ennui sur ennui". A theory which one has fully mastered does not, however abstract, normally give rise to ennui. The modifications which these men and the schools to which they

belonged effected in the theory of value were, however, by no means improvements. On the contrary, both of them expanded the classical attempts at generalization to exaggerated paradox. In the hands of the Socialists (especially Rodbertus, and Marx still more so) the theory of value became a terrible weapon against the existing order. It almost rendered all other criticism of society superfluous. Labour was conceived by them—Ricardo never meant or said any such thing—to be the sole creator of value—in other words, the source of value ; and thus all other factors of production existing in private hands were to be regarded as parasites on production, and their rewards a robbery at the expense of labour, which is alone entitled to remuneration. The fallacy of this reasoning will be made clear in what follows. The harmony economists, Carey, Bastiat, and their numerous disciples in different countries, believed, on the contrary, that they had found in the principle of labour as the only creator of wealth a highly effective weapon for the defence of the existing order of society. They attempted, indeed, to reduce all the shares in the product, even including the rent of land, to wages of labour (i.e. wages for the labour which had been employed on the land or in production in days gone by).

The absurdity of such arguments is obvious and has perhaps contributed more than anything else to the charge of dishonesty and subservience to the interests of the powers that be which has been levelled against scientific, or quasi-scientific, economics. In Karl Marx's theory of value the Socialists believed that they possessed a theoretical foundation as good as that which was offered by the harmony economists, and both sides considered that they were fighting, with as much or as little justification, under the banner of classicism.

The establishment of a new and better-founded theory of exchange value was, therefore, not only of abstract theoretical importance, but also of eminent practical and social interest, and the three men who almost simultaneously and independently succeeded in doing so—the Austrian, Carl Menger, the Englishman, Stanley Jevons, and the Frenchman, Léon Walras [1]

[1] To some extent, the German, H. H. Gossen, whose work appeared in 1854 but was entirely neglected during his life-time, ought to be reckoned a predecessor of all three. Yet neither Gossen—nor, for that matter, Menger—went so far as to establish the proportionality between the marginal utility of different goods, which, as we shall see, constitutes the law of free exchange and which is put forward in essentially the same form by Jevons and Walras.

—thereby paved the way, more than is usually supposed, for mutual understanding even in the social field.

2. *The Concept of Marginal Utility*

A presentation of the modern theory of value may, as has already been indicated, conveniently proceed from a revision and analysis of Adam Smith's thesis relating to the divergence between *value in use* and *value in exchange*—which he exemplified by water and diamonds (cf. p. 18). Literally interpreted, this thesis appears to be either meaningless or a contradiction in terms. In the first place, which value in use has he in view ? Evidently it cannot be the utility of water or diamonds in their totality, for even if it were at all possible to exchange all the water for all the diamonds in the world it would soon become clear that the former had an infinitely greater *exchange value* than the latter ; of course, the comparison must relate to manageable quantities, e.g. a litre of water or a diamond weighing one gramme. But, even in such a case, as Mill remarks, the value in exchange cannot possibly be *greater* than the value in use (though it may be less, according to Mill), for we should otherwise be confronted by the absurdity that a person would dispose of a more useful for a less useful commodity. In other words, the value in use, according to Mill, constitutes the upper limit of value in exchange. But on further consideration it appears that the value in exchange cannot be lower than the value in use either, for exchange presupposes two exchanging parties, and while no one will buy a commodity which has a value in exchange *higher* than its value in use, no one will sell a commodity whose exchange value is *lower*. We thus seem to arrive at the remarkable result that value in use is, at one and the same time, the upper and the lower limit of exchange value ; or, in other words, is its exact equivalent. This, however, is contrary to experience ; neither is it easy to understand how, under such circumstances, any exchanges whatever could be effected. The obvious explanation is the well-known fact that the *same thing* may possess *different degrees of utility* for different persons, so that the relative values in use can, at the same moment, be greater or less than the relative exchange values *for one or other of the exchanging parties*

respectively. If we follow up this train of thought, we shall easily see that a thing may have quite different degrees of utility for one and the same person *under different conditions.* The most important circumstance in this connection is evidently, at least in a primitive economy, the *quantity* of the commodity in one's possession—or of other commodities which can, to a greater or lesser degree, replace it. In a more advanced economy, the determining condition will be the possession, or accessibility, of a certain quantity of the *medium of exchange*—that is, of the commodity in exchange for which, as experience shows, other commodities can be obtained. But what sets the standard in both cases is, in the last resort, the quantities of the various commodities which the person in question is in a position to consume in a given unit of time.

Value in use is, therefore, by its very nature, something variable. Value in exchange, on the contrary, is always, or always tends to be, constant and invariable for each commodity throughout the market. The question then becomes: which of these possible, or conceivable, degrees of value in use determines (or, to express ourselves more cautiously, is related to) the actual exchange value of the commodity? The answer must evidently be: the degree of utility which it possesses for the exchanging parties at the moment the exchange is effected, whether that utility arises from their present or future needs. That, however, is evidently hardly ever the *maximum* utility which the commodity in question *might,* under certain circumstances, possess, nor even the *average* utility which such a commodity *usually* possesses, but rather the *minimum* utility which the commodity, or one unit thereof, under the given circumstances, *will possess* or may conceivably possess. This degree of utility is what is called the marginal (or final) utility of a commodity, and corresponds, therefore, to the least important of the needs satisfied by the acquisition of that commodity—and that is the same as the *most* important of the needs which are not satisfied if the commodity is *not* acquired, or is acquired in lesser quantities. As regards the commodities given in exchange, their marginal utility will correspond to the least pressing of the needs which will be satisfied if they are *not* offered in exchange, though as regards *very small* quantities this cannot be distinguished from the least pressing of the needs which, after a completed exchange,

remain unsatisfied. The result is that, after an exchange has been effected, the marginal utilities of both commodities *for each of the exchanging parties* stand in the same relation as their common exchange value. If this were not the case then, as we shall show later, one of the parties would desire to exchange further and, by offering a somewhat more advantageous price, would induce the other party to consent.

An easily comprehensible example of the variability of value in use is the well-known one given by Böhm-Bawerk (originally given in almost the same form by Menger). A colonist living alone in the virgin forest by agriculture has just harvested five sacks of corn (excluding that set aside for seed) which constitute his entire supply of foodstuffs until the next harvest. If he disposes of this stock in accordance with his previous consumption, every sack will have a different use and will therefore be of different importance to him, although physically they are all identical. The first sack is absolutely necessary for the maintenance of life and is therefore as valuable to him as life itself. The second sack is still of the greatest importance to him, because with it he can eat his fill and preserve his health and bodily strength. The third sack he will no longer consume directly but will use to keep fowl and thus procure a necessary change in an otherwise purely cereal diet. The fourth sack he may use for making spirits. For the fifth sack he can find no better use in his simple mode of life than to employ it for his own amusement in providing for a few parrots. If, by some accident, he should lose one of his sacks of grain, then it is clear that, under such circumstances, it would be the fifth sack which he would sacrifice, i.e. the least important from the point of view of the satisfaction of his needs. If he lost another it would be the one used in the making of spirit, but not one of those which was required for his real sustenance ; and so on. Strictly speaking, there also exists a certain gradation within the sphere of each of these utilities : it is quite possible that he would renounce a little of the satisfaction of the more important needs before he entirely abandoned those which, regarded as a whole, rank lower in the scale of utility. But we shall soon return to this point.

By means of this simple conception, the theory of value has obtained the clearness and coherence which it formerly lacked. The dualism inherent in the traditional conception of exchange value as requiring *two* qualities, utility and scarcity—though

it was never clear in what relation they stood to each other—now disappears, in so far as marginal utility actually represents a *synthesis* of utility and scarcity. Marginal utility becomes the degree of utility at which the consumption of a commodity must cease precisely because of its *scarcity*. The term scarcity (*rareté*) was used by Walras as exactly equivalent to marginal utility (his father, Auguste Walras, had earlier employed the same word); for he regarded a commodity as scarce only when it exists in insufficient quantities *in relation* to the need or demand for it—so that the degree of scarcity is indicated by the marginal utility. This is, of course, a matter of taste; but Walras' terminology is somewhat forced and has not found general support.

Thus, if a relatively scarce commodity (e.g. a choice wine) has a high exchange value, it is due to the fact that consumption must cease at a point where the least important of the needs satisfied and the most important of the unsatisfied needs or degrees of need (of choice wine as refreshment or as a stimulant) are still of great significance; whilst common commodities, such as bread, are usually consumed in such large quantities that the need which one more unit per consumption period could satisfy is of relatively little significance, or of none at all (as is usually the case with the free goods, air, water, etc.). It is of no importance, in this connection, that the category of needs which bread satisfies (the maintenance of life) is, as a whole, much more important than the category which is satisfied by wine, namely, the need for refreshment and the satisfaction of more refined appetites. The same conditions apply here—to use, once again, a simile from Böhm-Bawerk—as in the case of two mountain heights. One of them is, absolutely, much higher than the other, but this does not prevent a climber at a given moment from being situated much higher up on the lower mountain than another climber on the higher mountain.

It was this relation which Adam Smith overlooked. The value in use on which his gaze was fixed, and which in his view might often stand in inverse relation to exchange value, was evidently the maximum utility which the commodities compared (water and diamonds) could respectively attain under given conditions. But the parties to the exchange have nothing at all to do with this; they are, of course, only concerned with the

actual or prospective utility which the commodities possess for them at the moment of the exchange. Bearing this in mind, one is almost tempted to turn Adam Smith's thesis upside down and to say that those commodities which have a high exchange value thereby prove themselves to possess great value in use or high utility—i.e. high marginal utility. Yet such a formulation would not be quite accurate, for the individual differences among consumers, and especially their different financial positions, here play an important rôle. To the rich man, who can fully satisfy practically all his needs, all commodities must have a very low marginal utility: if a rich man spends hundreds of pounds on a single diamond, that does not prove that it has a higher value in use for him than for others. In most cases it only means that the commodities, the consumption of which he forgoes in order to procure the diamonds, possess for him little or no value in use. Indeed, as we shall see later, we find, in arriving at the laws of price formation under free competition, that the degrees of utility—the relative marginal utilities—of the same thing to two different persons are never compared, but only the marginal utilities of different commodities to a single individual. If, however, property and income were more equally divided, it would no doubt appear that the scale of values in use for most persons would more or less coincide—and this would produce the result that diamonds and many things now highly esteemed would fall in exchange value, and their production would decline—perhaps sufficing merely for the provision of enough diamonds for glass cutting and drilling. There was a striking example of this in the world crisis of 1907, when the world-wide reduction in profits led to a special crisis in the Dutch diamond industry.

A question which has, perhaps, already occurred to the thoughtful reader and to which we will not postpone the answer, is the following. It seems clear that marginal utility determines exchange value so long as it is only a question of obtaining, or disposing of, a small quantity of a certain commodity in exchange for a similar small quantity of another; and so far as one is already provided with a sufficient, or nearly sufficient, quantity of both. But actually, in a modern economic society, based on division of labour, we obtain practically all commodities, or at any rate a large proportion of them, exclusively by exchange. Thus

those commodities in fact satisfy *all* our needs—even those of the highest degrees of intensity. How then does it come about that exchange value as a whole is only regulated with reference to the last and least important of these degrees of need ?

This observation is fully justified. In actual fact, exchange value is by nature just as variable as value in use or utility. In isolated exchange there exists, as we shall soon see, fundamentally no such thing as a uniform exchange value. The more or less fixed proportions in which, as we know by experience, goods are exchanged for each other in the market, and which have given rise to the name and concept of exchange value, are something peculiar to the *market as such* or to the influence of the market—and not to individual exchanges independently of the market. That something is *free competition* on the part of either or both parties to the exchange. As Jevons expressed it, there is operating in the market "the law of indifference". It is a matter of *indifference* to buyer and seller alike with whom they do business provided that they obtain the same goods or the same price, as the case may be. For this reason there can be, roughly speaking, only *one* price in the market, for a given commodity at any moment of time.

Fundamentally, marginal utility and exchange value or price will stand in the same reciprocal relation of dependence as that which we have already found to exist between exchange value and marginal costs of production. If the exchange values are given beforehand, e.g. as they are given in a small economic unit, by the influence of the world market, then the marginal utilities will be regulated by them ; for the various goods will be consumed up to the point where, for each and every consumer, their respective marginal utilities stand in the same relation to each other as the exchange values or prices. If the exchange values are not given in advance, but are determined by the market proper then marginal utilities and prices will mutually determine each other in a single system of equilibrium and they can be symbolically or hypothetically expressed by a system of equations, in which the goods available in the market, or for the period of consumption, constitute the known quantities in the problem. But actually even these quantities are not given ; goods are in most cases constantly being produced and consumed and can, according to circumstances, be brought to market or withdrawn from the market in larger or smaller quantities. The final problem of equilibrium, *the problem of equilibrium between production and consumption by means of exchange*, therefore includes among the

unknowns the quantities produced and consumed and the relative exchange value of the goods, as well as the proportional marginal utilities for each particular individual. On the other hand, the definitely known quantities are the means of production existing at each particular moment: labour, land, and capital (and if the process extends over a longer period, factors affecting the accumulation of capital), as well as the individual dispositions of consumers. The exchange value must then be fixed at a level such that the forces on the two sides balance; i.e. the desire to consume (the utility or satisfaction of consuming) on the one hand, and the difficulty of producing, the inconvenience or discomfort of manufacture (sometimes called *negative* utility or disutility), on the other. That the marginal utility or disutility should be the decisive element is quite in accordance with a number of other apparently paradoxical phenomena of equilibrium (cf. the so-called hydrostatic paradox); but, at the same time—though this is unsatisfactory from the ethical and social points of view—it shows the purely *mechanical character* of the economic phenomena which occur under conditions of free competition.

We shall now endeavour to explain in more detail the complicated phenomenon of exchange equilibrium, following the principle strictly pursued throughout this book (as in Walras' work) of proceeding successively from the simple to the complex.

3. *Free Exchange and Market Value*

A. The Different uses of a Single Commodity

In the market, we observe a double phenomenon: the determination both of the *magnitude* of the volume of goods exchanged, and of the ratio in which they are exchanged. If there are only two commodities, this ratio is, as a rule, a direct consequence of the quantities of the goods exchanged; but not if there are more than two. But for the present we shall make the assumption that the ratio (or ratios) of exchange are for some reason given and fixed, so that it is only a question of determining the absolute quantities exchanged; if there are only two goods, their relative magnitude is thus already given.

The simplest conceivable form of exchange is that in which one and the same person chooses between different uses of

a single commodity. Let us, for example, return to Böhm-Bawerk's colonist in the virgin forest and his stock of five sacks of corn (see p. 31). But now suppose that he had only *two* uses to choose between: either *direct* consumption in the form of bread or cereal food, or *indirect* consumption in the form of meat which he obtains by using a part of his stock of corn for poultry breeding. For the sake of simplicity, we shall ignore the additional trouble and inconvenience which he incurs in following the latter alternative. We may then conceive his operations as a sort of exchange, in which the exchange value is determined by technical circumstances: by sacrificing the direct consumption of so many kilograms of corn he can, if he wishes, obtain one kilogram of eggs or fowl. The only question is what *quantities* of his original stocks will, economically speaking, be offered in exchange.

If we were to think of the utility (or value in use) of each article of consumption as a fixed quantity, we should arrive at the absurd conclusion that he must convert either all or none of his corn into fowl or eggs, according as the utility of the latter is greater or less than the utility of the former. The case is quite different if, in accordance with reality, we suppose the utility of a unit of goods to be a variable quantity, which, *ceteris paribus*, is reduced when the number of units available for consumption increases. The colonist had no need at all for the last sacks of corn as food; their utility for direct consumption was thus zero—or even negative. But the addition to his comfort and well-being resulting from the consumption of the first portions of animal food per unit of time—e.g. an egg or a roast chicken a week—is very considerable. Thus, if he converts the last sacks of corn into poultry, he adds considerably to the utility which would otherwise have been attainable. If he sacrifices another sack for the same purpose, his gain on the exchange will still be considerable, though not as great as from the first, because he might have derived a positive advantage from using this sack for direct consumption, and also because the desire for animal food is not so strong when it has already been partially satisfied. The same is true in an even higher degree of the third sack. The sacrifice of a part of this sack for poultry breeding might possibly increase its utility, but for the other part he would presumably prefer the direct use and

would consider that he had lost on the exchange if he used it for conversion into animal food. Economy demands a line of demarcation between the portion of the original stock of corn which is given up and that which is retained ; and this evidently lies—at least if we assume that the quantities in question are continuous variables—at the point where the last kilogram of corn has the same or about the same utility, whether it is consumed directly or converted into animal food. In other words, the *marginal* utility, the utility of the last kilogram consumed directly and of the last converted into animal food, must, in economically-regulated consumption, be *the same*. Or, in other words, if we assume that 5 kilograms of corn are required for the production of 1 kilogram of chicken or eggs, then the utility of the last kilogram of animal food would be five times as great as the utility of the last kilogram of cereal food, so that the *marginal* utility would be *proportional* to what we may here call (though not altogether appropriately) the *exchange value.*

The position would naturally be exactly the same if, instead of only two uses for the original stock of corn, there had been three, four, or more. However different the significance of the various uses—to sustain life and health, to improve diet, to provide enjoyment or trivial diversion—may be, one thing is certain : that, of the portions used for each of these different purposes, the last kilogram will procure for its owner, at any rate approximately, the same amount of satisfaction or utility. Otherwise it would be inexplicable why he did not, from the beginning, either use that portion for a purpose which would bring him greater advantage, or, if he had made a mistake from lack of foresight, did not rearrange his plan of consumption for the ensuing year accordingly. If, instead, we measure the various methods of consumption by their own particular units—1 kilogram of corn, of meat or of eggs, 1 litre of spirit or one parrot—then, obviously, their marginal utility will, in every case, be proportional to their relative "exchange values".

This provides the answer to some of the objections which were raised to the theory of marginal utility when it was first propounded, and which one still sometimes hears. To the ordinary mind, the utilities or values in use of various goods appear as something incapable of comparison, as incommensurate quantities ; they were thus described by Ricardo and, after him,

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by Karl Marx. To compare hypothetically the utility, or marginal utility, of various commodities, as the modern theory does, seems *a priori* absurd; and to try to *measure* utility exactly—to maintain that the marginal utility of an object or of a class of goods is so many times greater than that of another—is, at first sight, as absurd as to say, with F. J. Neumann, that "one person is one and a half times as polite as another". And yet, as the above example shows, we all make such a comparison at almost every moment of our life. Neither does the idea of exact measurement really involve an absurdity; if we can generally say that a certain unit gives a utility equal to, or somewhat large or smaller than, that of a different unit, then we can also say the same of two, three, four, or more units of the one kind in comparison with one or more units of the other. And, in fact, we meant nothing else but this when we said, in reference to corn and animal food, that the marginal utility of the latter was about five times as great as that of the former. It is true that one assumes that each of the 5 kilograms of corn, which are compared with 1 kilogram of poultry, has the same utility. But this assumption can be made without any risk with reference to small portions of a large stock, as indeed is often done in corresponding cases, in the natural sciences, when it is a question of continuous variables. Indeed, the arguments used in the theory of marginal utility strikingly resemble those by which, a couple of centuries ago, mathematical precision was given to previously vague ideas such as mass, force, velocity, acceleration, mechanical work, etc.—a precision which was only achieved for measures of heat, light, and electricity in quite modern times.

It should be observed, however, that the more or less precise comparisons which we are accustomed to make nearly always relate only to *small* quantities; precisely, in other words, to the *marginal utility* of the various commodities or goods. To determine whether the consumption of a particular commodity as a whole is productive of more or less utility, or how many times greater or less that utility is than in the case of another kind of commodity, is of course much more difficult—if not impossible: a fact which can best be proved by the many mistakes which we make when a more violent change in our habits of life is in question. Sometimes this comparison is even,

to a certain extent, self-contradictory, as when the consumption of a number of (commodities such as meat and corn in the above example) forms an inter-related whole—so that, strictly speaking, one can only speak of a certain total amount of welfare which is achieved by the combined consumption of a number of different commodities.

Graphical Version.—If there are only two ways of using the given stock of goods, then it is simple to illustrate the above argument graphically.

Let the horizontal line AB represent the original stock of corn. On each of the successive unit lengths along this line, counted

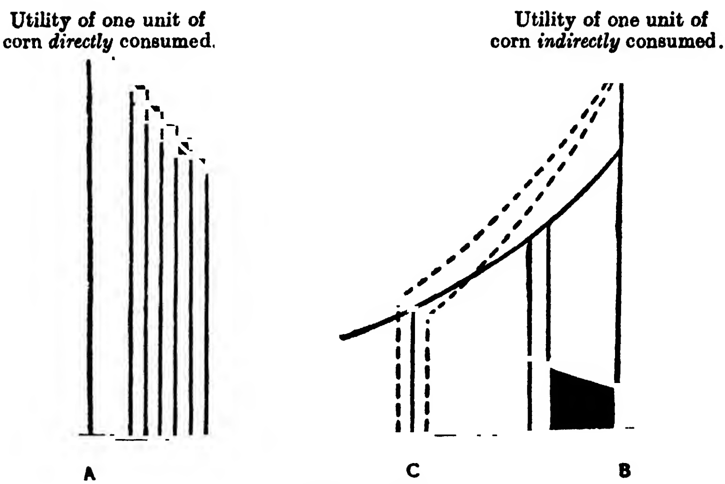


Fig. 1.

from left to right, we erect a rectangle; the areas of these rectangles represent the additional amounts of utility or satisfaction accruing to the colonist if his direct consumption of corn, during the period of consumption in question, is increased to one—from one to two—from two to three, etc.—units or kilograms. The upper limits of these rectangles form a stepped line, and for this, without introducing any material error, a continuous curve may be substituted. The area bounded by this curve, by the vertical line drawn through the point A, by the horizontal line and by a variable vertical line (or ordinate) represents the whole utility when the consumption of grain is restricted to that part

of the horizontal line which is cut by the variable vertical line. *Ex hypothesi,* the curve gradually approaches the horizontal line and will, sooner or later, intersect it ; for every consumption of corn over and above a certain quantity does not produce any extra utility.

It is clear, however, that the portions of this curve (or surface) which are furthest to the left do not really exist, for the colonist would starve to death if his annual ration were limited to only a few kilograms of corn. The curve only acquires real significance in the case of an increase or decrease of the stocks annually consumed. With every increase or decrease by one unit, there is a corresponding increase or decrease of utility, which is represented in the diagram by a narrow rectangle, or—since the base of this rectangle is one unit—by its height reckoned in linear units ; i.e. by the ordinate of this curve. This, then, will be the geometrical representation of the marginal utility of the corn when its consumption per unit of time or period of consumption is indicated by the corresponding section of the horizontal line, measured from A.

Let us now suppose that, on the horizontal line, we construct a similar figure, from B, going from right to left, and draw a curve, of which the enclosed surface and the ordinate represent the total and marginal utility respectively, of *indirect* consumption of corn (in the form of meat and eggs). One unit of length on the horizontal line will still represent one kilogram of corn, and the narrow rectangle (or trapezium) constructed upon it and bounded at the top by the curve—or alternatively the height of the rectangle, the ordinate of the new curve—will indicate the increased utility which would arise if the quantity of corn employed in feeding poultry were increased by one kilogram, supposing the colonist obtained it without cost. Since, however, it must be taken from the stock otherwise available for direct consumption, the actual increase of utility will correspond to that part of the rectangle, or of its height, which is bounded by the two curves. The new curve will obviously fall from right to left, and should, therefore, sooner or later, intersect the old curve. It is now easy to see that the most advantageous use of the original stock of corn will be found by dividing the line AB at a point C, which lies vertically below the point of intersection of the two curves. Here the two curves have a common ordinate, which is equivalent to saying that the marginal utilities of the corn consumed directly, and of the corn used as animal food, are the same.

Strictly speaking, however, our diagram only has this significance in so far as it relates to two kinds of consumption which are independent of each other—the utility or satisfaction derived from consuming a certain quantity by one method being equally great whether much, little, or nothing is consumed by the other method. This is never wholly the case—least of all as regards two such closely related kinds of consumption as vegetable and animal food. Consequently, the first curve represents the utility and marginal utility of the direct consumption of corn on the assumption that there exists no other use for it. But the right-hand curve would certainly have an entirely different shape if it really represented a consumption of meat *without* a simultaneous consumption of corn. It may be regarded as representing the utility and marginal utility of a consumption of meat which is carried on while, *at the same time*, the remaining stock of corn is consumed directly. Naturally, we might also have regarded the meat consumption as primary and the corn consumption as secondary. The two curves would then have assumed very different forms, but the result, i.e. the division of the original stock of corn, would remain the same on the supposition that, in this case, there is only one equilibrium position. But this assumption—as we shall see later—is by no means always true. (For an algebraic treatment of the problem, see p. 47 seq.)

A question of great interest, not only in relation to this special case, but for all that follows, is to what extent the division of the original stock (of corn) among various uses is altered if, for technical reasons, the quantity of the original commodity required for the production of a unit of the second commodity is also altered. Let us assume, for example, that, for the production of 1 kilogram of chicken or eggs, not 5 kilograms but (in consequence of more rational methods of feeding or of breeding) only 4 kilograms are necessary. In such a case, it is evident that the quantities of corn set aside for poultry food will yield a greater utility than previously. In other words, the curve of meat consumption (cf. Fig. 1) will begin higher up on the vertical axis than before. But, on the other hand, the demand for meat will, for the same reason, be satisfied relatively more rapidly since every unit of corn used will bring a greater increase of meat than previously. For this reason the curve of meat consumption will fall more steeply than before, and it is, therefore, not difficult to see that it may just as well

intersect the curve of corn consumption to the *right* as to the *left* of the former point of intersection. In other words, the technical improvements by which more meat is obtained from every unit of corn may, according to circumstances, lead either to an increased or to a diminished direct consumption of corn, and thus to a decrease or increase of the quantity of corn consumed in the form of meat.

On the other hand, it may be thought that, in such circumstances, the consumption of meat must necessarily be increased. For if it remained unchanged or were reduced, then in both cases more corn would be consumed than formerly, and the marginal utility of corn would fall; whereas the marginal utility of meat, one would suppose, would remain unchanged, or rise. Consequently, the marginal utility of the latter would *rise* in relation to that of corn, whereas equilibrium requires that it should *fall*, since more meat is now obtained per unit of corn than formerly. However, this conclusion is only justified on the assumption that the consumption of corn and meat are *independent of one another*. If we make the contrary (and more realistic) assumption, that they influence each other to a high degree, then it is conceivable that an improvement in the production of meat might lead to a *diminished* consumption. If, for example, as we have assumed, the consumption of meat remained unchanged and the consumption of corn rose in consequence, then, in reality—since human needs for sustenance are limited—the marginal utility *of both corn and meat* would fall, and it is, *a priori*, not impossible (though in this case improbable) that the latter would decline more rapidly than the former. We see from this what are the complications which may emerge from analysis of the simplest possible case of exchange, and how careful one must therefore be not to draw hasty conclusions in the much more complicated cases arising in a developed system of trade which will be the subject of examination in the following pages.

The relations between two or more commodities as regards consumption may, as Pareto remarks,[1] be of two essentially different, indeed contrary, kinds. They may be *complementary*—so that an addition to the one requires for its effective utilization an addition to the other, or others. Or they may be *competitive*—

[1] [*Manuel d'économie politique*, p. 251.]

so that an addition to the one renders a part of the other, or others, superfluous. This distinction is perfectly valid and has various interesting consequences, though the second type is seldom found in complete purity. In the case discussed above, the animal and vegetable foods are largely substitutes for each other, but, on the other hand, each also increases the satisfaction derived from the other. Perhaps some day the physiologists will succeed in isolating and evaluating the various human needs for bodily warmth, nourishment, variety, recreation, stimulation, ornament, harmony, etc., and thereby lay a really rational foundation for the theory of consumption.

B. *Exchange at Given Prices*

In the actual exchange of goods between individual buyers and sellers—and frequently enough in a larger economic unit, or even a whole country—the given market price, or the world price, has the same function as the technical rate of exchange in the examples discussed above. It is true that the individual who desires to make an exchange, himself exercises a certain influence on prices by virtue of his supply or demand, but, in most cases, this influence is, in itself, inappreciable and therefore, from his point of view, without significance. He plans his economic behaviour exactly as he would do if the exchange value of the goods was unalterably given and predetermined. Consequently, his offer of his own goods and his demand for those of others—assuming the exchange to take place within a given consumption-period—are determined in exactly the same way as in the previous case, in which it was a question of alternative uses of the same goods. If, for example, he has agricultural goods for sale but wishes to buy coffee, sugar, fish, manufactured goods, etc., he must regulate his offers and his demands in such a way that consumption in the period in question, both of the goods he gives up and of those he receives, will yield a marginal utility proportionate in each case to the given exchange value in the market for the goods in question. If, as is usual, the price is expressed in money and if the marginal utility of each commodity is compared with the price, then these ratios, or what are usually called the *weighted* marginal utilities (weighted according to the price) will always be equal. Hence the last shilling which our farmer expends, whether on coffee, sugar, clothes, or shoes, and also the last shilling's worth of corn, meat,

bacon, eggs, linen, wool, etc., which he retains for his own consumption—all taken on a given consumption-period, say one year—will bring him the same amount of utility or satisfaction; for otherwise economy necessarily demands that he increase his consumption of one or more of these goods, and reduce that of others.

Moreover, this is exactly the same condition as in the preceding case and can, especially if we restrict our observations to two commodities only, be represented by exactly the same diagram as before, in which, by the horizontal line AB (see Fig. 1), we now represent the quantity of goods in hand at the beginning—or, what amounts to the same thing, their exchange value (e.g. in money)—whilst the marginal utility of the goods, partly for direct consumption and partly in the "converted" form assumed by exchange—or the utility of the last shilling's worth of each commodity—is represented by the ordinates of the two curves.

Now we discover in this new case exactly the same peculiarities and apparent paradoxes with regard to the effect exercised by an alteration in the exchange value of goods, as determined in the market, on the supply and demand of the individual consumer. For example, suppose that a person has a stock of corn and wishes to exchange a part of it for coffee beans. If the market rate at some moment of time is 10 kilograms of corn for 1 kilogram of coffee, he will acquire the quantity of coffee he needs for a year, or half-year, by exchanging 100 kilograms of corn for 10 kilograms of coffee. But what will happen if the relative price changes so that for 1 kilogram of coffee he need only give, say, 9 kilograms of corn? In the present case, which relates to goods which cannot really replace each other in consumption, it seems probable that the change in price must lead to an increased consumption of coffee. On the other hand, it is uncertain at the outset whether it will lead to an increased or diminished supply and, consequently, to a decreased or increased consumption, of corn. For if, in consequence of the lower price, he increases his consumption of coffee by more than one-tenth to, say, 12 kilograms, then he will increase the quantity of corn which he must give in exchange for coffee to $9 \times 12 = 108$ kilograms; and consequently he will have 8 kilograms of corn less for direct consumption.

But if he increases his consumption of coffee by less than one-tenth—say only to 10·5 kilograms—then he need only offer 94·5 kilograms of corn, and will consequently have 5·5 kilograms *more* than formerly to consume directly. Each is consistent with the law of marginal utility, which only requires that the marginal utility of coffee in relation to that of corn shall fall until it accords with the new relative exchange value, and this condition may perfectly well be satisfied in either case. Indeed, it is even conceivable that the new price situation might possibly lead to a diminished consumption of coffee, in so far as an increased consumption of foodstuffs, such as corn, might perhaps reduce the need for coffee and thereby in itself reduce the marginal utility of coffee even although all other circumstances remain unchanged. This is, of course, as we have already pointed out, still more true of goods which can completely replace each other in consumption, such as the various kinds of animal and vegetable foodstuffs, etc.

The above conclusion, which is theoretically irrefutable, viz. that the supply of a commodity may be either increased or diminished when the price rises in relation to that of other goods, and *vice versa* when it falls, is seldom encountered in reality, because a rise in price nearly always leads to an increased, and a fall in prices to a diminished, *production* of the commodity in question. If this change in production cannot be effected with sufficient rapidity, or not at all—or if, as we shall show later, the two commodities are made from wholly different factors of production—then there is nothing to prevent such a result, though it is generally regarded as unexpected and paradoxical. Thus, for example, a chance rise in the price of agricultural products may very well induce farmers who had previously been compelled to deny themselves necessaries in order to pay interest and taxes to increase their consumption of the produce of the land, with the result that, in spite of the rise in price, less of those products, instead of more, will be offered on the market. If I am not mistaken, this actually happened in the later years of the world war.

Another very interesting case is that of the supply of labour, in so far as the regulation of hours of labour lies in the hands of labour itself. An increase in wages may cause more labour to be offered in the market, but it need not necessarily do so.

As we have already pointed out in connection with the consumption of goods, both possibilities accord with the principle of marginal utility: the labourer, if free to choose, extends his working day up to the point at which the effort of the last hour of labour approximately corresponds to the gain he expects from the wages offered for that hour. If wages are raised, it might be supposed that the prospect of increased well-being would be an inducement to greater effort; but, on the other hand, since the wages for each hour are raised, the whole standard of living of the labourer is changed. He can now satisfy his usual needs by less work than formerly, and the increased well-being which is now available to him can be realized in part by allowing himself more leisure and recreation than formerly. The vehement disputes often heard, as to whether a workman is made "more diligent" or "more lazy" by higher wages, cannot therefore be settled *a priori* either way. On the other hand, there can be little doubt that a percentage increase of wages for overtime leads to an increased supply of labour. For, in this case, the economic position of the workman remains essentially the same, and the increased wages for the last hour of work (overtime) will therefore have their full effect. This method of stimulating the worker to increased effort is, therefore, just as popular among employers as it is regarded with suspicion by the workers, because at first it is a temptation to over-exertion and then later it leads to periods of unemployment. A quite different question, of great practical importance, though we cannot pause to discuss it now, is whether higher wages may lead to greater intensity of work, by enabling the labourer to procure for himself better nourishment and a better technical education for his children, etc.

Algebraic Version.—It is now many years since the first attempts were made to express economic quantities and their relations in algebraic terms. After a period of poor success, the method has now become fairly well established in economic theory —chiefly as a result of the work of Jevons, Walras, and their followers. In what follows, we shall apply this method side by side with our ordinary discussion, and shall introduce it here for the first time.

If we suppose the consumption of each particular kind of commodity to be independent of every other simultaneous line of consumption, then we may regard the utility to a consumer

arising from the consumption of a given quantity, a, of the commodity (A), during a given period of consumption as a function $f(a)$ of the quantity, a function about which one can only say *a priori* (i.e. without a special investigation of each particular case) that it increases simultaneously with a but less than proportionately. If the quantity consumed is increased by a small addition, Δa, then the total utility or satisfaction is increased by a corresponding amount, which we may designate $\Delta f(a)$. The additional utility which arises when the quantity of the commodity is increased by one unit, i.e. the marginal utility, will then be expressed by the ratio $\frac{\Delta f(a)}{\Delta a}$. If we now suppose these quantities to become infinitesimal, the ratio will, as a rule, have a determinate limit which is the differential coefficient, or the first derivative of the function, $f(a)$, with respect to a. The latter, which is usually indicated by $\frac{df(a)}{da}$ or by $f'(a)$, is itself a function of a, and, in the present case, has the characteristic peculiarity of being a *diminishing* function of its variable, i.e. it diminishes when a increases. All this is, of course, only a symbolic expression of the theoretical argument already developed that the marginal utility falls—whilst the total utility obviously continues to grow, though in a diminishing degree—when the quantity consumed, per unit of time, increases.

If we now apply the above argument to all the other kinds of commodities, (B), (C), (D), etc., some of which the consumer possesses at the outset, and the remainder of which he acquires by means of exchange at market prices, then we can express symbolically the conditions of equilibrium for the economy of the individual which have been described above; on the one hand, the marginal utility of each commodity is proportionate to its price, and, on the other, the total exchange value of the commodities given up is identical with the total exchange value of the commodities acquired. If the market prices of a unit of each of the various goods (calculated, for example, in money) are p_a, p_b, p_c, etc., and if the quantities of these goods, which the person in question possesses *after* the exchange, whether he has acquired them or has possessed them from the beginning are expressed by x, y, z, etc., then, if $\phi\ (\ )$ and $\psi\ (\ )$ indicate utility functions analogous to $f\ (\ )$, the first condition will be expressed as follows :—

$$f'(x) : \phi'(y) : \psi'(z) : \ldots = p_a : p_b : p_c : \ldots$$

This is evidently equivalent to a system of equations whose

number is one less than the number of goods dealt in. The second condition we may simply express by the equation

$$p_a.x + p_b.y + p_c.z + \ldots = p_a.a + p_b.b + p_c.c + \ldots$$

in which a, b, and c are the quantities of the various kinds of goods possessed at the beginning (some of which may, of course, be equal to zero). In other words, the value, in money, of the possessions of the person in question is the same before and after the exchange. Consequently the number of equations is equal to the number of unknowns—x, y, z, etc., and the problem should be capable of a mathematical solution if the forms of the functions —$f(\ )$, $\phi(\ )$, $\psi(\ )$, etc.—which express total utility, and whose derivatives express the marginal utility for a given consumption of each and every kind of goods by the person in question, are precisely known. A closer study of the forms of these functions falls within the province of experimental psychology and of statistics of consumption; it may perhaps be of great importance in the future. For the present, we are only concerned with the attempt to investigate the inter-connection between the phenomena of consumption and exchange, and for this purpose we may be content with a general knowledge of these functions derived from our daily experience.

In reality, as we have frequently pointed out, the position is that the utilities and marginal utilities of the various kinds of goods are not independent but, on the contrary, influence each other in a greater or lesser degree. The only really rational procedure is, therefore, to regard the total satisfaction or well-being as a function of all the quantities of goods consumed *simultaneously* per unit of time, or during a certain consumption period, so that, if these quantities are a, b, c, etc., the function can be symbolically represented by $F\ (a, b, c\ \ldots)$. Of this function it may generally be asserted that it increases as soon as any of the goods consumed increases in quantity, the other quantities remaining unchanged, although, of course, in this case *a fortiori* the function increases in a much smaller proportion than the quantity of the single commodity. If, for example, the increase consists of one unit of the commodity (A), then the increase in utility (or marginal utility) of commodity (A) should be symbolically expressed by the first *partial* derivative of the function $F(\ )$ with respect to a, i.e. $\frac{\partial}{\partial a} F\ (a, b, c,)$ or, as it is frequently written, $F_a\ (a, b, c)$, which will thus be itself a function not only of the quantity a, but also of all the quantities of goods consumed. The same applies to the marginal utility of the goods

(B), (C), etc. Thus, according to this view, the conditions of equilibrium would be that the partial derivatives of the total utility functions with respect to the quantities x, y, z, etc., available for consumption, should after exchange be proportional to the prices of the goods. Thus :—

$$F_x : F_y : F_z : \ldots = p_a : p_b : p_c \ldots$$

to which must be added the same equation as above :—

$$p_a . x + p_b . y + p_c . z + \ldots = p_a . a + p_b . b + p_c . c + \ldots$$

which means that the total money value of the goods in the possession of the person is the same before and after the exchange.

C. *Isolated Exchange*

Before proceeding to show how the exchange values of goods, which we have hitherto regarded as data, are in reality determined by the competition of buyers and sellers in the market, we shall refer briefly to a kind of exchange whose direct practical importance is not as great as its theoretical interest: exchange between two isolated individuals. In reality, an exchange between two individuals is almost always effected under the influence of the market, even if not in the market itself. For the moment, however, let us abstract from this, and assume that, during the period of consumption in question, neither of the parties has any opportunity of trading with anyone but the other party. The problem of price formation in this case is far from being as simple as it may at first sight appear. We shall not treat it in more detail than is necessary to show by contrast the influence of competition on prices.

Let us suppose that a peasant from the plains and a peasant from the forest meet on the way to town. The former has a sack of corn which he has so far been unable to dispose of, the latter has half a load of wood which he intends to sell. Since each needs the goods of the other, they agree to exchange, and each of them is thereby saved an extra journey to the town. It may be that, if necessary, the peasant from the plains would give his sack of corn for a quarter of a load of wood; and the peasant from the forest, on his part, his half-load of wood for only half a sack of corn. Thus, if they exchange only with each other, they both consider that they have made a considerable gain on

the exchange ; but they might equally well have exchanged their stocks if the one had possessed $1\frac{1}{2}$ sacks of corn or if the other had had three-quarters of a load of wood, and so on. Again, if we suppose that the stocks in their possession had been greater and that they had only this one opportunity for exchange during a longer period of consumption—e.g. for a whole year in advance—then it is quite clear that the question how large a quantity of their respective goods they could and, from an economic point of view, should, exchange with each other is quite *indeterminate.* Within certain more or less wide limits, the question may be answered in an infinite number of ways, since it is only a question of satisfying the condition that the exchange shall benefit both parties ; and here there is no other necessary condition. So much only is certain, that if the exchange continues until equilibrium is reached for both parties, the relation between the marginal utilities of the corn and of the wood must be the same on both sides :—

$$\underset{\text{(for the peasant from the plains)}}{\frac{\text{marginal utility of 1 unit of corn}}{\text{marginal utility of 1 unit of wood}}} = \underset{\text{(for the peasant from the forest)}}{\frac{\text{marginal utility of 1 unit of corn}}{\text{marginal utility of 1 unit of wood}}}$$

Otherwise—at least theoretically—the exchange would proceed further ; or, alternatively, it would already have proceeded too far—in which case it would be to the advantage of *both* to re-exchange a certain portion. If, for example, after the peasant from the plains has exchanged a certain quantity of corn for a certain quantity of wood, it is more or less a matter of indifference to him whether he obtains two more logs of wood of ordinary size in exchange for 1 litre of corn, whilst the peasant from the forest still considers it advantageous to obtain in exchange a few more litres of corn at three or four logs of wood per litre, then the latter should, by offering this price, or one near it, be able to induce the other party to continue the exchange ; and so on.

But this is by no means the same thing as saying that the relation between the marginal utilities of the two commodities (which, in equilibrium, should be the same on each side) will be also the same as the proportion in which the *whole* quantities exchanged stand to each other and which, therefore, constitutes

the *average* ratio of exchange of these goods. In fact, this ratio can, within certain limits, vary indefinitely, and in each particular case the relation between the marginal utilities of the goods at the margin of exchange will be different, though always the same on both sides for the persons exchanging.

It is a pretty mathematical problem—which we will not pursue here—to investigate the law which these variations follow.[1] Here we shall content ourselves with establishing the fact that price determination in isolated exchange is an *indeterminate* problem; i.e. it cannot be solved solely on the assumption that both parties desire the greatest possible profit. This is a point whose great importance—even in practical affairs—we shall subsequently realize. Whenever isolated exchanges occur in practice, the actual determination of price will depend in a high degree on the personal characteristics of the contracting parties, their cunning and coolness, or on mutual goodwill, all of these being things intrinsically too complex and variable to be embodied in the schematic presentation of economic theory to which we must here confine ourselves. Certain related or at least analogous cases (where not two individuals, but two great organizations of buyers and sellers, or employers and employed, are opposed to each other) are evidently of the utmost practical importance; and it is, therefore, essential that the economist should clearly understand the extent to which his science can afford him any guidance in answering these questions.

One of the greatest difficulties with which the arbitrators between employers and employed have to contend is the absence of any scientific standard for the amount of wages or profits in a big conflict. What is usually called a reasonable wage, or a reasonable profit, proves on investigation to be not so much *reasonable* as *usual*, to be in fact the wage or profit determined by free competition under the prevailing conditions of time and place. If, therefore, the conflict only extends over a small area, such as a single factory, then the arbitrator has sufficient basis for his decision in the wages and conditions prevailing in other establishments in the same industry. But this is not the case if, as is more and more common in modern collective bargaining, a wage dispute rages simultaneously throughout the whole of an industry, or even a connected group of industries.

[1] This problem was first treated by Edgeworth (see Marshall, *Principles*, 4th ed., appendix, note xii (*bis*) and my *Uber Wert*, etc., p. 36).

D. Price Formation in the Open Market. Exchange of two Commodities

The more or less fixed ratios at which goods are exchanged on the market (usually by means of money) are not, as is often supposed, due to qualities inherent in the goods themselves; nor, at least directly, to their normal costs of production. As we have already indicated, they spring from the nature of exchange on a market (as opposed to isolated exchange); from what Jevons called "the law of indifference", which is, fundamentally, nothing else than the old "free competition".

According to this law, there cannot theoretically be more than one price in the market for the same commodity at the same time, or more than one ratio of exchange between two commodities. But in that case, it may be asked, could not the "sellers" (the holders of a particular commodity) hold back their supply at the beginning, thereby forcing up prices, and then afterwards lower them in order to dispose of the remainder of their goods, or so much of them as they do not wish to retain? Of course they could, and they often do. But there is always the risk that some sellers may succeed in disposing of the whole of their stocks while the price is still high, so that the others will either not be able to sell their goods at all or will have to be satisfied with a price much lower than they would have got if the equilibrium price had been fixed by competition from the beginning; since the purchasing power of the buyers who had already partially satisfied their needs at the higher price would then be less than it would have been if, from the beginning, they had bought the same quantity at a lower price; or since as a rule there would then remain fewer buyers able to purchase the goods. This is presumably the reason why so-called rings or cartels of producers or other sellers so often fail, when the participants have only agreed to maintain a high price, but have nothing else in common and have no organization controlling output and individual sales. If, on the other hand, organization has reached the point of forming a cartel or trust in the real modern sense, so that the maximum quantity of goods which each of the members may offer is determined beforehand; or if the members agree to compensate each other for possible losses, or to divide their profits or simply to set

up joint production or a joint selling organization under single control, then price formation will more or less approximate to monopoly conditions—of which we shall have more to say later.[1] Assuming that buyers (i.e. holders of the other goods) also combine, form trusts, cartels or rings, then there is no longer any purely economic law of price-formation—no law based on mutual desire for the greatest possible gain—and we revert to *isolated* exchange, in which, as has been said already, all possible rates of exchange are, within certain limits, conceivable.

If, however, we disregard this possibility and assume universal free competition, then, so far as genuine market transactions are concerned, the relative prices of commodities will more or less rapidly approach a certain equilibrium position, or else oscillate about it. At this equilibrium position, all holders of goods will be able to *exchange* up to a point of *relative satiety*, that is to say, they will continue to exchange so long as there is any advantage in doing so at *that* market price. We may assume, for the sake of simplicity, that this equilibrium price will be reached at the very outset. For the individual desiring to exchange his goods, the price relationships thus reached in the market will have exactly the same significance as the given prices in the case we discussed above.[2] He will regulate the supply of his own goods and his demand for other goods in such a way that the marginal utility of each commodity will be proportional to its price, or that the weighted marginal utility is everywhere the same (in other words, that for the last shilling he spends he will obtain the same additional utility from each commodity). To every price relationship, therefore, there corresponds for each individual a determinate combination of supply and demand, and of quantities of goods retained and acquired. The sum of the individual demands for each particular commodity evidently makes up the total market demand for the goods and, in the same way, the sum of the individual supplies constitutes the total supply of these goods. Market equilibrium is thus only possible with a price relationship at

[1] It is related somewhere in the Corpus Juris how two teachers of grammar in a small Roman town, instead of entering into mutually injurious competition, agreed to divide the profits of their lessons. On the same principle innumerable agreements have been entered into, in ancient and modern times, between sellers of all kinds of goods.

[2] [See page 43 seq.]

which the demand and supply are equal for each particular commodity. If we include in the demand for a commodity the quantities which a seller wishes, at a given price, to retain for his own use, then it may be said that equilibrium is to be found in a system of prices which, for each commodity, makes the demand equal to the stocks in the market, or to the total supply of that commodity. Thus, on the assumption that the market gravitates quickly enough towards equilibrium, it should be possible—if the given quantities of goods on the market for a certain period of consumption, and if the personal dispositions of all consumers, were known—to establish a system of logical relations (or what in mathematics is known as a system of equations) from which both the quantities of goods acquired or given up by each individual and also the relative equilibrium prices, would be determined. It is, however, in no way excluded —as we shall soon see—that the problem may, under otherwise identical conditions, have more than one solution.

Formally, indeed, this doctrine is only a repetition of the old thesis that the market price of goods is regulated by an equilibrium between supply and demand. In reality we have advanced considerably, for we have found in marginal utility the general principle which governs supply and demand under any price system. We are, therefore, in a position to carry the discussion of price formation in the open market considerably further than the earlier economists were able to do.

In accordance with our method of proceeding from the simpler to the more complex, we will begin with the case in which only two commodities are exchanged in the market. This case, moreover, is not so abstract and unreal as may at first sight appear. It is true that two particular commodities are very seldom exchanged directly. Nearly all actual exchanges are effected indirectly, through the mediation of money. Every commodity, or group of commodities, has its special market, in which it is exchanged for money, and the market price of this commodity is determined there with more or less regard to the simultaneous market prices of other commodities. But if we look at the problem broadly and consider, for example, the economic interests of a particular class of society, of a district, or country, as compared with those of other classes, districts, or countries, then it not infrequently happens that, omitting

intermediate links, we must regard as decisive the exchange of only two commodities, or of two related classes of commodities, whose price-ratio is determined almost without reference to other goods on the market, which are of comparatively minor importance. This is true where the interests of an agricultural population are opposed to those of an industrial population; where the commodity "labour" is confronted with the commodity "means of subsistence"; or where the economic welfare of a district or of a whole country depends on the price of its staple commodity in foreign markets in comparison with the price of its imports taken as a whole.

From the theoretical point of view, the exchange of two commodities has this peculiarity—that it is the only form of exchange which can normally take place by the direct barter of goods against goods. Not that two holders of the different commodities could always satisfy one another's needs by themselves—for this, in fact, occurs only in exceptional cases. As a rule, at least one of the parties to the exchange is compelled to deal with more than one holder of the commodity he wishes to acquire. But, nevertheless, it should in this case be possible to exchange goods for goods without the mediation of either money, credit, or any other intermediary; that being usually—as we shall soon see—an essential condition for the achievement of equilibrium as soon as the number of goods in the market exceeds two.

We assume, for the sake of simplicity, that at the outset the two commodities are held by different parties, so that no one at first possesses more than one commodity. Let us suppose the prices of the two commodities (A) and (B) offered in the market to be expressed in terms of one of them, (A), so that the price of a unit of (A) is, consequently, *invariably equal to* 1, and the price of a unit of (B) (which we indicate by p) *is variable*; it then follows, from what has been said above, that an arbitrary price (p) quoted in the market will call forth from each holder of the community (A) a certain demand (x units) for the commodity (B), and a corresponding supply of the commodity (A), which will then clearly equal $p.x$ units. The sum of all these demands (x) constitutes the total demand X for the commodity (B), which implies a corresponding supply, $p.X$, of the commodity (A). In the same way, the holders of the

commodity (B) offer, at the price p, a total supply, Y, of the commodity (B) and demand a corresponding quantity, $p.Y$, of the commodity (A). The condition of p being the equilibrium price is that the supply of and demand for the commodity (B) are equal, so that $Y = X$; from which it follows that demand for and supply of the commodity (A) will also be equal, for it follows that $p.Y = p.X$. Further, let all conceivable values of p, which, as we have explained, must be treated as a variable, be represented by distances from a fixed point (the origin) along the horizontal axis, and through each of these points draw a vertical line, on which are marked off two lengths, one representing the total demand for (B) on the part of the holders

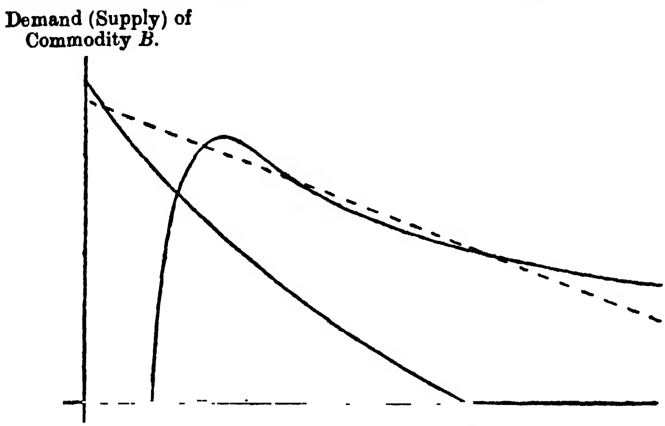


FIG. 2.

of (A), and the other the total supply of (B) by the holders of (B). We shall then obtain two connected curves, one of which represents the demand for (B) and the other its supply for every conceivable price-ratio. If these two curves intersect and so have an ordinate in common, then at that point demand and supply are equal; and the corresponding distance along the horizontal axis (the abscissa of the point of intersection) represents the desired equilibrium price.

If we begin by assuming that (A) and (B) cannot in any way replace each other in consumption, we can then describe the general course of these curves in the following way. If $p = 0$ —i.e. if (B) can be obtained for nothing or for a purely nominal

amount of (A)—every holder of (A) will demand (B) up to the point of *complete satiety*—i.e. until its marginal utility has fallen to zero. For this to happen, as a rule, only a finite, though sometimes a quite considerable quantity of (B) is required; hence the demand curve leaves the vertical axis at a finite distance above the origin. If p rises, the demand falls continuously; since the marginal utility of (B), relative to that of (A), must fall *pari passu* with its price. The curve therefore falls continuously towards the x-axis (though it may be convex or concave to the x-axis or alternately the one and the other) and finally meets it at a point corresponding to the price at which (B) ceases to be demanded by the holders of (A). This point may possibly be so remote that it does not, in practice, exist—in the case when (B) is an absolute necessity of life which would be in demand at *any* price.

The supply curve of (B) follows an entirely different course. If the price of (B) is zero, or very low, then there is no inducement for holders of (B) to offer their goods, and when they do begin to do so it will, at first, be only in very small quantities. The supply curve will thus begin at a point on the horizontal axis which is at a certain distance from the origin and will gradually rise *pari passu* with the rising value of p. But the increase in supply will not continue indefinitely; sooner or later a point will be reached at which an increased price will no longer induce holders of (B) to offer any more, but will, on the contrary, make them offer *less*, because at this higher price they can obtain with less sacrifice of (B) so much of (A) that its marginal utility will fall until it is equal to the marginal utility of (B), notwithstanding that the latter will also sink when the quantity of (B) retained is increased. The supply curve thus reaches a maximum, from which it falls again towards the horizontal axis; however, it never cuts the horizontal axis, but moves towards it asymptotically; for however high a price a person is offered for the commodity in his own possession, he will always be prepared to give up some small part of it in order to acquire other goods.[1]

[1] On the above assumption that the two commodities are independent of each other as regards consumption, the supply curve is subject to the further condition that the rectangle formed by the co-ordinates (i.e. the supply multiplied by the corresponding price) must continuously increase, since it evidently equals the demand for the other commodity (A), and this demand increases continuously

If we now remember that, on our assumption, the two curves are entirely independent, since the demand for and supply of (*B*) proceed from different persons—the supply curve is determined exclusively by the availability of (*B*) and the demand curve by the availability of (*A*)—then it is clear that there are as many possible kinds of equilibrium as there are possible kinds of intersection, for two curves drawn in the manner we have described. The point of intersection may lie to the left of the highest point of the supply curve; this is the case which was considered almost exclusively by the older economists. In such a case equilibrium is necessarily *stable*, for a slight increase of price would increase supply and simultaneously decrease demand; a slight fall in price, on the other hand, would increase demand and decrease supply, so that, if the price were by chance to be disturbed, it would automatically revert to its former position.

But the point of intersection—for the moment we may assume that there is only one—might also lie to the right of the highest point of the supply curve, so that equilibrium in the market would only be reached when supply had begun to be restricted by the rising price. This equilibrium is also stable; if in this case the price rises, then supply will indeed be reduced, but demand will be reduced even more, so that it will be less than supply—with the result that the price must fall again. If the price falls, then supply will increase but demand will increase more rapidly, for which reason the price will soon revert to its former level.

when the price of (*B*) in terms of (*A*) increases, and consequently the price of (*A*) in terms of (*B*) falls. If, on the other hand, the two commodities are to some extent substitutes, this condition need not be satisfied; for, in that case, a falling price of (*A*) in terms of (*B*), and accordingly a rising price of (*B*) in terms of (*A*), might conceivably cause a diminished demand for (*A*) and, accordingly, a still greater diminution in the supply of (*B*). It should be observed, moreover, that the rising portion of the supply curve may be absent if the commodity offered (*B*) has no appreciable utility for its holder, which is often the case with goods which are manufactured only for sale. In that case, the commodity (*B*) is offered—unless there is the possibility of withholding it until the market position is more favourable—to the maximum extent and at any price, so that the supply curve at the beginning is represented by a straight line parallel to the price axis, which later becomes a falling curve. As may easily be seen from what has been said, in such a case the demand curve for (*A*)—whose abscissæ thus represent the price of (*A*) in terms of (*B*)—will, in its lower course, become a rectangular hyperbola, with the axes as asymptotes, and will, therefore, not intersect the price axis. Holders of (*B*) will then demand (*A*) at any price of (*A*), though naturally in quantities which stand in inverse proportion to the price.

That the older economists so generally neglected this case—except occasionally in regard to foreign trade—is all the more remarkable, since it is evidently in full agreement with the well-known and frequently observed fact that the demand for a commodity which has risen in price (e.g. a necessity) may frequently fall in a lesser proportion than the actual rise in price. As against this particular commodity all other commodities constitute a group whose relative price has fallen. Their supply (in exchange for the former commodity) has, on the other hand, clearly risen; it thus rises with a falling price and falls with a rising price of that group of commodities (expressed in terms of the former commodity), and in one of these positions equilibrium between demand and supply will be reached.

Finally, there is nothing to prevent the two curves having several points, and (if so) at least three, in common. In this case, the curious position arises that both the point of intersection to the extreme right and that to the extreme left indicate a stable position of equilibrium, whereas at the intermediate point of intersection a so-called unstable equilibrium prevails; the equality of supply and demand at this price is merely accidental. A disturbance of the price equilibrium in this case has no tendency to an automatic restoration but, on the contrary, produces an uninterrupted shifting of the price in one direction or another until stable equilibrium is reached at one of the two extreme points of equilibrium either to the left or the right.

This very remarkable phenomenon was first pointed out and analysed in detail by Walras.[1] Walras himself, however, seems inclined to under-estimate its practical importance, and appears to be of opinion that, under actual conditions, where a large number of articles are exchanged for each other, only one position of equilibrium would really be possible in the same market. But in that he is mistaken. We have already seen examples, derived from exchanges between employers and employed and between farmers and industrialists—and we shall

[1] In Marshall's *Principles* (4th ed., p. 525, *et seq.*) there are curves of supply and demand which resemble those discussed here. They relate, however, to a different case, namely the number of positions of equilibrium which with an unchanged (or only slightly changed) disposition on the part of the buyer might occur as regards such commodities as follow the so-called law of increasing returns; commodities which can be produced and sold at a lower cost, if their output is large than if it is small; e.g. newspapers, books, railway journeys, and others.

later add a famous case of international exchange—which show that equilibrium may very well occur under circumstances where a price increase would cause a reduction and not an increase of supply, and *vice versa* a reduction of price an increase of supply. From this it is only a short step to the admission of several possible equilibrium prices in the same market, as a glance at Fig. 2 will show.

We arrive at still more remarkable results if we assume, in accordance with what often occurs, that the two commodities may, to a greater or lesser degree, be capable of acting as substitutes. In that case, as we have already indicated, the demand curve of either commodity may also have both a rising and a falling section and the chances that both curves will have several points of intersection, or even that they may approximately coincide over small stretches, are quite considerable. It is not impossible that puzzling disturbances in the market, which frequently occur without any known cause, may be properly attributed to the hitherto neglected fact that a particular state of equilibrium may not be the only one which is possible under the given conditions, and that a state of equilibrium chosen at random can just as well be unstable as stable, or may for some insignificant reason be converted from one into the other.

An admittedly artificial example of this (cases more or less similar to which are, perhaps, not so rare in reality) is the following :—

A person, A, possesses a stock of wheat, another person, B, a stock of rye. For the sake of simplicity we will assume that rye and wheat have the same nutritive value per pound (this, however, is not essential to our argument). We assume, however, that wheat (owing to its better taste) is preferred by both parties ; yet each of them endeavours primarily to obtain *the maximum nourishment*; but only up to a certain limit, say a thousand pounds, beyond which any additional nourishment cannot in general be utilized and is, therefore, without value. If A at the beginning had 800 lb. of wheat, then, as the price of rye varied, his demand for rye would clearly be determined in the following manner. If the price is zero, i.e. if rye can be obtained for nothing, he will provide himself with 200 lb., neither more nor less, because this will fully satisfy his requirements for this kind of nourishment. If the price rises above zero he will be compelled, in order to acquire the necessary nourishment, to dispose of a part

of his stock of wheat, but in that case he will evidently be forced to consume more rye than before. In other words, his demand for rye will increase when the price of rye increases. If p is the price of rye, expressed in wheat (or in the money price of wheat as a unit), then, as will easily be seen, his demand x will be such that it satisfies the equation

$$800 + x - p.x = 1{,}000,$$

so that

$$x = \frac{200}{1 - p}.$$

The limit is reached when p equals $\frac{4}{5}$, when he will have to

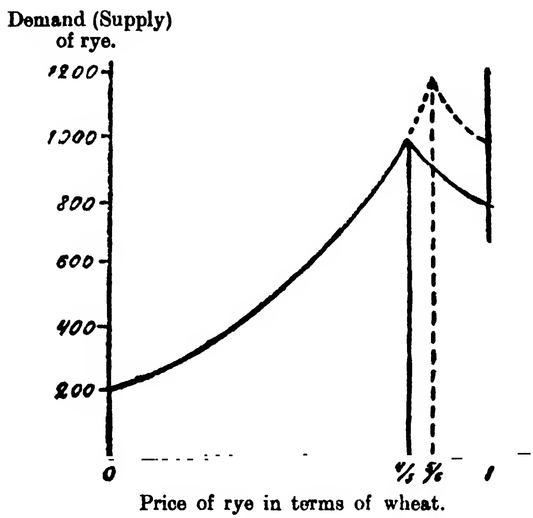


FIG. 3.

exchange the whole of his stock of wheat, 800 lb., in order to get a sufficient amount of nourishment, i.e. 1,000 lb. of rye. If the price of rye rises still further he cannot in any way acquire full satisfaction, but will endeavour to obtain as much as possible, which he will do by continuing to offer the whole of his wheat for as much rye as the market determines. His demand for rye will thus now $= \frac{800}{p}$. Only when $p = 1$, and rye consequently commands the same price as wheat, would an exchange be purposeless for him. At this point he ceases to demand rye.

His individual demand curve will thus assume the following

form: it begins at a point on the vertical axis, the distance of which from 0 corresponds to a demand for 200 lb. of rye. It then describes an hyperbola which has for its asymptotes (*a*) the horizontal axis and (*b*) a vertical line which intersects the horizontal or price axis at a distance of one unit from the origin. This hyperbola, however, terminates at a point whose distance from the horizontal and vertical axes corresponds to a demand of 1,000 lb., or a price of rye, $p = \frac{4}{5}$. The demand curve next describes a descending hyperbolic curve, whose asymptotes are the horizontal and vertical axes. At a distance, along the horizontal axis the curve suddenly descends from a height, corresponding to a demand of 800 lb. rye, towards the horizontal axis.

The amount of rye offered by *B* will clearly depend on the size of the stock he holds. We will assume that it is exactly 1,200 lb. If the price of rye is zero he will, of course, have no inducement to exchange; but as soon as rye, expressed in terms of wheat, is worth something, however little, he will immediately exchange the whole of his worthless surplus, 200 lb., of rye in order to obtain at least some wheat. If the price of rye is raised, he will be in a position to acquire more and more of the desired commodity wheat, and in order to obtain as much as possible he will still continue to offer so much rye that his total stock of food will amount to exactly 1,000 lb., neither more nor less. If we call his supply of rye y we shall arrive at the equation :—

$$1{,}200 + p.y - y = 1{,}000$$

where $y = \frac{200}{1-p}$ —or exactly the same as we previously found for *A*'s demand for rye. The only difference is that *B*'s supply of rye will continue to increase, even after the price reaches $\frac{4}{5}$; for so long as wheat can be obtained in the market there is no reason why *B* should not procure more than 800 lb. of it. Only when the price of rye has risen to $\frac{5}{6}$ of that of wheat can *B*, who at that price will offer the whole of his stock, 1,200 lb., no longer increase his supply, and indeed has no reason for doing so, since at a higher price he could obtain the necessary 1,000 lb. of wheat even for a fraction of his stock of rye.

In this case, the curious fact emerges that the supply and demand curves of the two individuals coincide for a large part of their course. In other words, for every price of rye between

zero and $\frac{4}{5}$, A's demand for rye and B's offer of it are exactly the same—and consequently, for the same reason, their respective supply of and demand for wheat.

This example should show to what a large extent the simple scheme of the variations of supply and demand with which economists have hitherto contented themselves, requires to be developed and completed in order to correspond with the varying phenomena of reality.

E. Continuation. Exchange of Three or More Commodities

As soon as there are *more than two* commodities on the market, complete equilibrium cannot as a rule be reached by direct exchange alone, but *indirect exchange* must supplement it. This is seen in its simplest form in the extreme case where direct exchange is altogether excluded. A country (say Sweden) has timber for sale and sufficient corn for its own needs, but must buy fish. Another country (Norway) can supply fish and has sufficient timber, but must buy corn. Finally, a third country (Denmark) has a surplus of corn and sufficient fish, but lacks timber. Evidently no direct exchange can take place here, but an indirect exchange may; if, for example, Denmark as an intermediary buys up Norway's surplus of fish in exchange for its own surplus of corn, in order, in its turn, to sell the former to Sweden and thereby satisfy its own requirements for timber. Or the same result might have been achieved by the use of a special medium of exchange, money or credit, as we shall soon see.

But even if, in a three-cornered exchange, each party was a purchaser of the products of both the others (so that, up to a point, direct exchange could take place) even then, so far as the exchange values of the goods were regulated only by mutual supply and demand in direct exchange, a final price equilibrium would not, as a rule, be reached. As between each pair of commodities, the price ratio would be determined in a separate market, isolated from the other two, and the resultant three relative prices would not usually be correlated, i.e. they would not be such that each would be the ratio (or product) of the other two. If, for example, in a direct exchange of the commodity (B) (fish) for the commodity (C) (corn) the equilibrium price were

such that one unit of (*B*) were exchanged for two units of (*C*), and on the market for (*C*) and (*A*) (wood) the price is four units of (*C*) for three of (*A*), then *if the prices are correlated*, two units of (*B*) must be exchanged for exactly three units of (*A*). It may, however, happen that, in the direct exchange of (*A*) for (*B*), a different equilibrium price would obtain, so that either less (say one and a half) or more (say two and a half) units of (*B*) would be exchanged for three units of (*A*). Whichever occurred, it would then be profitable to enter into a so-called *arbitrage* transaction. Thus, in the latter case, a holder of (*A*) desiring to acquire (*C*) would first buy a suitable quantity of (*B*) and subsequently exchange that (*B*) for (*C*). In this way he would obtain five units of (*C*) for three of (*A*), whereas by

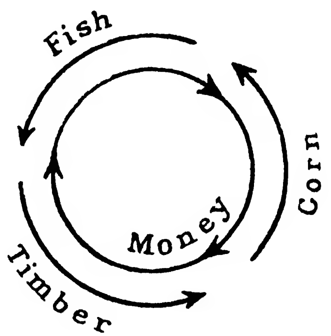


FIG. 4

direct exchange he would only have obtained four units of (*C*), and similarly if the price of (*B*) in direct exchange for (*A*) had been *lower* than the correlated price. If, therefore, full equilibrium is to be reached in such cases, at least a part of the commodities in the market must necessarily be the object of indirect exchange.

The commonest procedure in such cases is for the exchange to be effected with the assistance of a special medium of exchange, money, which only formally appears in the market as an object of exchange. In the extreme case which we mentioned by way of introduction, Sweden, for example, buys fish from Norway for *money*; Norway uses this money to buy corn from Denmark, and Denmark in turn uses it in payment for timber from Sweden, so that in the end Sweden gets its

money back. We can visualize the position by means of a diagram in which each commodity moves one-third of the circumference of an outer circle, whilst money makes a whole revolution in the opposite direction in an inner circle, and thus finally *returns to its starting point.* The result is, or may be, that after the conclusion of the business only the goods have changed hands, whilst the sums of money employed are in exactly the same hands as at first. Thus, in fact, goods have been exchanged for goods, not directly, but, in part at least, indirectly. The law of marginal utility has been none the less effective. Under ideal market conditions, in which the final price equilibrium is established from the very beginning, the exchange values and the marginal utilities of all commodities must be proportional for each of the exchanging parties taken separately. As far as money is concerned, as we have said, its rôle is purely formal—or may theoretically be *conceived* as such. Indeed, a sum of money, however small, may effect an indefinitely large exchange of goods, if it circulates frequently between the exchanging parties. The importance of this observation will become clear when we come to treat of the functions of money. However simple and commonplace the above consideration may appear, it constitutes in reality the master-key to a proper understanding of the peculiar problems of money.

It is not easy to give a graphical version of this problem—exchange that, in part at any rate, is indirect. If there are only three commodities, then it is possible to represent the position by a three-dimensional figure—if we want to do so—but even this method breaks down when the number exceeds three.

On the other hand, we can easily express the conditions of equilibrium by algebraic symbols and thereby set out the logical relations or equations which determine the equilibrium price. It is simplest to conceive demand in the wider sense already indicated, including the quantities of the various goods which the original holder wishes to retain for his own consumption at a given system of prices. In equilibrium, demand in this sense must be equal, not to the amount offered in exchange, but to the whole of the *stocks* available in the market for consumption in a given period. Of course, we might have used this method for *two* commodities; and this would have given us a more satisfactory expression of the position where, for example, one person is in possession of *both* of the traded commodities from the start, and appears

according to circumstances as a buyer or seller of either. But the discussion of that case was simplified in other respects by using the more limited conception of demand.

For every conceivable system of prices, in accordance with the law of marginal utility, each person in the market will have a certain demand for each commodity; indicating either that he wishes to acquire, or if he possesses it already, to retain, a particular quantity. If his total utility function is expressed, as before, by $F(x, y, z \ldots)$, then we have the equations, already set forth on page 49 :—

$$F'_x : F'_y : F'_z : \ldots = p_a : p_b : p_c : \ldots \text{ and}$$
$$p_a.x + p_b.y + p_c.z + \ldots = p_a.a + p_b.b + p_c.c + \ldots$$

altogether n equations in which all the letters have the same meaning as before except that the commodity prices p_a, p_b, etc., are no longer to be regarded as given, but as unknown quantities. These prices may also be regarded as expressed in terms of one particular commodity selected as a unit of value: in which case p_a (say) is constant ($= 1$), or else in terms of a measure of value, such as money, which takes no part in the real exchange. In both cases, if the form of the function $F(\ )$ is assumed to be known, all the n unknown quantities of goods x, y, z, etc., can be obtained from this system of equations; if one of the commodities is itself the standard of value, the quantities are expressed in terms of the $n - 1$ prices of the remaining commodities, still unknown for the present; otherwise they are expressed in the $n - 1$ ratios between the money prices of the n commodities. *For each person in the market there is an analogous system of n equations*, from which the quantities of all goods demanded may be expressed in terms of the $n - 1$ relative prices of the commodities.

We have now to describe the position of equilibrium, where the sum of all the demands for the commodity (A) must equal the total quantity in the market, A, and the same as regards (B), etc. Thus, if we treat each of the parties to an exchange in the same way, and mark them out by the suffixes 1, 2, 3, etc. (x_1, x_2, $x_3 \ldots a_1$, a_2, a_3, etc.), which for precision we ought to have used before, we obtain the equations :—

$$\Sigma(x) = A, \quad \Sigma(y) = B, \quad \Sigma(z) = C \ldots$$

in which $\Sigma(x)$ stands for $x_1 + x_2 + x_3 + \ldots$, etc.

The number of these equations is n; but only $n - 1$ of them are really independent; one of them can always be derived from

the others by means of the equations already set out. Thus if we add together the equations (on p. 66),

$$p_a.x + p_b.y + p_c.z + \ldots = p_a.a + p_b.b + p_c.c + \ldots$$

and all the corresponding equations relating to the other persons in the market, we shall obtain :-

$$.\Sigma x + p_b.\Sigma y + p_c.\Sigma z + \ldots = p_a.A + p_b.B + p_c.C + \ldots$$

And since this equation could also have been found by the addition of the corresponding members of the equations $\Sigma(x) = A$, $\Sigma(y) = B$, etc., after multiplying each of them by p_a, p_b, etc., the above assertion becomes obvious. It is also deducible *a priori*, for if goods are only exchanged for goods (so that money, if it is used at all, functions in a merely formal manner) then, if the demands for all the commodities with one exception are equal to the existing supplies, the same must apply to the last commodity (what the holders do not wish to retain has, of course, already found purchasers). But these $n - 1$ equations are sufficient for the solution of the problem, for all the quantities involved—x_1, y_1, z_1 . . . x_2, y_2, z_2, etc.—can, as has been shown, be expressed in terms of the $n - 1$ relative prices of the commodities, so that finally we shall have as many equations as unknowns. Thus the problem is perfectly determinate.

If, on the other hand, we had imposed the further condition that the exchange must only take place directly, in other words, that the quantity of commodity (B) which is demanded by the holder of (A) should pay in full for the quantity of (A) demanded by the holder of (B), then the problem would have given us more independent equations than unknowns and would thus have become over-determined; unless at the same time we had foregone the demand for correlation between the commodity prices, in which case the possible exchange ratios between n goods would be not $n - 1$ only, but $\frac{1}{2}n\ (n - 1)$, i.e. for three commodities 3, for four 6, etc.

In any case, by the method we have followed, we can only arrive at the relative exchange values of the goods or their relative prices—not at their actual money prices, which must remain quite undetermined; this is obvious so long as we regard the functions of money as purely formal. If, after the exchange is over, all the money employed has returned to the hands of its first owner, it is a matter of complete indifference to him, as to everybody else, whether in the actual exchange transaction,

one unit of goods was exchanged for more or less units of money; in other words, whether, in order to effect the transaction, the money circulated a greater or lesser number of times among the parties in the market before it ultimately returned to its starting-point. In reality, of course, this is never a matter of complete indifference. In every market, there are persons for whom money is something more than this; who exchange goods for money or money for goods in order to obtain at a later date new goods for the money they have acquired. To them, clearly, the exchange value of money—and especially its fluctuations—are by no means unimportant; and the function of money in any particular market transaction becomes, in actuality, not merely formal but also real. In other words, money prices, as such, have their laws and their conditions of equilibrium; but we cannot develop them here because they are very closely connected not only with the nature of money as a commodity and with the conditions of its production, etc., but also with the time-element whose importance in human economy we have not yet considered—in other words, with the theory of capital and interest.

4. *Objections against the Theory of Marginal Utility. Exceptions to the Theory*

The objections which were made in various quarters against the theory of marginal utility when it was first propounded, were largely due to a misunderstanding of its real meaning and may, for that reason, be ignored. In the main, they were based on the fact that its advocates held too one-sided a view of the continuity of economic quantities, of the simplicity and flexibility of the economic system, etc.; on the other hand, the critics exaggerated the discontinuity of the quantities and the complexity of their interaction, and also exaggerated the power of *economic friction*. That, in fact, discontinuity occurs at many points, and must occur, scarcely any adherent of the theory of marginal utility has denied; it exists, after a fashion, whenever the price of a commodity is so *high* that some buyers cease to purchase it or some sellers dispose of the whole of their stocks; or when the price is so *low* that some sellers will not dispose of any of their stocks, whilst not yet appearing as purchasers, etc.

In such circumstances, of course, marginal utility has ceased to regulate the quantities of goods demanded or supplied by such persons. Yet the mathematical treatment of the problem raises no difficulties, for these quantities now enter into the equations as constants. A still more obvious case of discontinuity arises when the commodity which is the object of exchange only occurs in large indivisible units—such as houses, ships, etc. In some of these cases, the determination of a market price in the ordinary sense is impossible, and business is reduced more or less to isolated exchange, in which, as we have seen, the price is, from the point of view of abstract theory, indeterminate. In others of them, as in Böhm-Bawerk's often-quoted example of a horse market (cf. *Positive Theory of Capital*, pp. 203–13), an equilibrium price will be reached, at any rate approximately, which will be determined by the *marginal pair* of buyers and sellers. But it is only for these that the marginal utility (which in this case is roughly equal to the total utility) will correspond with the price. All other buyers and sellers will acquire the commodity at a price more or less below—or sell at a price above—its utility to the person in question.

In reality, however, there is one circumstance which, even in these cases, imparts to the law of marginal utility a wider and more individual application than one would at first sight suppose, namely, that most goods on the market are supplied in a number of *different qualities*. At a horse fair, for example, there is usually not merely one kind of horse, but horses of the most varied kinds as regards age, strength, swiftness, endurance, etc. For example, suppose a buyer has to *choose* between three horses, at 500, 550, and 575 shillings. At these prices he may prefer the second horse to both the cheaper and the dearer one: in other words he values the *difference in quality* between the first and the second at *more* than 50*s*., but that between the second and third at *less* than 25*s*. If every conceivable price and quality were to be found in the market, every buyer would certainly extend his demand up to the point at which a further addition in quality would exactly correspond to the additional price asked. If we conceive this difference of quality (looked at subjectively) as being the marginal utility of the commodity "horse" (which would be in full accordance with the genesis of the concept) then, here also, the marginal utility, at least for

buyers, would be approximately the same as the price or, at any rate, proportional to it. (Something similar also applies to sellers if they deal in horses on a large scale, so that each of them has several horses to sell.) On the other hand, the *total* utility will not, as is usually the case, stand in any definite relation to it. For the horse which the buyer now considers too dear at 575*s*. he would gladly pay 6–700*s*., perhaps 1,000 if it were the only one in the market and he had to have a horse. And the same applies to a number of similar cases.

On the other hand, it often happens, even in the case of goods which are physically perfectly divisible, that individual consumption is not expanded or contracted by every change in price. A very important case is the consumption of necessities. Adam Smith remarked that the human need for food is limited by the size of the stomach, and subsequent investigations have shown that a person under given conditions, doing ordinary manual work, consumes almost constant quantities of the principal foodstuffs—namely, about 120 gr. of albumen, 50–60 gr. of fat, and about 500 gr. of carbohydrates. With exhausting work (e.g. soldiers on the march, etc.) more is consumed, especially more fat. Any material reduction of these quantities would produce the most serious consequences [1] and would sooner or later render the person in question unable to carry on his work. An *excess*, on the other hand, has no value at all and would, in the long run, cause sickness and discomfort instead of added strength and well being. Here, evidently, is a case in which consumption essentially *lacks elasticity* ; or, what comes to the same thing, in which the total utility and the marginal utility *are themselves* discontinuous quantities, so that the latter falls rapidly, from a very high value to zero, or even becomes negative. If each of the three foodstuffs were only found separately in one kind of commodity, then, no doubt, there would be striking peculiarities in the price-formation of articles of food. In reality, all three are to be found, though in different proportions, in most edible commodities, and in addition, as everybody knows, even the commonest foodstuffs exist in different qualities, according to the

[1] To what extent more recent investigations concerning the possibility of substituting carbohydrates for albumen may change the above view, I shall not discuss here.

degree of digestibility, taste, perishability, etc. Hence there is room for the law of marginal utility to operate in individual consumption. Moreover, as we have already pointed out, foodstuffs not only serve directly as human nourishment, but also have indirect uses—especially as fodder for animals, etc.

Two objections mentioned above are of greater weight. It is only too true that concrete economic phenomena are infinitely too complex to be adequately explained by any theory—including the theory of marginal utility; for, in addition to purely economic forces, such as the quest for the greatest possible personal gain, there are others of a different kind: mutual goodwill, general philanthropy, social considerations, etc., which nearly always play some part. As a first approximation, however, we are justified, as we have said, in ignoring all other factors. It is by no means certain that, with the adoption of the principle of marginal utility, even (for example) the altruistic elements in social life would not also permit of analogous treatment, to the extent to which they must be regarded as relevant to the question of price-formation. The attempts made by recent writers to give a rational account of the theory of public finance seem to show that this is really the case.

On the other hand, what is called economic friction (*caused by habit and inertia*) so far as its effects extend—and they are very significant—constitutes an exception to our conclusions. It is indeed true that habit is, with most of us, the fruit of economic observation or instinct. It arises because, under given conditions, it proves the best means of achieving a desired end; but these conditions often originate in the remote past and have, perhaps, now given way to something quite different. During periods of great material progress all institutions based on custom may, therefore, easily appear as anomalies and even as non-economic phenomena, injurious both to the individual and to society, and yet persisting. The Italian economist, Pareto, in his earlier work, *Cours d'Economie politique* (vol. ii, p. 9 *et seq.*, and p. 281 *et seq.*) gives an interesting, though somewhat incomplete, theoretical analysis of economic friction—or, more correctly, of economic inertia, which plays much the same part in relation to other economic forces as does the so-called principle of inertia in mechanics.

But the most important objection to the theory we have so

far developed is no doubt the fact that our assumption of *free competition* is, and can be, only *incompletely realized* in actual life. The field in which it particularly prevails is, as everybody knows, that of wholesale trade; but consumers and owners of goods do not then, as we have assumed, come into direct contact with each other, and consequently the interests of consumers in price formation only become effective at a later stage, and are not direct. On the other hand, in the field in which consumers appear directly (i.e. in retail trade) the law of free competition only operates with certain limitations. Still more striking exceptions are afforded, of course, by industrial monopolies in the narrow sense.

Before we pass on to a more detailed consideration of these exceptions, some of which are of the greatest interest, we shall consider a question, the real significance of which can only be understood after detailed inquiry in the social section of our work, but which, even from a purely theoretical point of view, is of such importance that it cannot be entirely ignored at this point. I refer to the question of the *economic advantages* of free exchange or of free competition in general—a question which is beloved of writers on the theory of value, but of which, unfortunately, not very much has actually been made.

5. *The Gain from Free Exchange*

It is a corollary of the economic principle which underlies all our studies, that we only exchange for the purpose of gain and, under given conditions, we always endeavour to exchange in such a manner, and in such quantities or proportions, as will yield the greatest possible gain. The doctrine that marginal utility is proportional to price; that the subjective utility of the last unit acquired is equal to that of the last unit disposed of; and that the increase in utility at the margin of exchange is zero, are all different ways of expressing this postulate, and closely correspond with the criterion which indicates a maximum or minimum value in mathematics. It is easy—though it would involve a serious confusion of ideas—to cite this as a proof that free exchange brings a maximum satisfaction of needs to all participators; that is to say, as great a measure of satisfaction as is generally consistent with the prevailing conditions of

property or ownership—from which, of course, we must proceed in a theoretical consideration of price-formation. As we know, it was not the advocates of the theory of marginal utility who first advanced this view. It is rather the fundamental principle and dogma of free-traders—the physiocrats and their descendants of the so-called Manchester school—both in the field of production and of trade proper. The well-known saying, "*laissez-faire, laissez-passer*"—actually "*laissez nous faire*" ("let us manufacture our products freely and without restraint") and "*laissez passer les marchandises*" ("let our goods freely pass the boundaries of the province or the state"), which epitomized the principles of industrial liberty and free trade—became, as we know, the motto of this school, which was guided by precisely the above argument. If anybody may freely dispose of his possessions and his productive powers, he will undoubtedly seek to make the best possible use of them; it was assumed, therefore, that both the individual and society will be guaranteed the greatest possible advantage—always, of course, with the very important qualification: so far as existing proprietary rights permit. The harmony economists, who endeavoured to extend the doctrine so that it might become a defence of the existing distribution of wealth (itself a product of free competition and consequently the best possible distribution), cannot, in this respect, be regarded as representative of the views of the physiocrats and the classical free trade school.

Although the propounders of the theory of marginal utility were certainly not responsible for this all-too-optimistic view of the advantages of free trade, yet some of them cannot be entirely absolved from the charge of having helped to maintain faith in it by their support, and their apparently logical proof, of its doctrine. This is especially true of Léon Walras and his immediate disciples. Walras himself relates [1] that, in his youth, he was once helpless in the face of an onslaught on the foundations of free trade theory made by the Saint Simonist, Lambert Bey, who maintained that the exchange values arising from free competition were neither the only ones, nor the best. Walras realized that the theory, if it was to be maintained at all (which he himself never seems to have doubted), must be proved more satisfactorily than had hitherto been

[1] *Études d'économie politique appliquée*, p. 466.

done. "Il faudrait prouver que la libre concurrence procure le maximum d'utilité." And this view was in fact the starting-point of his own work in economics. It is almost tragic, however, that Walras, who was usually so acute and clear-headed, imagined that he had found the rigorous proof, which he missed in the contemporary defenders of the free trade dogma, merely because he clothed in a mathematical formula the very arguments which he considered insufficient when they were expressed in ordinary language.

In the following words—which he italicizes—Walras sums up his investigations into free exchange, especially exchange of two commodities: "Exchange of two articles in a market where free competition prevails is an operation by which all holders of either of these two articles, or of both, can obtain"—in the first edition he wrote only "obtain" and not "can obtain"—"the greatest possible satisfaction of their needs consistent with the condition that they must dispose of the goods they sell, and accept those that they buy, in one and the same proportion for all".[1] Although it is possible that this somewhat vague formulation may be interpreted in a way which can be defended, yet in fact both Walras and his disciple and successor, Pareto (in his earlier work already quoted [2]) employ it precisely in the sense that, under free competition, and under the existing laws of property, each of the exchanging parties obtains the maximum amount of satisfaction for his needs, with *any* system of *uniform* prices in the market. The latter condition must, of course, not be forgotten. The objection which has sometimes been made to this theory—namely that if free competition produced the maximum satisfaction of needs, it would be impossible to increase the available sum of this satisfaction by *gifts*—does not, at least in Walras' opinion, affect the essence of the argument. The "exchange conditions" which prevail in the case of gifts, where one party receives no material compensation, could not in general prevail in the market—not

[1] L'échange de deux marchandises entre elles sur un marché régi par la libre concurrence est une opération par laquelle tous les porteurs, soit de l'une des deux marchandises, soit de l'autre, soit de toutes les deux, peuvent obtenir (obtiennent) la plus grande satisfaction de leurs besoins compatible avec cette condition de donner de la marchandise qu'ils vendent et de recevoir de la marchandise qu'ils achètent dans une proportion commune et identique. (*Éléments d'économie politique pure*, 4me éd. 10me Leçon.)

[2] Concerning his later views on this question, cf. pp. 82–83.

even by the strictest orders of the authorities; for the holders of the goods for which only thanks would be received in payment would, as a rule, prefer to retain them for themselves.

Nevertheless, Walras' theory, as generally understood, and even as applied by himself, is undoubtedly wrong; and it is the more incomprehensible that he should have propounded it, since he himself had proved a few pages earlier that, in the exchange of two commodities, *many* equilibrium positions are possible. In the sense in which the word is here used, all of these cannot simultaneously represent positions of maximum satisfaction. What distinguishes prices fixed by free competition from all other prices, the thing which finds a mathematical expression in Walras' formulæ, is simply and solely this: that, under competition, each of the exchanging parties can and does go on exchanging up to the point of what we have called *relative* satiety—relative, that is, to the existing system of prices—so that *at those prices* none of them wishes to exchange any more. But this cannot be the case where, for example, by decree of the authorities, some other uniform price system is established in the market—which was formerly very common. There will then always be persons who, on ceasing to exchange, have not yet reached the point of satiety, though at these prices they would gladly exchange *more* of their own goods for a corresponding amount of other goods, if only these could be obtained at the established price; and what is more—they might even be inclined to lower the price of their own commodity or to offer higher prices for the commodities they desire, if this were not forbidden by the authorities. Further reflection shows that this must occur to all those who are so favoured by the official regulation that they obtain a higher price than they would have obtained under free competition. On the other hand, those who are handicapped by the prescribed prices, in so far as they might have obtained better prices under free competition, will continue to exchange to the point of satiety. However, if the owners of goods who are favoured by the prescribed prices are obliged to discontinue selling their goods sooner than they would wish, because they can no longer find purchasers, there is nothing to prevent them receiving in payment a larger quantity of other goods than they would have received under free competition, even though, under competition, they would have found

made on the distribution of property, or at any rate on the effects of this distribution.

The theoretical aspect of this somewhat difficult problem will be made clearer if we begin by taking a concrete example; for which purpose we will select the commodity "labour" and its corresponding price "wages". We assume that the supply, demand, and price of labour have hitherto been determined by free competition, and that the average working day has been fixed at 10 hours and the average wage at 1*s.* 8*d.* per hour. Even if this equilibrium position were the *only* one and therefore necessarily stable, so that a fortuitous rise in wages would cause the supply of labour to exceed the demand, and so on, we may assume that the workers by means of their organizations, or the help of legislation, succeed in forcing a reduction of working hours by half an hour to 9½ hours per day. This will inevitably have the same effect on the market as a diminished supply of labour,[1] and will result in a rise in wages *per hour.* If time-wages rise more rapidly than working hours are shortened, for example 1½*d.* 2*d.* or 2½*d.* (which is conceivable, though not very probable), then it is clear that the workers would reap a distinct advantage from the change. If, on the other hand, the rise in wages stopped at 1*d.*, or even ½*d.* per hour, it might at first sight be thought that the workers would lose by the change—for their daily wages would fall to 16*s.* 7½*d.* or 16*s.* 3*d.* instead of 16*d.* 8*d.* Here it should be remarked, however, that if the original working day, as we suppose, was established under free competition, then the labour and inconvenience of the last half-hour must have approximately corresponded to the wages offered for it, i.e. 10*d.* If not, it is difficult to see why, at that wage, the worker did not voluntarily prolong his working day. We may, therefore, assume that the half-hour of leisure gained for the worker has a value of about 10*d.* (in any case it has at least the money-value which the worker, by virtue of reduced muscular exertion, saves on his daily expenses). The slight reduction in his daily wages is therefore more than compensated by the increase of leisure time; in other words, the increase in wages of 9½*d.*, or 4¾*d.* respectively which the worker now obtains for his 9½ hours' work per day is to be regarded for him as a *pure net gain.*

As may be seen, this reasoning is general. There is no doubt that sellers of any commodity whatever can, by common agreement, obtain an economic advantage; but it should be noted

[1] As it is only our intention here to illustrate a theoretical principle, we ignore the otherwise important circumstance that shorter hours of labour usually give rise to a greater or less increase in the efficiency of labour.

that we can only definitely assert this on the two assumptions we have made: that the previous price relations are determined under free competition, and that the new price or supply does not vary too much from the old. Otherwise, we cannot always assume that the quantity of goods (in this case increased leisure) which the seller himself retains as a result of a decreased supply (or in consequence of higher prices, if this was a primary cause) has for him even approximately the same value as their price.

On the other hand, to what extent this undoubted gain for one class of society is a gain for society as a whole naturally depends upon whether it is greater than the loss which falls upon other classes of society—in this case primarily the employers, and through them the consumers; and, in the last resort, the other factors of production: land and capital. For them also marginal utility and price are equal under free competition, and their net loss is therefore simply the higher price which they must now pay for the labour which they demand. They lose, in other words, exactly as much in exchange value as the workers gain, and the only question is whether a penny or two *more* per day in the hands of the workers is of greater advantage than a penny or two in those of the propertied classes—a question which must certainly be answered in the *negative*, if we are to maintain the dogma of the unqualified social utility of free competition. The further objection which might be made, that a decreased profit in the hands of employers would lead to a decrease in capital accumulation, and would thereby indirectly injure the workers, will be examined at a later stage.

Treated generally, in algebraic form, the problem presents itself in the following manner. Let $\phi(x, y)$ be the total utility which one of the parties to the exchange, who originally possessed the quantity b of the commodity (B) can count upon after a completed exchange; it is expressed as a function of the quantity acquired, x, of the commodity (A) and of the quantity y of (B) disposed of; or respectively of the quantity $(b - y)$ of the commodity (B) retained. The price p of the latter commodity we suppose to be expressed in terms of (A), so that $x = p . y$.

A slight change, Δp in the price p would thus produce the corresponding changes Δx and Δy in the quantities x and y exchanged, these being connected by the relation $\Delta x = y . \Delta p + p . \Delta y$ in which Δx and Δy evidently have opposite signs. As an expression of the change which the total utility undergoes we obtain

$$\Delta\phi = \frac{\partial\phi}{\partial x}\Delta x + \frac{\partial\phi}{\partial y}\Delta y = \frac{\partial\phi}{\partial x}(p\Delta y + y\Delta p) + \frac{\partial\phi}{\partial y}\Delta y.$$

But in consequence of the fundamental condition of free exchange we obtain :—

$$p\frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial(b-y)} = -\frac{\partial\phi}{\partial y}$$

in which ϕ is, of course, a function which diminishes with respect to y. The above formula may therefore be simplified to

$$\Delta\phi = \frac{\partial\phi}{\partial x}.y.\Delta p$$

which indicates that, with a sufficiently small change in price, the seller obtains practically the whole of the increase in price (of his own commodity) as a net gain.[1] If we now add the analogous expressions for all parties to the exchange and count the quantities of (A) sold (and consequently the quantities of (B) acquired) as *negative*, we obtain

$$\Delta p.\Sigma(\frac{\partial\phi}{\partial x}.y),$$

in which by the summation sign we understand a summation of the bracketed expression for each of the indices 1, 2, 3, etc., so that the $\frac{\partial\phi}{\partial x}$ with the appropriate index indicates the marginal utility of (A) after exchange to each of the exchanging parties taken in order. The sum in question is evidently independent of Δp and in general is *not* equal to zero. As we can give Δp either a positive or a negative value, the whole expression can always be made positive—which proves that in normal cases there can always be found a system of uniform prices at which exchanges will produce a larger sum of utility than at competitive prices.

If, on the other hand, after exchange is completed, the marginal utility of one commodity (and consequently also of the other) were the same for all the parties to the exchange, then the above expression can be reduced to

$$\Delta\phi = \Delta p.\frac{\partial\phi}{\partial x}.\Sigma y,$$

and this is always zero, since Σy, the algebraic sum of the total quantities of the commodity (B) disposed of or acquired by the

[1] y is the quantity of his own commodity (B) which he originally sells; $y.\Delta p$ is consequently the additional quantity of the commodity (A) which he would obtain as a result of the increase in price if he could continue to sell the same quantity y of his own commodity; $\frac{\partial\phi}{\partial x}$ is the marginal utility of (A) and hence $\frac{\partial\phi}{\partial x}.y.\Delta p$ is the gain in utility derived from the increase in (A).

parties to the exchange, must be equal to nothing. This condition of equal marginal utilities implies—approximately, but not exactly—a position of economic equality between persons; and in that case—though not otherwise—free competition would secure a maximum satisfaction to all parties to the exchange.[1]

There is no need to emphasize the fact that an encroachment on free competition, if it is to yield the above result, must be effected in *the right direction*. Unrestricted liberty is in general infinitely to be preferred to a misguided system of restriction and compulsion. In so far as the government of a country is

[1] As an example of how even an experienced mathematician may be led to erroneous conclusions in this field, we may mention the argument of Launhardt (*Mathematische Begründung der Volkswirtschaftslehre*). He assumes two parties to an exchange, one of whom from the beginning possesses a units of the commodity (A) and the other b units of the commodity (B) and, for the sake of simplicity, he supposes the total utility derived by each person from the commodity (A) to be expressed by the same function, $f(\;)$; and similarly $\phi(\;)$ for the commodity (B). If they then exchange the quantities x and y the total utility received after exchange by both parties together is expressed by $N = f(a - x) + \phi(y) + f(x) + \phi(b - y)$. In order that this expression should be a maximum we must have :—

$$[-f'(a - x) + f'(x)]\Delta x + [\phi'(y) - \phi'(b - y)]\Delta y = 0 \ldots\ldots\ldots(1)$$

But in equilibrium we have

$$\frac{\phi'(y)}{f'(a - x)} = \frac{\phi'(b - y)}{f'(x)} = p, \text{ also } \frac{\Delta x}{\Delta y} = p$$

where p is the price of (B) in terms of (A). Thus the above equation is satisfied, *and consequently* Launhardt concludes, the equilibrium price determined by free competition is the one which, among all uniform prices, produces the greatest additional utility for the two (or for all) parties to the exchange.

The proof is evidently false. If we desired to discover the absolute maximum of N we should have made x and y independent and would then have obtained

$$f'(x) = f'(a - x) \text{ and } \phi'(y) = \phi'(b - y)$$

These equations are clearly satisfied by the values $x = \frac{a}{2}$ and $y = \frac{b}{2}$; in other words, the parties should simply exchange half their stocks. Since this result is not generally consistent with exchange at a uniform price (and is perhaps outside the possibilities of free exchange) we must impose the condition that *one* of the parties (the one who is at a disadvantage in regard to price) continues to exchange to the point of satiety. We thus obtain the equation

$$\frac{x}{y} = p = \frac{\phi'(y)}{f'(a - x)}$$

By differentiation of this equation and elimination of Δx and Δy with the help of (1) we obtain, according to circumstances, a maximum or a minimum of N, but in neither case an exchange at an equilibrium price.

By way of further proof, Launhardt tries to show, by means of an arithmetical example, that a price which would produce the greatest possible gain for either of the parties would, nevertheless, yield to them both a smaller surplus utility than would the equilibrium price. But a close examination will show that this result is due simply to the fact that he has unconsciously gone beyond the right maximum.

based on democratic principles, there is a certain, though not always reliable, guarantee that such measures will be introduced only when they are to the advantage of the vast majority; whereas when commercial and industrial policy are in the hands of a privileged minority there is a strong presumption to the contrary.

It may also be observed that a restriction of free exchange, of freedom to enter into labour agreements and of the right to free disposal of property—either by means of government intervention or by mutual agreement between buyers and sellers, employers and employees, etc.—is nevertheless a retrograde step, in so far as it usually tends to reduce the sum total of the means of satisfaction physically attainable—even if, under certain circumstances, it may lead to a socially more desirable distribution. We shall return to this important and difficult question at a later stage (p. 142 *seq.*).

In a word, free exchange in economics may be compared to the method of "trusting to nature" in medicine—when the doctor really does nothing, but leaves nature to effect its own cure. The term "physiocracy" means precisely this. In a state of perfect health, which corresponds to a system of economic equality, this is certainly the only correct treatment. Even in ill-health it certainly has a great advantage over bad treatment and dubious medicines. On the other hand, it cannot compare with a really scientific treatment which assists nature in a reasonable manner. And, in the last resort, the effects of even the most brilliant cure cannot be compared with those of rational hygiene, which aims at preventing disease and preserving health The application of the first part of the simile should be clear from what has been said; the latter will be elucidated when we come to deal with the social section of political economy.

In his last work, the *Manuel d'économie politique*, as well as in various earlier essays in the *Giornale degli Economisti*, Pareto returned to a detailed consideration of the problem of the "maximum d'ophélimité", as he calls it, which would result from free competition. He defines this maximum as the point or position, from which it is impossible to move while ensuring a gain in utility or *ophélimité* for *all* participators in the market.

With such a definition it is almost self-evident that this so-called maximum obtains under free competition, because *if*,

after an exchange is effected, it were possible by means of a further series of direct or indirect exchanges to produce an additional satisfaction of needs for the participators, then to that extent such a continued exchange would doubtless take place, and the original position could not be one of final equilibrium. The same would also be true of production. As soon as a change in production is more profitable *both* for producers and for their customers—or, from one point of view, for *all* owners of the means of production, workers, landowners and capitalists—then it is difficult to understand why, assuming general mobility, it should not happen. But this is not to say that the result of production and exchange under free competition will be satisfactory from a social point of view or will, even approximately, produce the greatest possible social advantage.

Hence, even in this new guise, Pareto's doctrine contributes nothing. And—what is worse—it tends to obscure the fact, which we have already pointed out and which we shall develop, that social production under free competition (with certain reservations) does really lead to a maximization, in the usual and proper sense, of the means of satisfying human wants. In this respect, therefore, and of course disregarding the distribution of the product, it achieves as much, or almost as much, as we can imagine under rationally organized production in a collectivist society.

6. *Pricing under Imperfect Competition*

A. *Joint Supply and Joint Demand*

We must now give an account of the principal cases in which perfect competition between the holders of a particular commodity does not exist, either because of natural circumstances or legislative regulation; and of the effect on pricing of such restrictions. We may begin with the case already mentioned, in which two commodities are bound together, either on the demand side (where the consumption of a certain quantity of one is a necessary condition for the consumption of a certain quantity of the other); or on the supply side (where the technical conditions of production are such that the one must always be produced simultaneously with the other in more or less definite proportions). The former, which Marshall called *joint demand*, may, however, without difficulty be treated as a special

case of the laws governing market prices which we have already formulated; and may, therefore, be passed over. Well-known examples of such a demand occur in the case of commodities dependent on each other either in consumption or individual production, such as nails and wire; knives and forks; lamps, wick, and oil; ink, pens, and paper, etc. Because of this relation, the consumption of ink depends in a much higher degree on the price of writing paper and postage than on the actual price of ink—and so on. Actually, as we have already observed, nearly all demand is joint in the sense that different commodities affect each other and are therefore, to some extent, mutually conditioned. That they should be demanded in absolutely fixed proportions may be regarded as a special case, which is of minor importance.

The second group of phenomena, which has been called (also by Marshall) *joint supply*, really belongs to the theory of production, and the regulation of exchange values under the influence of production, which we have still to describe. But it seems to be desirable to touch upon this question here because the related phenomena have been taken by some economists as a pretext for an attack on the whole classical theory of exchange—not so much with the object of criticizing it in the manner we have done in the preceding pages, but of replacing it by a very peculiar theory of pricing, which has never been very clearly formulated. Thus, the series of supposedly new price categories, which F. Neumann set up in his articles on value and price, in Schönberg's *Handbuch*, are really nothing but various examples of joint supply. If, before the advent of lifts, town flats commanded a lower price the higher up they were, then according to Neumann this would constitute an exception to the principle that prices must correspond to costs of production. Costs of production, he says, are higher for the upper storeys since, in building them, the material must be carried to a greater height, and the weight of these storeys renders it necessary to make the supporting walls thicker than would otherwise be the case. But the obvious explanation is that, in addition to floors, walls, and ceiling, a house must have land on which to stand and a roof to cover it—of which the former, particularly, is usually very expensive to buy (or, as in England, to lease). These costs, or the interest on them, must be distributed over the rent of all the flats and it is not possible

to determine *a priori* by what principle this should be done. As we have already indicated in an analogous case, the rent of the different flats is simply regulated by demand, that is to say, mainly by their respective comfort and suitability for various purposes; or, in the last resort, by their marginal utility. All that really matters is that the total rent should be sufficient to pay interest on all the costs of building, including the cost of the site. The high cost of building sites in towns has led, as is well known, to the erection in recent times of lofty steel and glass structures on the model of the American skyscrapers; otherwise all buildings would presumably be erected only one or at most two storeys high—as in country districts. It is the same with all other examples adduced by Neumann. As an example of "joint price", he describes how the shares in the cost, which are borne by the participants in a common drainage scheme, are not proportional to the actual cost of cutting the ditch through their respective plots of land. This is true enough up to a point, but it is entirely due to the fact that the latter costs cannot be ascertained or imputed, for the ditch might have had exactly the same length, breadth, and depth, whether one or more of the interested parties had participated in the enterprise or not. If, on the other hand, the individual costs can be ascertained—if, for example, in order to satisfy the wishes of some particular landowner, it is necessary to follow an otherwise unnecessarily circuitous route in the construction of the ditch, or if the enterprise is involved in other special costs which would not otherwise have arisen—then it is clear that these would usually have to be defrayed by those who cause them. Usually, however, such an imputation of costs is impossible, and in that case there is no other way out than to see that the total costs of construction correspond to the total contributions and to distribute the latter equitably. The generally accepted principle (for example, that of the Swedish Ditching Law of 1879) that each shall contribute in proportion to the objective utility, i.e. the increase in yield or rent which the enterprise brings to him, is by no means the only conceivable one—or even the best or most reconcilable with economy and justice. If, for example, one of four interested parties has gained a capital value of £1,000 and the three others only £100 each, whilst the total cost of the enterprise was £500, then the first would gain more than any of the

others—more than all of them combined—if he paid the whole cost himself and the others did not contribute a farthing.

In this case—unlike the preceding one—there is no automatic economic law of price formation ; for it is really a case of *isolated* exchange. Nevertheless the discussion which springs from such a price-problem is full of interest. An analogous case of the widest implications is presented in a field which may at first sight seem far removed, namely, in the theory of equity in *taxation.*

B. Pricing in Retail Trades

Retail prices are frequently regarded as exceptions both to the law of costs and generally to every rational process of price formation, which is all the more remarkable since these prices are the only ones which are of direct interest to the consumer and which are directly influenced by consumption. Yet the laws of retail prices are perhaps not so difficult to ascertain and do not seem, in the main, to depend on any other factors than those which we have already treated, except that they are more complex and more difficult to unravel. To a considerable extent, the apparent divergence of retail prices from the law of costs and from wholesale prices is to be regarded as an example of the phenomenon of *joint supply*—which we have just considered. Unlike the wholesaler, whose general costs for his whole business constitute only a small part of his annual turnover, the retailer's general costs for premises, heating, lighting, advertisement, wages for his assistants and for his own labour, etc., are very considerable. The first item in particular assumes large proportions since, for the convenience of his customers and for purposes of advertisement he must seek to acquire business premises which are as central as possible. What proportion of these general costs shall be apportioned to each parcel of goods, over and above the purchase or wholesale price, cannot be determined *a priori,* but depends upon a number of variable circumstances. It is of great importance in this connection that certain kinds of goods require much more *expert knowledge* for their valuation than others ; the latter, such as sugar, flour, etc., the quality of which anybody can easily judge, yield, if I am not mistaken, a comparatively small profit.

With the former goods, on the other hand, the buyer, if he is not exceptional in possessing such knowledge, will, in order not to be sold inferior goods, deal with a seller in whom he has confidence. The service which the retailer thus renders him is that of an expert buyer, and the customer quite reasonably has to pay him a relatively higher price.

The desire for stable retail prices must also be taken into account. For many customers it is of great importance to be able to determine their household expenses well in advance. Retailers, who usually have a fixed circle of customers, therefore endeavour to afford this advantage of approximately fixed prices, which they calculate so that the profit and loss of good and bad times to some extent cancel out. Naturally, greater and more permanent variations in wholesale prices are ultimately reflected in retail prices—though, as a rule, later and in a modified form—just as a thermometer buried deep in the ground responds slowly to changes of temperature on the surface.[1]

In conclusion, we should not forget that practically every retailer possesses, within his immediate circle, what we may call an actual sales *monopoly*, even if, as we shall soon see, it is based only on the ignorance and lack of organization of the buyers. He cannot, of course, like a true monopolist, raise prices at will—only in places remote from trade centres can a considerable local rise in prices occur—but if he maintains the same prices and qualities as his competitors, he can almost always count upon his immediate neighbourhood for customers. The result is not infrequently an *excess of retailers*, apparently for the convenience, but really to *the injury, of the consumers.* If, for example, two shops of the same kind are situated at

[1] In an essay in *Ekon. Tidskrift*, October, 1908, and also in his work, *Den ekonomiska fordelningen och Kriserna*, Brock has sought to prove that the above conception of the relation between retail and wholesale prices is not correct. Retail prices, in his view, show a strong tendency to follow wholesale prices *upwards*, but very little tendency to follow them *downwards*. The statistics (from America) on which Brock bases this assertion would seem to show merely that of recent years retail prices have, on the whole, risen as compared with wholesale prices; a fact which, owing to the great relative increase of retailers, is in itself probable and is quite in accordance with what we are about to say. As a general doctrine, Brock's view (and that of Lexio and others) is clearly absurd; it would imply that retail prices would diverge more and more from wholesale prices at each cyclical fluctuation—which would lead to absurd consequences. Obviously, we do not attribute any altruistic motives to retailers when we speak of their endeavour to keep prices as steady as possible for their customers' convenience. It is well understood that it is in the interest of every business man to satisfy his customers.

different ends of the same street, it would be natural that their respective markets would meet in the middle of the street. Now if a new shop of the same kind is opened in the middle of the street each of the others will, sooner or later, lose some of its customers to the new shop, since the people living round the middle of the street believe that if they get the same goods at the same price they are saving time and trouble by making their purchases at the nearest shop. In this, however, they are mistaken, for the original shops which have now lost some of their customers without being able to reduce their overhead expenses to a corresponding degree, will gradually be compelled to raise their prices—and the same applies to the new competitors who have been obliged from the beginning to content themselves with a smaller turnover. This should explain the observation which is said to have been made on the abolition of the *octroi* —the tax on the entry of goods into a town, common on the continent—that the expected reduction in prices never took place, though the number of retailers considerably increased. The correct remedy, unless one of the competitors (such as a great store) manages to overshadow all the others, is clearly the formation of some form of organization among buyers. But so long as such an association does not exist—and between persons in different positions in life and without more intimate bonds it is extremely difficult to establish—the anomaly must remain that competition may sometimes raise prices instead of always lowering them, as one would expect.

C. Monopoly Prices

A still more pronounced divergence from the formation of prices under free competition is provided by monopoly prices proper. *Monopoly* involves the absence of competition, either *absolute* for a certain class of goods, such as a state fiscal monopoly (of liquor, tobacco, salt, etc.), patents of industrial inventions, etc.; or only *relative*, in a definite geographical area and within certain price limits. Every limitation of supply or of productive power does not necessarily create a monopoly —for in that case every price would, strictly speaking, be a monopoly price, since none but free goods occur in unlimited quantities. The ownership of land, for example, is certainly

the privilege of a more or less limited class, but so long as active competition exists between landowners, this possession is not a monopoly and does not lead to monopoly prices for the product of agriculture, either individual or collective. The difference lies in the fact that a commodity or factor of production, whose supply is limited, but which is not the subject of a real monopoly, is offered *as a whole* at the price it can fetch, or at any rate up to the point at which the owners themselves prefer to retain it for their own use. The monopolist, on the other hand, artificially restricts the available market supplies of the commodity or factor of production in his possession. His supply is not regulated by the coincidence of marginal utility and price. If, indeed, it should happen that he were to offer the whole of his stock of goods or means of production, up to the limit determined by this condition, he might nominally *have* a monopoly, but the price would not be monopolistically determined, but would follow the ordinary laws of supply and demand. His profit would then depend solely upon the natural scarcity of the commodity. Frequently, however, the monopolist's stocks are unlimited—as in the case of a patent the use of which might be extended without special expense to all consumers who would in any way profit by it. But if this is to happen, either some customers must pay *more* than others, or there must be a zero price for all; i.e. the invention would be on the same footing as a free good—which is actually the case when patent rights run out. The high price of patented goods is therefore due exclusively to an *artificial* restriction of output, as Adam Smith remarked.

In exceptional cases, as has been said, competitive prices may prevail under an actual monopoly. Thus the Standard Oil Company of America, which has absorbed practically all the petroleum refineries of the U.S.A., fixes its prices, by measuring the yield of the wells during the preceding days or weeks, at the level at which consumption is expected exactly to equal production. Generally speaking, in a case of this kind, it would often be possible to obtain a larger profit—perhaps a much larger profit—if the price were raised, in spite of the fact that this would reduce consumption. But in that case, the wells already opened would have to be partially closed down, or their contents allowed to run to waste—which would presumably

cause dissatisfaction among the public, and might lead to the intervention of the authorities.

If no such considerations exist, it will be to the advantage of the monopolist to fix his prices so high that he will obtain the maximum net profit. Every rise in price causes, we may assume, a falling off in demand. But so long as the falling off in demand is less than proportionate to the increased profit per unit of the commodity resulting from the higher price, the total net profit (the product of these) will increase. But when the decrease in sales is more than proportionate to the increased profit per unit, any further increase in price will be disadvantageous. The ideal monopoly price is thus to be found precisely at the meeting point of both these tendencies—the point at which demand is reduced in the same proportion as the net profit is increased in consequence of the higher price.

We shall endeavour to represent the position by an arithmetical example in tabular form. Suppose that a monopolized commodity costs the monopolist £2 a unit to manufacture. And assume for the sake of simplicity that the relation between price and sales is such that, with a price of £12, 1,000 will be sold in a unit of time; and that every increase or decrease in price by £1 causes a decrease or increase in sales by exactly 100 units. We may then set out the following table :—

Selling Price. £	*Costs.* £	*Profit per Unit.* £	*Turnover.* *Pieces.*	*Total Net Profit.* £
22	2	20	—	—
20	2	18	200	—
18	2	16	400	3,600
16	2	14	600	6,400
14	2	12	800	8,400
13	2	11	900	9,900
12	2	10	1,000	10,000
11	2	9	1,100	9,900
10	2	8	1,200	9,600
8	2	6	1,400	8,400
6	2	4	1,600	6,400
4	2	2	1,800	3,600
2	2	—	2,000	—

In this case, a price of £12 is, therefore, the most advantageous to the monopolist. He would get less profit if he either raised or lowered the price.

It is easy to represent the fundamental features of monopolistic pricing graphically, or algebraically. It we mark

off the various unit prices, p on the horizontal axis and the corresponding quantities y, sold per unit of time, on the vertical axis, then the locus of these points will generally describe a curve $y = f(p)$. The rectangle $y.p$ represents the gross receipts, and that part which lies to the *right* of a line at the distance a from the vertical axis—where a is the unit cost of production, i.e. $y(p - a)$ represents the net profit.

The expression is maximized when its first derivative with respect to p is zero. We thus obtain

$$(p - a) f'(p) + f(p) = 0,$$

a condition which is satisfied, as will easily be seen, when that part of the tangent to the curve which lies between the above-mentioned vertical line and the horizontal axis is bisected at the point of contact. If $y = f(p)$ is a straight line, as with our figures, we have simply to take *half* the maximum *net* price, where sales will be half the maximum which can be marketed without a loss. Other questions relating to monopoly prices are similarly capable of an easy mathematical solution. Thus, *inter alia*, there can be deduced from these figures, or formulæ, answers to such questions as the various influences of general and special costs, various forms of taxation, etc., considered on p. 72.

It is important to note that the amount of overhead costs (i.e. costs which remain constant whether output is large or small) has no influence whatever upon the level of the most advantageous monopoly price. Whether, for example, a private railway company has to pay a large or a small amount of interest on the capital invested in construction, the height of its charges cannot be affected, so long as these are fixed on the principle of maximum net profit. This is obvious: if, in the table, p. 90, we deduct a fixed amount per unit of time (say £1,000) from the monopolist's net profit, then all the figures in the right-hand column will be reduced by 1,000. Obviously, even after this reduction, the previous maximum profit would still be a maximum; so that the most advantageous selling price would still be exactly £12. It is evident that this would still apply if, for any reason (say income tax), the net profit were reduced in proportion to its size—and even if the deduction (as in the case of progressive income tax) increases more than proportionately to net profit, so long as the rate of progression is such that the residue (after deduction) continues to increase wherever the profit (before deduction) would have increased.

But different considerations apply in the case of prime costs—which increase with the output. For the sake of simplicity we will assume that the increase of costs is exactly proportional, so that every new unit of commodity increases costs by as much as the preceding unit; and so on. If, for any reason, the cost

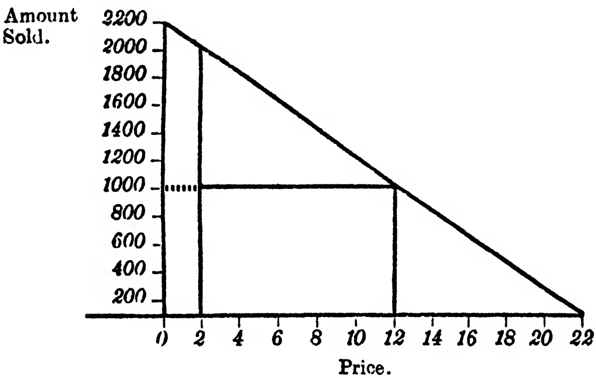


FIG. 5.

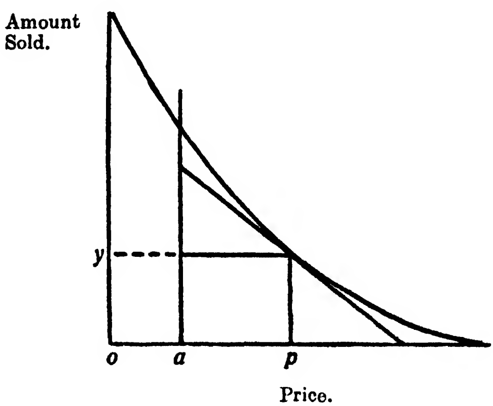


FIG. 6.

of a unit now increases—as for example by reason of a consumption tax, or excise duty on the quantity manufactured or offered for sale—then in our table the net profit per unit will be reduced by the amount of the additional cost, and it is obvious that this will cause the monopolist to raise his price in order to obtain the maximum total profit. The increase will not be as great

as the additional costs of production but usually less. With a simple linear law of demand (on which our table is based), the most advantageous increase in the monopoly price would be exactly half the increased cost per unit, so that if, for example, the increase were £2 and the monopolist's cost of production were thus to become £4 per unit, the best selling price would be £13.

These propositions, which are due originally to Cournot,[1] but have been developed subsequently by Pantaleoni, Marshall, Edgeworth, and others, are of great interest both for the theory of taxation and for the solution of the pressing problem—which is daily becoming more important—of a rational regulation of industrial monopolies, whether legal or merely *de facto*.

The mathematical treatment of monopoly profits and their taxation abounds in interesting and often very surprising features. Suppose, for example, that a railway company which has a monopoly in passenger traffic, with only two classes, second and third, is taxed on the basis of the number of second class tickets sold. Who would suppose, at first sight, that this taxation might make it economically advantageous for the company to *reduce* the price of both second and third class tickets? And yet Edgeworth has fully proved [2] that, on certain assumptions, this can be the case.

This can, if necessary, be understood without the use of higher mathematics. For the sake of simplicity we shall assume —an assumption very far removed from reality—that *ceteris paribus* the number of second class passengers is determined exclusively by the price *difference* between the two classes; in other words, the passengers would travel in any case, though the difference in price decides whether they will travel second or third class. In such a case it is in the interest of the railway company to *increase* this difference in order to force some passengers to go over from second class to third class—and thereby save in taxation. That this can always happen without a corresponding reduction in the total revenue is implied in the very concept of maximization—at least in most cases. A slight change in the most advantageous price combination produces a relatively very small reduction in traffic revenue, whereas the

[1] See *Principles mathématiques de la théorie des richesses.* This work was first published in 1838, but was not generally known until much later. Translations into English and various other languages are now available.

[2] *Papers relating to Political Economy,* vol. i, pp. 143–151, and *Economic Journal,* 1899, p. 286.

corresponding saying in taxation is considerable. Now a given increase in the price difference can be brought about in *three* different ways :—

(*a*) by a moderate increase in second class fares and a reduction in third class fares ;

(*b*) by a greater increase of the former and a slight increase (or, at any rate, no reduction) in the latter ; and

(*c*) by a slight reduction (or, at any rate, no increase) in second class fares and a greater reduction in third class fares.

By all three methods the railway company makes an equal saving in taxation. It remains an open question, therefore, which of the three will produce the least decrease in the traffic revenue. As a rule it would be the first method, but in special cases the second and even the third may be preferred, in that order.

Thus, if second class traffic is very considerable and third class traffic not particularly elastic, it may happen that the most profitable course would be to increase both fares (although, apart from taxation, this increase must always reduce the traffic revenue, since it alters the combination of prices existing before the imposition of the tax, which must be assumed to be, in those circumstances, the most advantageous). But if third class traffic is very elastic—so that reduced fares would attract a number of new passengers (to the third class)—and the second class traffic is not very great, then, however paradoxical it may at first sight appear, the last of the three methods will be the most advantageous to the railway company.

Alternatively, we might approach the problem in the following way. Let us draw up a series of combinations of prices which, apart from taxation, would yield the company *a certain given net income* slightly less than the maximum. Geometrically, this series could be represented by a closed curve (roughly elliptical in shape) enclosing the maximum point ; we have then to find the point on this curve at which the difference between the co-ordinates (the difference between second and third class fares, and consequently the saving in taxation) is a maximum. This point is clearly the point of contact of the upper of the two tangents to the curve which make an angle of 45° with the axes (cf. Fig. 7). The same construction may then be repeated with a succession of new curves (new series of price combinations) the process being continued so long as the saving in taxation increases more than the traffic revenue decreases. If the maximum point is taken as the origin (with the direction of the axes retained) it will easily be seen that the new point of equilibrium may be situated in the

first, second, or third quadrant—but of course never in the fourth—according to the form and position of the curves, of which nothing is previously known.

It must, however, not be overlooked that the study of monopoly is peculiarly liable to be disturbed by great differences between "theory" and "practice"; and that for many reasons.

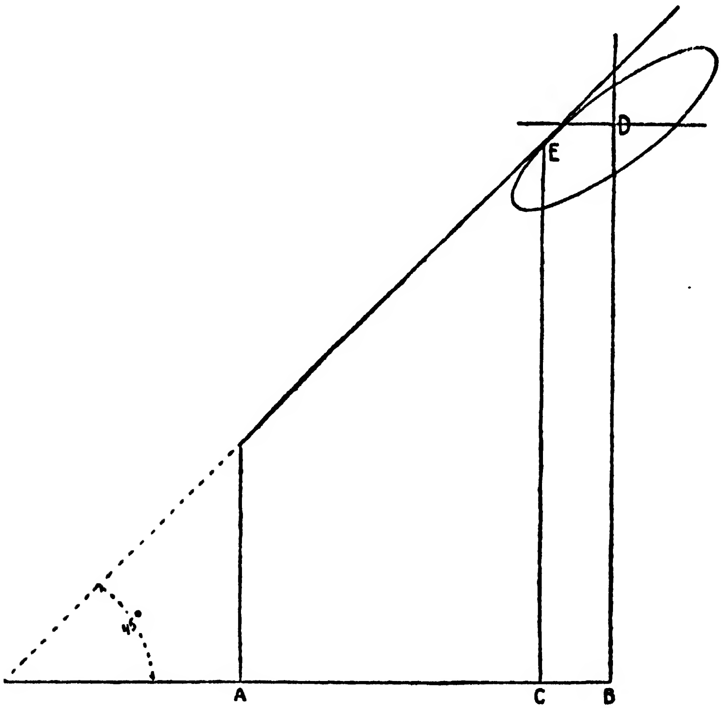


Fig. 7.

AB =	Price of	III	class	tickets	before	taxation.
BD =	„	II	„	„	„	„
AC =	„	III	„	„	after	„
CE =	„	II	„	„	„	„

The monopolist is not obliged to keep so close a watch on prices as a seller or producer working under free competition, especially since most monopolies are in the hands of great companies, or corporations, or States, and are managed by salaried officials who are usually much more anxious to avoid loss by incautious experiments than to increase their profits. Another circumstance,

which should not be overlooked, is that the growth or decline of net profit in the immediate neighbourhood of the theoretically most advantageous selling price is very small. This feature is common to all real maximization, and we may easily convince ourselves of its correctness here by reference to the above table.[1] It is, therefore, largely a matter of indifference to the monopolist whether his price is a little above or a little below that which is theoretically the best—however important the matter may be to the consumer.

Finally, it may be pointed out that the sharp distinction between monopoly prices and competitive prices which we (in common with other economists) have drawn here scarcely ever exists in reality. Not infrequently, two or more monopolists in the same branch of production, or in closely-related branches (e.g. owners of various patents in the same industry) actually compete with each other.[2] We have already pointed out that there also exists in the ordinary free competitive market a sort of monopoly for each individual producer, and even for every consumer—dependent upon their various *geographical positions* relatively to each other and to the centres of business activity, with consequently differing *transport costs.* But economic theory has paid very little attention to this aspect of the problem of pricing.[3]

If there are two equally powerful monopolists in the same branch of production then, *if they operate independently*, they will doubtless depress prices, but, as Cournot observes, *only up to a certain limit*—namely, the point at which each obtains the maximum profit, under the assumption that the other neither increases nor decreases his output beyond that limit. This new equilibrium position can be determined without difficulty, if a is the cost of production, by the equation

$$2(p - a) . f'(p) + f(p) = 0,$$

where p is the common selling price and $f(p)$ the *combined* sales of the two monopolists. The tangent referred to above (Fig. 7) will be divided at a point *one-third* of the way along it, and in our table (p. 90) the selling price would be reduced to £$(2 + \frac{1}{3} \times 20)$

[1] Cf. also my *Finanztheoretische Untersuchungen*, p.12, *et seq.*

[2] The theory of pricing under "duopoly" or "polypoly", as they were formerly called, was developed by Cournot (see below) and deserves attention.

[3] A. Weber's *Der Standort der Industrie* may be described as such an attempt.

= £8.67 a unit with a total sale of 1,333 units—or 666 to 667 for each monopolist. In the same way, if there are three or more monopolists, the price will fall further, until it finally sinks to the bare cost of production ($p = a$) as in free competition. The public will, therefore, gain by the competition of the monopolists, but the monopolists will lose. Their own interests compel them to combine and divide the profits—in which case monopoly prices and sales will again be the same as when there is a single monopolist.[1]

7. *Pricing under the Influence of Production*

Transition to Part III

Although hitherto our purpose has been to describe the origin of market prices, on the assumption that goods exist in given quantities for a certain consumption period, yet we have on several occasions touched upon the effects of *production* on pricing; or rather on their influence on one another. We shall now concern ourselves directly with this problem, and shall consider it in detail in the next section. The older economists drew a distinction between market price, regulated solely by demand and supply, and "natural price", about which the market price always oscillated, and which is itself determined by the cost of production of the commodity. In actual fact, the formation of prices is essentially the same in both cases, except that the relation between supply and demand, effective on the market, is replaced in the latter case by the relation between production and consumption. If price equilibrium in the market demands equality of supply and demand, then in the long run the prices of the various commodities will be stationary at, or

[1] Edgeworth, in his *Mathematical Psychics* (1885) and in an essay in the *Giornale degli Economisti*, 1897 (and also the mathematician, Bertrand, in the *Journal des Savants*, 1883), criticized Cournot's reasoning, but, in my opinion, on insufficient grounds. It is certainly true that the problem, as Edgeworth says, will to some extent be indeterminate in the case of two, or generally of a limited number of monopolists, whether in the same or in different branches of production. But Cournot's further assumption, quoted above, seems to me much more reasonable than the one selected by Bertrand and Edgeworth. The latter involves the assumption that each monopolist aims at the maximum net profit on condition that the other does not change his price—an assumption which seems to me quite unjustifiable where they both produce the same commodity. [See Wicksell's review (*Economisk Tidskrift,* 1925) of Professor A. L. Bowley's *Mathematical Groundwork of Economics*; a German translation of this review subsequently appeared in the *Archiv für Sozialwissenschaft,* 1927.]

oscillate about, the point of equilibrium between production and consumption—in other words the point where *production exactly covers consumption.* We may add, in passing, that this simple relation is all too often overlooked as, for example, when we speak of a permanent over-production or under-consumption of some, or even all, commodities. If this means that production permanently exceeds consumption—and what else can it mean? —then it is manifestly absurd. After all, the capacity of our warehouses is limited!

If it were true that the manufacture of a commodity always required a certain definite quantity of each factor of production (i.e. a certain quantity of homogeneous labour, a certain area of land of given physical properties and finally a certain use, and corresponding using-up of capital goods—factories, railway material, ships, tools, machinery, etc.), and that production did not require any *time* (or, more correctly, that the time actually required need only be regarded, economically speaking, as *quantities* of services of labour and land, which could just as well be supposed to be applied simultaneously as successively) then we should have every reason to agree with Walras' assertion that the determination of prices, taking production into account, constitutes essentially the same problem as the formation of prices in the market; or is, as it were, only a variant of it. Anyone who demands a given quantity of a given commodity will *implicitly* demand a given determinate amount of each of the factors required for the production of that commodity. On the other hand, each owner of these factors—the labourer, the landowner, and the capitalist—offers a certain quantity, the amount of which depends *ceteris paribus* partly on the market price (i.e. on the rate of wages, rent and interest, etc.) and partly on the prices of the goods which the owners of the factors wish to acquire in return. Or, in accordance with what we have already said, we may regard the problem from a somewhat different point of view: the owner of a factor of production has himself a certain direct use for it, so that what he wishes to retain for himself may be regarded as his contribution to the general demand for that factor. The supply must then be regarded not as the amount which he and other owners offer, but as the whole quantity in existence—for example, in the case of labour, the whole twenty-four hours of the day—which

in extreme cases might find productive employment. If we start from a hypothetically given system of prices of all the factors of production, then, in the first place, we can on our assumption deduce the corresponding prices of the finished goods (if we regard their costs and selling prices as equal). For every such system of prices we can then obtain, directly or indirectly, a determinate demand for and supply of each particular factor; and it only remains to state that, in equilibrium, demand and supply must coincide, or—if we take the word demand in its wider sense as including the quantity which the owners of the factors wish to consume directly at the given price—that demand exactly equals the quantity available.

Working under this assumption, we should actually have to deal with *two* factors of production only, land and labour, since machinery and other capital goods can ultimately be reduced to products of land and labour. If time did not play any economic rôle, the employment of, and demand for, capital could be regarded as an indirect demand for labour and land. But it is precisely at this point that the weakness of the argument appears; for, since the indirect productive services must be rewarded in the same way as the direct, the share of capital in production would consist only of successive repayments of the capital itself, and not of any addition in the form of interest. This agrees with the Socialist view, according to which the remuneration of capital consists exclusively of "unpaid labour"; i.e. is an economically unjustifiable robbery of the fruits of production. We must either accept this view—which, however, Walras and his school refuse to do—or we must admit that the reasoning which leads to this result (which really ignores the existence of interest) overlooks an important element in the explanation of the phenomena of the real world.

This view of the position is evidently far too imperfect to be even an approximation to reality. In the first place, the proportions in which the various factors of production contribute to the manufacture of any commodity are by no means given or determinate, but may vary within certain (sometimes wide) limits; or, as it is sometimes expressed, *one factor of production can always, to some extent, be substituted for another.* This is particularly true of the production of foodstuffs, which are obtained, in a fairly uniform quality, either by extremely extensive agriculture (for example in the "robbery cultivation"

—rightly or wrongly so called—of the Western States of America or in the practice, common in Sweden, of burning off woodland in order to secure arable land) or else by a highly developed intensive cultivation as in China, Belgium, and the plains of Lombardy. But, even in manufacturing industry, the various factors of production, such as human labour and machinery, may be substituted for each other to almost any extent. That is to say, direct human labour is replaced by natural forces (in combination with the employment of capital) and *vice versa*. A further factor, which at bottom has a close connection with the above, is that the time-element in production, so far from being a matter of indifference from the economic point of view, is of the very greatest importance. We cannot—at least in the last analysis—conceive the commodity market, on the one hand, and the market for factors of production or productive services, on the other, as lying alongside one another, so that they could theoretically be regarded as one. In point of *time* the latter always *precedes* the former, and this circumstance—as we can easily understand *a priori*, and as we shall show in more detail soon—is of the greatest importance in actual pricing. Before we can hope for a final solution of the pricing problem we must first consider both sides of it more carefully: the ability of the different factors of production to replace each other, and the time-element —or, what amounts to the same thing, the economic significance of capital. We shall consider these matters in the next part and shall, at the same time, endeavour to solve the problem of *distribution* under free competition—a problem which would already be solved if the shares of labour, land, and capital could be determined as simply as has been indicated above. That such is not the case, and that the time-element plays a decisive part in distribution, and especially in the determination of wages, was what John Stuart Mill wished to express by his statement, "Demand for commodities is *not* demand for labour"—a statement which, though fundamentally correct, has been widely challenged and frequently misunderstood.

PART II

THE THEORY OF PRODUCTION AND DISTRIBUTION

Bibliography.—There still exists no exhaustive presentation of this subject on modern lines; at least, not in an elementary form. Walras in his *Éléments* once and for all correctly formulated the solution to the problems of production, distribution, and exchange as a whole, but his treatment of the economic function of capital is hardly satisfactory. Böhm-Bawerk, on the other hand, whose work *Kapital und Kapitalzins*[1]—and especially its latter part, *Positive Theorie des Kapitals*[2]—is the chief source for the modern theory of capital, did not concern himself with the synthetic treatment of the problem of production and distribution as a whole. An attempt to combine the work of both these writers into a single whole is to be found in my essay, *Über Wert, Kapital und Rente*; and also in the elegant but unfortunately unfinished articles of Enrico Barone, "Studi sulla Distribuzione" (*Giornale degli Economisti*, 1896). P. H. Wicksteed's succinct *Co-ordination of the Laws of Distribution*[3, 4] (London, 1894) is interesting and rich in ideas—but not easy to read. Jevons' *Theory of Political Economy* contains many instructive, though scattered, remarks on production. The most exhaustive treatment of the subject in English, from the modern point of view, is to be found in Marshall's *Principles of Economics*, an abridgment of which was published under the title *Elements of the Economics of Industry*.

An original writer, unfortunately to a large extent self-taught, is the German, Effertz, who in several works (of which the earliest is contemporary with the *Positive Theorie des Kapitals*) develops views similar to those of Böhm-Bawerk; they are often very well stated.

We have hitherto examined, as far as it has been possible to do, the process of valuation of the material objects or direct

[1] [*Capital and Interest.*]

[2] [*Positive Theory of Capital.*]

[3] [This is now published in the series of Scarce Tracts, published by the London School of Economics.]

[4] In his *magnum opus*, *The Common Sense of Political Economy*, he declared —for reasons difficult to understand—that he desired to withdraw this work. He devotes a chapter to the subject in the *Common Sense* which does not cover the same ground.

personal services with which we satisfy our needs. We shall now consider how the available stocks of goods (and, strictly speaking, personal services also, in so far as the supply of services presupposes a supply of consumable goods) are maintained, renewed, and replaced. In other words, we shall now consider *production.*

As has already been indicated, the problem of value and exchange cannot be finally solved unless attention is simultaneously paid to production. Production, on the other hand, as it actually takes place, cannot be understood except in association with the laws of exchange and exchange value. In reality, exchange, and consequently valuation, enter into *all* production. Even in an individual's production with his own resources for his own needs there is always, at least in the wider sense of the word, an exchange (or choice); the resources can be used *either* in direct consumption *or* in indirect consumption—through the medium of production. Thus, for example, anyone who has labour available, so long as he is a free human being, has the choice of using his working hours either for rest or diversion, or for productive employment in the ordinary sense. The element of exchange naturally appears even more clearly in production which is carried on in association with outside labour or other factors; or when the product is intended for consumption by others, as is the case nowadays with the vast majority of goods produced. In the former case, there is, of course, a direct exchange of factors of production—land, labour, and capital—against their necessary remuneration—wages, rent, and interest. In the latter case, production proceeds with constant reference not only to the *volume* of the output which can be obtained, but also to the exchange *value* anticipated or already determined on the market. In the majority of practical cases, both of these considerations are present.

Production and exchange can only be separated by a process of abstraction; but such abstraction is an invaluable aid in the survey and examination of what at first sight appear to be hopelessly complicated phenomena. For this reason, we have hitherto assumed, in our examination of the principles governing market values, that the supplies in the market to meet the needs of consumers in a given period are given in advance; although, naturally, these supplies are continuously affected in reality

by new production—especially in modern times with highly developed communications. In the same way we can, and shall for a while, in our treatment of production and distribution, ignore the *changes in the exchange value of goods* which are constantly brought about by relative changes in production and consumption. In other words, we assume, in the first instance, that for the society in question these exchange values are given—as they approximately are in reality for every individual producer, in his relation to the market as a whole. A concrete case of this kind would arise if a country or some smaller area produced only one or a very few staple commodities and imported everything else it required; so that all exchange values could be assumed to be determined in advance by the market of some larger area, or even the world market.

For a first approximation, we may also introduce another important simplification. As we have already said, every owner of a factor of production can choose between two methods of employing it: directly or in the service of production. Even if the relative exchange values of goods are given in advance, the need will constantly arise for the individual to weigh up against one another, on the one hand, the goods which he obtains, or can obtain, in return for his productive services and, on the other hand, the enjoyment he obtains from being able to dispose of them freely on his own account; as, for example, by having more leisure. We shall, however, assume for the present that the utility of the various factors of production, after a certain amount has been set aside for the owner's direct consumption, becomes so insignificant for this purpose that it need not be taken into account in comparison with the indirect utility derived from their productive employment. And this assumption may be made without danger in the case of several factors of production. Private owners of building sites in cities do not usually leave any part unoccupied in order to retain it as a promenade ground. No landowner—unless he were a very exceptional person—would allow arable land to lie waste or would use it as a hunting ground. Still less has the owner of capital any choice in this respect; in order to obtain any yield from his capital he must employ it productively or, what generally amounts to the same thing, lend it to someone else. The personal, unproductive use of capital would almost necessarily be

tantamount to its partial destruction. Dwelling-houses occupied by the owner constitute no exception to this rule, for the only possible productive use for such capital goods is that they should be occupied as dwellings.

Hence it is approximately true of land and capital—that is to say, of the capital existing at any given moment of time—that they enter as a whole into production. On the other hand, we cannot reasonably say the same thing about labour. It is a physical impossibility to work regularly for the whole twenty-four hours of the day, and even if working hours were limited to the maximum time which *can* be devoted to work in the long run, the labourer's position would still be so miserable that only the most acute necessity would keep him from converting a little of his working time to leisure purposes. To the older economists, who generally held that the natural and average wages of labour exactly corresponded to the minimum of subsistence of the labourer and his family, it was natural to regard individual labour and hours of labour as a fixed and definite quantity, the limits of which were set only by the physical powers of the labourer. It is characteristic that when Adam Smith discusses the problem whether labourers are likely to respond to a rise in wages, by devoting more time to leisure he only does so in order to absolve them from this charge. Nowadays, when wages have fortunately risen somewhat above the subsistence level and when the limitation of working hours in order to give the worker an opportunity for educational and cultural activities has become one of the most eagerly sought objectives, especially on the part of the workers, this assumption. is no longer permissible. Our use of it here will be only provisional, in order to simplify the argument. We must also remember that, in certain occupations (particularly the manufacturing industries), the amount of time devoted to production (especially the length of the working day) is largely determined independently of the individual worker, by collective agreements—which may be denounced collectively, but not individually, excepting in so far as an individual may occasionally "take a day off".

We also ignore here the practically very important circumstance that the mental and physical health and strength of the worker, and consequently the efficiency of labour, are

largely dependent on the wages received and, within certain limits, rise and fall with the wage.

Changes in the supply of labour due to movements of population—natural increase, emigration, immigration—are quite different in kind from these and may be disregarded here. For the most part, they are due to other than purely economic causes and only rarely do they cause the supply of labour available at a given moment, or in the near future, either to increase or decrease.

In the long run, of course, not only the total supply of labour, but also that of capital, and indeed of land also—or at any rate the available supply—will be subject to more or less extensive changes. The same is also true of labour on the qualitative side, in so far as changes in the manner of living, improved education, and upbringing may cause considerable changes in the efficiency of the available supply of labour. In a complete analysis of economic phenomena, these changes must of course be duly noted; for the moment, however, we shall content ourselves with what has been called the *static* aspect of the problem of equilibrium, i.e. the conditions necessary for the maintenance, or the periodic renewal, of a *stationary state of economic relations*.

If the country or area which was mentioned above were a unified economic unit, in which everything was produced and exchanged with the outside world on common account, the whole problem of production would be a purely technical one. Given the supply of factors, it would merely be a question of maximizing the production of the particular commodity produced by the country. If several commodities were produced—all of which were, in some measure, sold abroad at given prices—the object would be to maximize *exchange value*. Again, the *distribution*, whether of the direct output or of its equivalent obtained by exchange, would be an independent question and would be regulated by other than purely economic considerations.

The problem is different, at least at first sight, when production proceeds, as it does in reality, under free competition and private enterprise. In this case it is everyone's business to produce, not as much as possible, but as *cheaply* as possible, i.e. in such a way as to maximize his net profit. This again depends upon his costs of production or, in other words, on the

share of the product demanded by the factors of production. It is therefore bound up with the problem of distribution. For example, suppose a man has a large landed estate, but no capital. If he were to farm the land without capital—by his own labour and that of his family—then of course the product, relatively to the size of the estate, would be extremely small. He therefore borrows capital and employs labour. But the extent to which he does so obviously depends upon the remuneration demanded by capital and labour in the form of interest and wages. If he can get both for nothing, or for next to nothing, then he will carry on his farming more intensively, using more capital and more labour than he would do if the share in the product demanded by capital and labour were so great that—as a result of the law of diminishing returns, which we shall shortly consider—they gradually absorb the whole surplus and perhaps leave him almost nothing. Rents would have a similar significance to a person who possessed capital, and possibly skill at farming, but had insufficient land to be able to make use of them.

Again, if the producer can choose between the manufacture of various kinds of goods—whose market prices are given, but whose manufacture demands different proportions of land, labour, and capital—then it will be his object to select the branch of production which is most profitable ; and here again the relative levels of rents, wages, and interest will, of course, be decisive. Only when, by the influence of supply and demand, these have reached such a relative position that two or more of these commodities are equally profitable to manufacture, will they be simultaneously produced. In practice, as we have already emphasized, the problems of production and distribution cannot be separated, but are essentially one ; production is not a technical problem only, but technical and economic at the same time.

Another question of great interest—which we propose to examine later—is whether (as has often been maintained by Socialists) collectivist production would, in a physical sense, be superior to individualist production—leaving aside the question of distribution ; or whether we should not, from a technical point of view, regard both systems as leading to essentially the same result.

The agents of production have usually been divided into three main groups—land, labour, and capital—of which the first denotes the external natural forces at the service of man. In a narrower sense, however, "land" may be taken to include only those natural resources which renew themselves continually, for the actual ingredients of land (such as clay, ore, peat, coal, etc.) in so far as they are employed in production and consumption have rather the characteristics of capital. By labour, again, we mean exclusively human labour, whether manual or mental. The concept of capital requires a closer analysis—and we shall return to it later. Further, there exist important factors of production, essentially of an immaterial kind, which cannot well be subsumed under any of these categories, but which are *sui generis*, even though labour, capital (and land) are required for their production. To this class belong technical inventions, so long as they are patented or are trade secrets (otherwise they become free goods) and also—if the term production is taken in the wider sense, to include the distribution and marketing of products—well-known trade marks, the goodwill of a business, and so on. For the sake of simplicity, however, we will keep to the three main groups—especially since all the others, strictly speaking, presuppose a restriction of free competition. In accordance with our usual method we shall postpone discussion of the difficult problem of capital; and shall at first concern ourselves only with land or natural resources—assumed to be in private possession—and human labour; their co-operation in production and their shares in the product, under free competition.

Marshall, in his *Principles*, has endeavoured to set up a fourth class of agents of production, beside land, labour, and capital, namely *organization*, to the important functions of which in the modern mechanism of production he has devoted several long and suggestive chapters of his book. But, however important it may be to determine the economic rôle of intellectual progress and of inventions and discoveries (which earlier economists not infrequently confused with capital itself), this classification suffers from the inconvenience that the new agency thus introduced, unlike the old, lacks *quantitative precision*, except in some special cases. Such a case would arise when organizing talent or technical discovery is incorporated in certain individuals of outstanding

gifts or specialized education. But in that case, "organization" cannot be distinguished from "labour"; it is only a special form of labour, and has always been so treated. Further, if inventions exist, like a treasure of new knowledge and experience which, by their very nature, are accessible to all, then they can only acquire economic significance if they are preserved as trade secrets or are protected by patents, etc.; or unless they have given rise to an actual monopoly for the first user—as happens in certain cases in large-scale manufacture. In the contrary case, they are to be regarded, as we have said, as free goods—such as air, water, sunlight, etc. These enhance the whole of production and, thereby, *ceteris paribus*, raise human well-being to a higher plane, whilst themselves making no claim to a share in the product. They have, therefore, no influence on prices.

It seems to me not altogether impossible that this defect in scientific classification is associated with certain somewhat hasty conclusions of Marshall which we shall discuss later.

1. *Non-Capitalistic Production*

Let us assume, in the first place, that production is non-capitalistic—without implying that there is no capital whatever in existence. As a rule, production without the use of any capital is impossible, though the most primitive form of production—mere collection of wild fruits—is a possible exception. For our purpose it is sufficient to assume that on account of a lack of technical knowledge, very little capital can be employed; but, that it is available in such large quantities relatively to the state of technical knowledge, that, as a first approximation, its share in the product can be ignored. (We shall examine later the exact conditions under which this can happen.) We might assume, for example, that all production—as was probably roughly the case in the earliest agriculture in primitive clearings—is carried through in the course of a single year, during which the few simple tools and utensils employed are also made and completely worn out. For the sake of simplicity we will also assume that finished products only become ready at the end of the year, that all wages are paid at the end of the year, and that the workers maintain themselves during the whole of the succeeding year on their wages so acquired. (It might be argued that they themselves must, therefore, be regarded as a sort of

capitalist class, but on our assumption the advantage thus gained is so small that it need not be taken into consideration.) All agreements between workers and landowners, or between these two and a third party as entrepreneur, are thus based on a division of the product at *the end* of the current production year. On what principles will this distribution take place?

We have here two opposing groups of contracting parties—the owners of labour, and the owners of land—who, on our assumption, are on a footing of equality when making a business agreement between themselves or with a third party. The landowner, it is true, has hands; but he may be unable to use them for labour, owing to old age or from his being unaccustomed to manual work. And, in any case, if the land is considerable in extent, his own work may well be insufficient to produce enough even to repay him for his trouble and to meet the taxes on the land. He is therefore not less dependent on labour than labour on him. Neither are the labourers dependent on any other entrepreneur, since, on our assumption, they are able to maintain themselves during the whole period of production. We may, therefore, assume either that the landowner will hire labourers for a wage, paid, let us say, in kind at the end of the period of production, or that the labourers themselves will hire the land for rent which again will only be paid when the product is completed; or, finally, that a third person, an entrepreneur, hires both labour and land—but still on condition that wages and rent shall only be paid after the completion of production.

In order to prevent any misunderstanding, it may be pointed out that this device is simply a logical construction without any counterpart in reality, either at the present day or at any previous time. On the contrary, it is reasonably certain that individual ownership of *moveable property* (i.e. capital) and the possibility in one form or another, of interest, preceded historically the private ownership of land and, therefore, the possibility of (private) rent. However insignificant the quantities of capital-goods may have been, which could find employment with a primitive technique of production, yet probably capital accumulation and saving were, for many reasons, even less developed. Thus, a superfluity of capital, even a relative superfluity, seldom occurred. On the contrary, there was, as a rule, a marked shortage. The fact that *usury* was forbidden in the Middle Ages did not prevent interest from being taken in some disguised form.

Moreover, loan interest is only one of the many possible forms of interest.

If we revert to modern times, we shall find that nearly every square yard of land in most countries is in private possession (or if in public hands is no longer available for free use), and rents are, on the whole, steadily rising even though they fluctuate. At the same time, however, interest is nowadays probably a greater source of income than rent. Technical inventions, combined with a rapid increase in population, still prevent the rate of interest from falling below a certain amount and this yield has to be multiplied by a quantity of capital which has grown enormously —even in proportion to the simultaneous increase of population.

Nevertheless, the above assumption of production without capital, or rather of production in which capital is to be regarded as a free good, is logically conceivable and is, therefore, an abstraction which is permissible for purposes of exposition—in much the same way as it is permissible in Ricardo's theory of rent, of which we shall shortly speak, to regard cultivation as proceeding from "better" to "worse" land, even although, historically, the development may in many cases have been in the opposite direction.

A. The Landowner as Entrepreneur.

We will first assume that the landowner is the entrepreneur. The conception "landowner" presupposes that all land—or at least the more fertile land and land more favourably situated for trade—is already in private ownership, which is nearly always the case in older countries. But, at the same time, the limit has long been passed within which every new labourer will produce the same additional product, or possibly even, by better organization of labour (i.e. division of labour) a larger product than that produced, on the average, by the labour already employed on the same area of land. So long as this remains the case—even with private ownership of land, and on the assumption of active competition between landowners—there could scarcely be any rent, properly so called, and landowners would only receive a wage for their personal participation in production, for example, as managers of labour. It is quite otherwise where, as is usual in modern society, agriculture and its related industries have already, owing to the growth of population, reached such a degree of intensity of production

that every additional labourer employed on the same area of land can only produce an additional product which is *smaller* than the average.

The fact that the total product of the same area of land increases more slowly than the number of workers employed has been put forward as a law which applies especially to agriculture and the production of raw materials: the law of diminishing yield, or *diminishing returns.* Yet this law is universal in its application as soon as one or more of the factors of production necessary for any particular manufacture is increased beyond a certain limit, while the other factors remain unchanged. That it has been possible to establish a contrary *law of increasing returns*, valid for at least some branches of industry, is entirely due to the implied assumption that the raw materials required are to be found in practically unlimited quantities at an unchanged, or almost unchanged, price. If the same assumption were made with regard to agriculture—in other words, if there were a superabundant supply of the best quality of land—then the law of "increasing", or at any rate of "constant" returns would apply there too.

To claim, as Marshall does, that the former of these two "laws" applies to *nature* and that the latter is characteristic of the contribution of *human labour* to production seems to me to be hardly logical. The two contributions can never be separated altogether, but can only be differentiated at the margin of production, as we shall show later on. The so-called law of increasing returns is, fundamentally, another way of looking at the advantages of large-scale production over small-scale or isolated production, and it applies, in general, to all fields of production, though in varying degrees. The law of diminishing returns is even more universal in its application, as soon as we assume a one-sided increase of *some* of the factors of production only. In a conflict between these tendencies, therefore, "increasing" returns may well prevail for a time, though "diminishing" returns will prevail in the long run.

To the landowner, it can evidently never be economically advantageous to pay an additional labourer *more* in wages than the additional product obtained from employing him. But since there is free competition between labourers, and since (as we assume for the sake of simplicity) one labourer is as good as another, none of the labourers previously engaged can claim higher wages than the last one engaged; for in that case it

would be more advantageous for the landowner to dismiss him and fill his place by the new labourer, who must be satisfied with the lower wage. On the other hand, if there is perfect competition between employers, wages cannot sink materially *below* the amount by which an additional labourer employed would increase production; or (which is much the same thing if the number of labourers is large) below the amount which would be lost if one of the labourers already employed were dismissed and his work distributed over the remainder. So long as the landowner, by engaging one more labourer, obtains a greater increase in production than the amount by which wages are increased, it will be to his advantage to do so, and the dismissal of a labourer already engaged will be, *a fortiori*, a disadvantage. But if the same applies over the whole range of producers, their competition for labourers must force up wages until the difference between the additional product obtained and the wages paid for the last labourer engaged eventually disappears. One may therefore say, in theory, that *the additional product of the last labourer engaged* will, in general, regulate wages; which can neither rise above it nor fall below it. At the same time, it may be assumed that, owing to competition, this additional product will be the same in all branches of production, either in the *physical* sense, if only one commodity or one particular group of commodities (such as agricultural products) is produced in all undertakings—or, if several different kinds of commodities are simultaneously produced at given prices, then the values of the additional products must be equal. And, theoretically, at these wages *all* the labour in the market *will just find employment*.

It is easy to see that what has been said above is, fundamentally, an application of the principle which has already guided us in the determination of market values. Here also, there is a sort of exchange between the product and the wages of labour—though not an exchange in the strict sense, since the latter are a condition of the actual production of the former. And the correspondence between wages and the additional product of the last worker—or, as we shall henceforth call it, the *marginal productivity* of labour—is evidently analogous to the equality of marginal utilities for each of the parties to an exchange—which regulates market price. But they are not quite

the same thing; the difference being that, in the case of wages, the equality is objective, but, in the case of direct exchange, the equality of marginal utilities is subjective only.

After the payment of the wages so determined (an analogous remuneration for the employer's own work being supposed to be included) there remains, as a rule, a surplus for the landlord, which is greater or less according to the quality and size of his holding. This surplus, whether we regard it as pure rent or as rent and entrepreneurial profit combined—of which more later—will thus, on the given assumption, be the share of land, or of its owner, in the product. In modern terminology: after the share of one factor of production, labour, has been independently determined (by its marginal productivity), the second factor of production, land (or the landowner), is the *residual claimant* who has a claim on what is left.

All the labourers are regarded as possessing the same skill and strength. A merely quantitative difference in physical strength, however, can easily be taken into account, if we treat a particular labourer as equal to 1·1, 1·2, etc., or 0·9, 0·8, etc., of the average labourer. On the other hand, a higher quality of labour cannot, as was once supposed, be reduced to terms of simple unskilled labour; in fact, at least at any given moment, the different classes of workers represent distinct groups, each of which is paid according to its own marginal productivity.

In order to emphasize this we will take a concrete, though somewhat artificial, example. We will assume an area of 10,000 square miles—about the area of Wales—entirely devoted to agriculture, and with a working population of 160,000 adult men. Suppose this territory divided up into 10,000 estates of 1 square mile each, all equally good, i.e. containing in about the same proportion the usual kinds of land: fields, meadows, woodlands, etc. It will then be clear that, in equilibrium, exactly sixteen men must find employment on each one of these estates. This distribution of labour, however obvious from the data, comes about in reality as the result of competition on two sides, in the way described above. So long as wages are materially *lower* than the marginal product of the sixteenth labourer, it will be to the advantage of every landowner to employ more than sixteen labourers. But all the landowners cannot simultaneously succeed in this object, and consequently their endeavour must result in a rise of wages. Again, if wages are *higher* than the marginal

product, each of the landowners will content himself with less than sixteen workers, which will result in unemployment and a fall in wages through the competition of the unemployed. The final wage, equal for all the labourers, must therefore lie somewhere between the marginal product of the sixteenth and that of an imaginary seventeenth labourer on any one of the estates in question.

Everything now depends upon the size of this marginal product—on the law of variation of the total product of an estate of a given area, when the number of labourers and the intensity of agricultural work increases or decreases. Unfortunately, this law is practically unknown and its mathematical expression is certainly very complicated. If, however, as is nearly always the case in practical economic questions, it is only a question of small variations, we can, as a rule, content ourselves with a comparatively simple expression; we may therefore begin by supposing the product to vary as a root (e.g. the square root) of the number of labourers. If experience showed that, with the actual labour force of sixteen workers per square mile the average harvest was 1,600 hectolitres of corn, and the price per hectolitre 10*s.* then we can draw up the following table :—

HARVEST PER SQUARE MILE

Number of Labourers.	*Volume of Product. (Hectolitres.)*	*Money value of Product.*
1	$400 \times \sqrt{1} = 400$	4,000 shillings
4	$400 \times \sqrt{4} = 800$	8,000 ,,
9	$400 \times \sqrt{9} = 1{,}200$	12,000 ,,
16	$400 \times \sqrt{16} = 1{,}600$	16,000 ,,
17	$400 \times \sqrt{17} = 400 \times 4{\cdot}123 =$ 1,650 (approx.)	16,500 (approx.)

Naturally, one would not expect that this simple relation would, in reality, apply throughout the table. But that it does not lead to absurd results seems to be shown by those parts of the world where good land is still employed in very extensive agriculture, as in newly settled countries. According to a writer in *Schmoller's Jahrbuch* (1902), in Santa Fé and Cordoba (in the Argentine), a colonist employing only one labourer was able to plough and sow about one square mile and to harvest about 1,000 decitons of wheat annually. For this case our table would give ($400 . \sqrt{2} =$) about 570 hectolitres (per square mile) as the total product. But, of course, in this case no small part of the product would be deducted as *interest* on capital in the form of machinery, transport, buildings, etc.

If we now assume that wages are determined by the imaginary 17th worker's additional product, which according to what has been said would, under these circumstances, be the *minimum*, then there would be 500*s*. per annum per worker, or 8,000*s*. per sixteen workers; so that the landowner's remainder would also be 8,000 and the rent 80*s*. per hectare. This equality between the total shares of the product of the workers and the landowners is no accident, and would be the same with *any* degree of intensity as soon as the law of returns has the particular form assumed. (See p. 116.)

The following is a simple way (and one often used nowadays) of showing the mutual dependence of rent and wages, and the determination of their relative magnitudes: the successive labourers employed on a given area of land are represented by

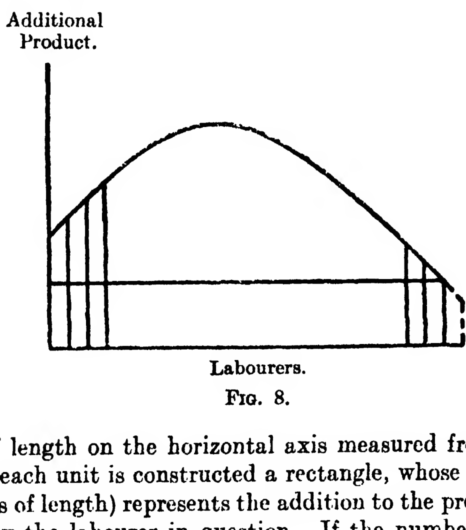


Fig. 8.

units of length on the horizontal axis measured from the origin, and on each unit is constructed a rectangle, whose area or height (in units of length) represents the addition to the previous product made by the labourer in question. If the number of labourers is large enough, the upper limit of these rectangles may be replaced without serious error by a continuous curve—the curve of productivity or gross yield. The area under this curve (bounded by the axes and a variable ordinate) represents the whole of the gross product secured as the number of labourers increases. The additional product of the last labourer is represented by the last rectangle to the extreme right, or by its height; and since this additional product determines both the *wages* of the last labourer and those of all others, the total *sum of wages* is represented by a rectangle of the same height and with a base consisting of the

whole distance from the origin (the total number of labourers). The remainder of the gross yield, or the upper portion of the area under the curve, represents the *rent* of the whole area cultivated.

If the number of labourers is a, then the gross product P may be represented algebraically as a function, $f(a)$ of the number a. The wages of the last labourer, as of every other labourer, is then represented approximately by the differential coefficient $f'(a)$. We then obtain as an expression for the rent :—

$$R = f(a) - af'(a)$$

If, in addition, we were to assume, as in the numerical example above, that this production function was simply a fractional power of the number of labourers, so that $P = f(a) = k.a^{\alpha}$ in which k is a constant and $\alpha < 1$ then the expression for rent is reduced to

$$R = P.\,(1 - \alpha)$$

that is to say, the index α also expresses the relation in which the gross product is divided between labourers and landowners. If, for example, as we have assumed, $\alpha = \frac{1}{2}$, then both would receive equal shares ; if $\alpha = \frac{2}{3}$ the labourers would receive two-thirds of the product and the landowners would keep only a third.

The above theory of the relation of wages to the rent of land was developed (so far as its fundamental principle—the determination of wages by the marginal productivity of labour—is concerned) as early as the beginning of the nineteenth century by the German economist and landowner, von Thünen. But even earlier there had been propounded by Anderson (an English contemporary of Adam Smith) and afterwards, quite independently, by Malthus and West, a theory of rent, which was adopted and developed by Ricardo in his *Principles*, and which is usually associated with his name. All these theories are fundamentally the same. In spite of the remarkable simplicity of von Thünen's theory, it coincides completely, at least as regards the explanation of the origin of rent in the narrower sense, with Ricardo's theory. The latter is based, as is well known, on two assumptions : *either* that agriculture is extended successively to less fertile or less advantageously situated land, so that the owner of the better land retains the difference in productivity in the form of rent ; *or* that the land already under cultivation is more intensively worked by the employment of increased amounts of labour and capital, so that a similar differential

rent arises from the diminished return (marginal product) of the labour and capital later employed. In Ricardo, however, capital is taken as representing a certain quantity of labour, directed and maintained by this capital. He makes no mention, at least in this connection, of increase or decrease in the length of the *period* of production, which, as we shall see later, is of decisive importance in determining the share of capital in the product. We may, therefore, regard this part of his theory as identical with that of von Thünen.

Fundamentally, however, the same applies to the first part of Ricardo's theory, for whether the additional product of the last worker engaged arises from the cultivation by him of poorer land previously uncultivated, or by more intensive cultivation of land already in use, is a matter of indifference in theory. Which of the two occurs may be regarded in reality as the sole concern of the entrepreneur. If the estate in question, as often happens, includes both good land and inferior land he will in each case select the method which is technically most advantageous; with essentially the same result, namely, that every new labourer engaged, employed in the best possible manner, will produce a smaller addition to the product. Differences of situation with regard to marketing can, as von Thünen clearly shows, always be reduced to differences of costs of transport, that is say, to costs of production, since production must not be regarded as finished until the goods have been brought to the market where they are to be sold.

A Closer Examination of Ricardo's Theory of Rent

Ricardo assumes for the sake of simplicity that wages, reckoned in *products* or *means of subsistence*, are *constant*; because if they should happen to rise the number of labourers would increase to such an extent that wages would again fall either to the absolute minimum of subsistence or to the standard which the labourers regard as their normal standard. At that wage, the capitalist-farmer—whom, in accordance with English conditions, he assumes not to be identified with the landlord—hires labour as far as his capital permits. On the other hand, the product becomes his property and constitutes, after the deduction of the capital paid out in wages, his (gross) profit. If there is a superfluity of good land, then owing to competition among landowners, there cannot be any considerable rent. But as soon as

capital, and consequently also the working population, increases to such an extent that *poorer* land must be taken into cultivation, rent immediately appears; for this poorer land yields a smaller product to the same capital, and consequently (since wages, reckoned in the product, remain the same) also a smaller profit. But, owing to competition among capitalists, all capital, even that which is employed on the better land, must now be satisfied with this smaller profit, and the remainder will accrue to the *owners* of the better land.

Simultaneously with the progressive cultivation of poorer land and the consequent rise in the rent of the better land (i.e. of all land under cultivation except the very worst) it will usually be profitable to employ *more labour* (and capital) on the better land already in cultivation. But since every additional quantity or "dose" (as James Mill called it) of labour and capital yields a smaller and smaller product, and the new capital must thus content itself with a lower rate of interest, interest will fall all round, even on capital previously invested and still employed, and the surplus product which thereby arises will go to landowners as rent.

As will be seen, the rôle of capital, in Ricardo's opinion, is mainly to advance wages (and to provide the necessary agricultural implements, etc.). But since we have assumed that the labourers are able to maintain themselves during the period of production (and to prepare the necessary implements), it is clear that the theory we have advanced above as regards the landowner's share in the product is exactly the same as Ricardo's. How the share of the product which does not pass to the landowner is in fact divided between the labourers and the capitalists is a question with which we shall deal later. On the other hand, Ricardo and the classical economists in general pay no regard at all to the fact that capital in many cases also advances rent. A farmer who breeds cattle for meat, for milk, or for draught, must pay rent for his pasturage for many years before he can employ or advantageously dispose of the animals in question. The same applies to an even greater extent to a person who engages in viniculture or fruit-growing on rented land. It may therefore be said, on the one hand, that Ricardo's theory of rent is too complex in relation to the single principle which it seeks to explain, and, on the other hand, much too simple when compared with reality. Nevertheless, his theory marked immense progress as compared with the obscure ideas on the subject previously extant—even in Adam Smith.

The objections which were raised against this remarkable theory in various quarters, especially in earlier times, scarcely deserve notice. The best known is the objection of the American economist, Carey, that, historically, cultivation did *not* proceed from better to poorer land, but, on the contrary, from the poorer to the better, i.e. from higher and therefore more easily cultivated, though less fertile land (as for example a sandy tract) to lower land more difficult to work, but more loamy and therefore more fertile. This may to some extent be true, but it has no bearing on the theory in question; for Ricardo was only concerned with the land which is cultivated or which can be profitably cultivated at a certain stage in the development of cultivation. Technical improvements, discoveries in agricultural chemistry, and so on, may well completely revolutionize an older system of agriculture and cause what was formerly the best land to decline in value, or perhaps even to be abandoned altogether. But the law of rent retains its validity, even although the assumptions under which it operates may have changed. The curve of returns referred to above assumes a new form, but retains its characteristic features.

We need not waste many words, either, on the attempt of the German, Rodbertus, the predecessor of Karl Marx, to replace Ricardo's theory of rent by a better one. Like Marx later, and partly on the basis of the theory of value he inherited from Smith and Ricardo, Rodbertus assumed that the value of the product was wholly determined by the amount of labour employed in its production. According to this theory, labour "as itself a commodity" only obtains as a reward under free competition "its costs of production", i.e. the minimum of subsistence for the labourer and his family; the remainder—which Marx calls "unpaid labour"—is taken by the capitalist. With free competition among employers, says Rodbertus, the degree of exploitation will be about the same. In industry proper, however—and this is the essence of Rodbertus' theory—the capitalist-entrepreneur considers his profit as interest on two amounts of capital: that needed for the maintenance of his labourers, and that needed for the raw materials which he must purchase—the value of which he has advanced for the period of production. But the producer of raw materials (the landowner) has no material expenditure of the latter kind. With an equal amount of "unpaid labour" he therefore obtains a larger amount of interest on his

actual capital, since it only consists of the maintenance of his labourers. If, however, he only reckons on that capital the same amount of interest as does the industrial capitalist, there will be a surplus, which he will consider as the rent of his land. The most obvious objection to this theory, which appears at once extremely artificial, is that it implies that interest and rent must always move in the same direction, must rise or fall together—which is contrary to all experience. That this may sometimes appear to be the case is simply due to the fact that, with falling interest, land, other things being equal, is capitalized at a higher value than previously and consequently, with unchanged rent, has a lower yield per cent on its capitalized or selling value; but naturally this is an entirely secondary phenomenon.

In point of fact, Rodbertus' theory of rent argues in a circle. There is no reason why the "degree of exploitation" in different trades between employers under free competition should be the same, other than the assumption that the value of the product is always proportional to the quantity of labour employed. But this in its turn presupposes precisely this—that the degree of exploitation is the same. In reality, the so-called "degree of exploitation" is very *different* in different trades, in accordance with the different amounts of capital invested relatively to the number of labourers employed, or (which comes to the same thing, as we shall see) the difference in the average *period* of the investment of capital. The same applies to the value of the product in relation to the amount of labour employed in its production.

It is evident that the Ricardo-von Thünen theory of rent described above is too abstract for us to be able to expect any *direct* verification of it by studying the world of reality. In addition to all other simplifying assumptions, the part played by *capital* in production, and its share in the product, find no place in the theory as presented by von Thünen; and Ricardo's treatment of the capital aspect is too rudimentary and incomplete. In addition, we must bear in mind that the assumptions of perfect competition and mobility and divisibility of the factors of production only very imperfectly correspond to reality. In small-scale agriculture, for example, the "last" worker employed is, frequently enough, the *only* one—for the simple reason that the area of land is so small that it does not permit the employment of more than one labourer in addition to the owner, and sometimes not even one. On the other hand, of course, we must not forget

the heterogeneity of human labour and the possibility of some substitution of the labour of women and children for that of men.

Nevertheless, experience seems to show that the range of applicability of von Thünen's *law of wages* is considerable, even in industries other than agriculture. Nothing is more common than for employers to reply to an increase of wages forced upon them by a labour organization by sooner or later dismissing some of their labourers, because it is no longer profitable for them to carry on at full strength. If the labourers do not support their unemployed comrades at the union's expense—as is common, in such cases, among English trade unions, though it is possible only up to a certain point—then their competition must undoubtedly force wages down again to the previous level—i.e. to equality with the marginal productivity of labour as it is when all labourers are employed.

Further, as far as this "law of wages" is operative, the *growth of population* will obviously exercise a most damaging influence on the position of labour and of the propertyless classes as a whole. Particularly will this be the case under the existing system of private ownership of land. The consequence of an increase in the number of labourers is not only that the new labourers will find it more difficult to earn a livelihood than the old ones, but also that there will be a lowering of wages *all round* owing to their mutual competition; so that the landowners' share of the product will be correspondingly greater. It may be thought that experience often runs counter to this view: wages sometimes remain unchanged, or even rise, despite a considerable increase in population. But the real cause here is that the conditions of production have been materially changed, in consequence of technical or scientific progress, and not least under the influence of capital accumulation, which we have not yet considered. Similarly, entirely new sources of supply may have been discovered. If, under such circumstances, population remained unchanged, the marginal productivity of labour, and consequently wages, would normally rise very considerably. If population increases, however, both will sink to their original level. In other words, technical progress, so far as the labourers are concerned, only protects them against the absolute fall in wages which would otherwise be inevitable,

whilst at the same time in increasing, frequently to a high degree, the surplus accruing to the landlord.

The principle on which the whole theory of rent is based—the decline in the average yield of labour when the number of labourers is increased (the so-called law of diminishing returns)—has, at all times and not least in our day, been vigorously disputed. From the point of view of pure theory this is a matter of indifference; for those who deny the existence of the law must, if they are consistent, *deny the existence of rent*, which they often do when they assert that the landowners' share of the product is only a compensation for the labour and capital invested in the land by them or their forefathers and is therefore *interest on capital*—possibly in part a repayment of that capital—and not rent of land. The existence of rent would still remain, even on this view, a proof of the applicability of the law. Owing to the extreme practical importance of the question, however, we will proceed to examine it in greater detail.

It may be thought that nothing could be easier, once attention has been drawn to it, than to verify such a simple rule as the relatively diminishing return of land under more intensive cultivation—if in fact it is valid. It must, indeed, be quite easy to prove it by *direct experiment*, and in so far as such experiments have been made—unfortunately all too few and on too small a scale—the results undoubtedly tend to confirm the law. On the other hand, it is very difficult, if not quite impossible, to confirm the law by observing the actual yield of agriculture on different estates. If one estate is as fertile and as rationally cultivated as another, then the intensity of cultivation in both will be carried to the same point, and both will naturally yield the same return. On the other hand, every difference in the fertility of the two estates under rational cultivation must give rise to a difference in intensity of cultivation; but the result of this differentiation will be in *apparent contradiction* to the law of diminishing returns. Thus if, in equilibrium, the *last* dose of labour and capital on the better land yields about the same return as perhaps the first and only dose on the poorer land (and previous doses on the better land therefore yield a higher return), then *on the average* the more intensive cultivation will yield a higher return for each unit of labour ("labour and capital") than the more extensive. It may consequently *appear* as if the law of diminishing returns had ceased to operate and had been reversed, although this result is really a consequence of the law. The same applies to a comparison of the yield of an estate at

different points of time if, in the interval, more intensive cultivation has been introduced, in consequence of technical progress in agriculture, or of a rise in the price of the product.[1]

It is very common, even among professional economists, to confuse the relative yield of agriculture with its profitability. They are, however, two entirely different things. The former is the ratio between the gross yield and the amount of labour (or labour and capital) employed; the latter is the *difference* between that yield and the amount of wages paid (or of wages and interest). They may therefore vary in quite different ways, and even in opposite directions. For example, with the law of productivity which we took as an example, according to which the gross product increases as the square root of the number of labourers, or $P = k \, . \sqrt{a}$, the relative yield would be $P : a = k : \sqrt{a}$, and would thus continuously decline as the intensity of cultivation increases, while the rent, as we have seen, would be equal to $\frac{1}{2}P = \frac{1}{2}k \, . \sqrt{a}$, so that the profitability to the landowner would continuously increase with increasing intensity.

As regards the *point* at which the law of diminishing returns begins to operate, we must distinguish between the individual and the collective, or social, points of view. From the individual point of view, the law presumably operates from the beginning, or at any rate from the time when the spontaneous products of nature, such as meadows, trees, etc., obtain an exchange value. For these products, which are obtained without labour, represent in proportion to the labour employed an *infinitely* great value, and in comparison with them every product obtained by labour will represent a diminishing return. In other words, for the person who has at his disposal a certain area of land, it must always be possible by the employment of a small quantity of labour to obtain a relatively greater return than by the employment of a larger quantity of labour.

From the collective point of view, on the other hand, the services which pioneers in newly settled countries can render each other by co-operation in defence against wild animals or hostile tribes, by the building of roads, and by the establishment of schools, and the advantages to be derived from combination and division of labour must, with an increasing population, outweigh the inconvenience of a smaller average allocation of land to each individual. The point at which the two opposing influences are

[1] This *apparent* failure of the law of diminishing returns, to hold within certain limits, in the case of land under more intensive cultivation has been treated by the author in greater detail in *Thünen-Archiv*, vol. ii, p. 347 *et seq.* and 568 *et seq.* (1907–8).

balanced, and consequently the *optimum* density of population, can of course only be determined in each particular case after consideration of the total resources of the country.

B. The Labourer (or a third party) as Entrepreneur. The Profits of the Entrepreneur.

We might equally well have begun by regarding the labourers themselves as entrepreneurs. The circumstance which in reality prevents them from assuming this function, namely, their lack of capital, would, on our assumption, be absent, since we suppose every labourer to be provided with the means of maintaining himself during the current period of production, and nothing more is required. They are therefore free to enter, either singly or in combination, into agriculture or any other productive enterprise by hiring the necessary land from the landowners against payment in kind at the end of the period of production. The process by which equilibrium would finally be reached in this case is fully analogous to the process described above; or rather it is its exact counterpart. The more land the labourers procure, the greater will be the product; though it will not increase proportionally to the land taken into cultivation, but *more slowly,* so that each newly-acquired acre will yield, with an unchanged supply of labour, a smaller and smaller return. In other words, the law of diminishing returns applies to a one-sided increase in the amount of land. The labourers must, therefore, if they act economically, extend their demand for land to the point at which the additional return of the last acre exactly corresponds to the rent demanded for it. We must, however, assume here—as we did in the case of labour—that *all land* capable of employment is of *equally good quality.* This assumption would not, indeed, be of much importance if we could assume that the different kinds of land could be regarded as of the same quality, whatever is the degree of intensity of labour, so that better land could always be represented by a particular multiple of the poorer land. As, however, this is not the case, the various kinds of land must be treated in the same way as the various qualities of labour, i.e. as so many different kinds of means of production. "Land" and "labour" are only to be taken as types of two independent factors of

production. This method is valid, at least, for any given moment; the possibility of converting one kind of land into another is a question that must be kept separate: in the same way as we keep separate the conversion of one kind of labour into another, by training and education.

If all the land is not at once taken into cultivation, or if, conversely, the demand of all the groups of labour for land is not satisfied, then it is clear that competition, in the former case between landowners and in the latter between labourers, would cause a fall, or a rise, in rent until complete equilibrium was restored. In a word, rent is here determined by the marginal productivity of land, and conversely wages are determined by the surplus product divided among all the labourers in the group —the labourer becoming the residual claimant.

For the analysis of this problem, it is possible to employ exactly the same diagram as in Fig. 8 with the difference that the units on the horizontal axis (abscissae) now represent the number of acres of *land* successively taken into cultivation by a constant number of labourers, and the corresponding ordinates (or rectangles) the marginal products obtained. The ordinate to the extreme right thus represents the return of the last acre (the marginal productivity of the land) or, what comes to the same thing, the rent of land per acre. The large rectangle represents the total rent and the upper part of the area under the curve the total wages; just the reverse of the previous case.

If the number of acres is b, the total gross product $P = \phi(b)$, the rent per acre is $\phi'(b)$; then the total share of labour in the product will be

$$L = \phi(b) - b\phi'(b).$$

If, for example, the function $P = \phi(b) = k\sqrt{b}$, in which k is a constant, then $L = \frac{1}{2}k\sqrt{b} = \frac{1}{2}P$, or the same result as we obtained on the assumption that the gross yield varies as the square root of the number of labourers. The reasons for this agreement will soon be made clear.

An interesting question now arises, to which we may turn our attention: will the distribution of the product between landowners and labourers be the same on each of our assumptions? Or, putting the same question in another way, if the entrepreneurs are a *third* category of persons who hire labourers and land, and pay both in accordance with the law of marginal productivity,

will the total of rent and wages swallow up the whole of the product, so that nothing is left over for the entrepreneur as such ?

This may seem evident, at least in abstract theory; and most economists who have employed marginal productivity as the foundation of their theory of distribution have thought so. On our assumptions, both labourers and landowners are free, as they prefer, to employ their labour or land on their own account or to hire it out to others. If the share of labour in the product is different in the two cases, the difference, it may be thought, will soon be cancelled out by competition, and similarly for the share of land. At the same time, it will be obvious that the profits of entrepreneurs as such must always tend towards zero. For the work and thought which the entrepreneur devotes to the management of production he must, of course, receive his wages like any other mental worker. If, in addition, he also employs property in the service of production (property which may be land or capital, though we are not yet concerned with the latter), then he will of course, for that reason, obtain his share of the product (rent or interest) like any other landowner (or capitalist). If, on the other hand, he could obtain a share of the product merely in his capacity of entrepreneur (a share not based on either labour or land) then it might be thought that everybody would rush to obtain such an easily earned income.

But on the other hand, as has been sufficiently demonstrated, the marginal productivities of labour and land do not stand in any definite relation to the total product or to each other. If, nevertheless, they possess this peculiar property that the wages and rent thus determined together add up to the whole product, then clearly some other condition must be satisfied. Such a condition exists, and is of the utmost importance, although it has been somewhat neglected by economists. This condition may be *either* that large-scale and small-scale operations are equally productive, so that, when all the factors of production are increased in the same proportion, the total product also increases exactly proportionately; *or* at least that all productive enterprises have already reached the limit beyond which a further increase in the scale of production will no longer yield any advantage. Were it otherwise, we could no longer invoke, as we have done, the levelling influence of competition; for under

such conditions, as we shall soon see, free competition cannot exist.

That the first condition is sufficient (though not necessary) for the operation of the law we will first show by means of an example. Imagine a firm, say an agricultural enterprise, in which 100 labourers are engaged on an area of land which we will imagine to be divided into 100 units—no matter of what size. We represent the annual product by P and proceed to examine what addition to this product will occur if we successively increase the volume of production by adding first one more labourer and then one more unit of land. The first additional product is the marginal productivity of *labour*, in so far as we may regard the additional product created by the 101st labourer on the given area of land as roughly the same as that created by the 100th labourer—a product which would be lost if one of the 100 labourers were dismissed or gave up working. We represent this quantity by l since, on our assumption, it would determine the amount of wages paid. If the land under cultivation is now increased by one unit of equally good land, so that the 101 workers may be spread over 101 units of land, then evidently the product will be increased, and this increase is just what we have called the marginal productivity of *land*; for just as with labour, we can see that the increased return which arises when the area of land worked by 101 labourers is increased from 100 to 101 units does not materially differ from the increase which would have taken place if the area of land worked by 100 labourers had been increased from 99 to 100 units. But since the yield of the last unit would, on our assumption, determine the rent of the land, i.e. constitute the rent of one unit of land, we will represent it by r and then $l + r$ will represent the sum of the additional product. On the other hand, the total production has been uniformly expanded both as to the area of land and the number of workers, and *on the above assumption* the product should consequently have been finally increased by exactly 1/100th, so that we obtain :—

$$l + r = \frac{P}{100} \text{ or } 100l + 100r = P.$$

In other words, the wages of 100 workers and the rent of 100 units of land together exactly correspond to the original total product.

A more general proof is the following. If we regard the product P as a function of the number of labourers, a, and of the number of units of land, b, both a and b being regarded as

continuous, then the marginal productivities may be expressed by the partial derivatives of P with respect to a and b; therefore, if the condition is to be satisfied, we must have

$$a\frac{\partial P}{\partial a} + b\frac{\partial P}{\partial b} = P,$$

a partial differential equation, the general integral of which is known to be :—

$$P = a.f\left(\frac{b}{a}\right),$$

in which $f(\ )$ is an arbitrary function, i.e. P must be an homogenous and linear function of a and b. Among the infinite number of functions which satisfy this condition, we may give as an example $P = a^{\alpha}.b^{\beta}$, in which the indices α and β are two constant fractions whose sum $= 1$. If we substitute ma for a and mb for b, then P becomes mP, i.e. large-scale and small-scale production are equally productive.

If, on the other hand, P retained the same form, but $\alpha + \beta > 1$, so that P was a homogenous function of a and b but of a *higher* degree than the first, we should obtain

$$a\frac{\partial P}{\partial a} + b\frac{\partial P}{\partial b} > P.$$

In other words, if, in an enterprise which becomes more productive the larger the scale of operations, the labour and land employed were both paid in accordance with the law of marginal productivity, then the sum of their shares would *exceed* the whole product, so that the entrepreneur would suffer a loss.

This result is connected with the circumstance that under such conditions equilibrium is impossible. Large scale operations, being more profitable than small scale, can here offer better terms to landowners and labourers (or cheaper goods to the consumer); and if the smaller entrepreneur seeks to compete, his profits will in fact be negative; that is, competition will drive him out. But the same will also happen in the case of the large-scale enterprise as soon as another on a still larger scale is established.

The converse will be the case if $\alpha + \beta < 1$; in other words, if an enterprise is more profitable the *smaller* the scale of its operations. We shall then obtain

$$P > a\frac{\partial P}{\partial a} + b\frac{\partial P}{\partial b};$$

that is, the entrepreneur as such will necessarily obtain a profit, but for that very reason everybody will want to be an entrepreneur, with the result that all enterprises will ultimately be split up into small individual units.

The first assumption, that the relative yield of production is independent of the scale of operations, is, of course, very seldom realized as a general principle in a given branch of production; the scale on which an enterprise operates nearly always has some influence on its average product. This is not to say, however, that its influence always works in the same direction. On the contrary, as a rule the best returns are obtained at some particular scale of operations for the firm in question; if this is exceeded, the advantages of centralization are outweighed by the increased costs which are encountered when larger areas must be exploited for the provision of raw or auxiliary materials, or else for the marketing of the product. This scale of operations is, under the given circumstances, the "optimum" towards which the firm must always, economically speaking, gravitate; and as it lies at the point of transition from "increasing" to "diminishing returns" (relatively to the scale of production) the firm will here conform to the law of *constant* returns.[1] Wages and rent will continue to be determined by the law of marginal productivity and the profits of the entrepreneur must tend towards zero—all on the assumption that the enterprises in question, in one and the same branch of production, are sufficiently numerous to compete with each other effectively.

Let a and b represent respectively the number of units of labour and land employed in the enterprise in question, and l and r the wages and rent actually paid, expressed either in money or product; and let P represent the annual product expressed in the same unit of value. Then, the ratio, k, between returns and costs of production in this enterprise will be :—

$$k = \frac{P}{a.l + b.r}.$$

If an additional labourer is employed, this equation will be changed to :—

$$k_1 = \frac{P + P_a}{(a + 1).l + b.r}$$

[1] This simple method of presentation was pointed out to me in a letter from Professor Davidson.

where P_a is the marginal productivity of labour in a firm of this particular size. If the supply of land is now increased in its turn by one unit, we obtain :—

$$k_2 = \frac{P + P_a + P_b}{(a+1).l + (b+1)r}$$

where P_b is the marginal productivity of land. So long as this fraction can be continually increased by the introduction of one more labourer or one more unit of land (so that $k < k_1 < k_2$, etc.), the enterprise has evidently not yet attained its optimum size. The latter is first reached when k can no longer be increased—which clearly occurs only when the numerator and the denominator of the fraction are increased in the same proportion, i.e. when :—

$$k = \frac{P}{a.l + b.r} = \frac{P_a}{l} = \frac{P_b}{r} \qquad (1),$$

where P_a and P_b represent the additions to the product P which arise from the employment of one more labourer, or one more unit of land—in other words, the (variable) marginal productivities of labour and land. Even *if* there is a profit for the entrepreneur ($k > 1$) wages and rent must be *proportional* to the marginal products ; as is evident, since labour and land are assumed to be substitutable at the margin.

If, even when the firms have reached the optimum scale, they are still numerous enough for perfect competition to be maintained, then wages and rent must be forced up to the point where the entrepreneur's profit becomes zero, either because new entrepreneurs enter the industry, or because those already engaged in it will establish more than one concern each. Indeed, strictly speaking, this must take place whenever there appears the smallest possibility of a profit. (This change will not affect the most profitable size of the firm, for since P, P_a and P_b are functions of a and b only, the same values a and b will satisfy the equations (1) even if l and r are increased or diminished in the same proportion. Full equilibrium is thus only reached when $k = 1$ and when, consequently, $l = P_a$ and $r = P_b$; when further

$$P = a.l + b.r.$$

This is the result previously obtained on the assumption that the average product was entirely independent of the scale of production. With the firm at its optimum size, the entrepreneur no longer receives a profit ; but he is secured against the loss in

which he would be involved if he were to expand beyond that size, or not to expand up to it.[1]

If, on the other hand, the law of increasing returns applies without qualification—or, what amounts to the same thing in practice, if the optimum scale of the enterprise is so high, and the number of such enterprises consequently so small, that the owners can easily combine in a *ring*, *trust*, or *cartel*; then there no longer exists any equilibrium of the kind we are here considering. The whole industry will be dominated by a more or less completely monopolistic association and all smaller concerns will disappear.

In reality this is not exactly what happens, but for several reasons, and especially because of the local character of the firm and its market, a small firm situated, it may be, in some geographically remote place, may sometimes exist alongside much larger firms in other places. This, however, will not prevent the larger firm from enjoying advantages due to its better organization and division of labour, which the smaller firm lacks, and from yielding on that account, in addition to wages and rent (as well as interest) a true profit, or perhaps more correctly, a monopoly profit. The large firm cannot be deprived of this profit, because any attempt on the part of the smaller enterprise at effective competition outside its own local area would be fruitless. If, on the other hand, the smaller enterprise, by a great economic effort, were to establish itself on the same footing as the large enterprise, this would only lead to the ruin of both, since there would be no room in the market for two such large concerns in the same industry. Thus the large enterprise has an actual monopoly simply because it came first on the scene, and this monopoly may be as good as a monopoly which is legally established.

We must not forget that the modern development of communications necessarily increases the advantages of large-scale operations and tends to hasten their ascendency. Agriculture is the industry which, both in the past and in the present, has offered most resistance to this tendency, though there are some indications that future developments in this industry may also be in the direction of large-scale operation.

[1] The basis of this argument is due to Enrico Barone; cf. Walras, *Éléments d'économie politique pure*, 3rd edition, p. 489 *et seq.*

The objection which has been raised to the effect that small farming on co-operative lines—by the establishment of buying and selling associations, co-operative dairies, the use in rotation of expensive machinery hired or purchased by the association—is a means of overcoming these difficulties is rather an argument in favour of the above assumption; for these associations in fact bring about a kind of large-scale operation, and this first step towards association, once taken, will, in all probability, soon be followed by others.

But, although more or less monopolistic enterprises constantly gain ground, there still remain fields of activity in which free competition prevails—either where large and small-scale operations are approximately equally profitable or where the most profitable scale of production is, on the whole, fairly small. In such fields our theory applies fully; there is normally no entrepreneur's profit in the narrow sense. In production without capital, wages and rent would alone share the product and their respective shares would be determined by the marginal productivity of labour and land—whether labourers, landowners, or anyone else, act as entrepreneurs. And, so long as such a field of activity of any considerable dimensions exists, it will set the standard of wages and rents in the whole field of production, since the entrepreneurs who enjoy monopolistic advantages will not give to labourers or landowners more than they would be forced to give under competition. In the latter concerns, moreover, the law of marginal productivity still applies, in the sense that the shares of labour and land remain *proportional* to their marginal productivity (cf. the paragraph in small print on p. 129).

Between rent and wages there is thus, in every case, a practically complete parallelism. No special theory of rent is necessary, but every acre of land may be treated in just the same way as a labourer; the owner of land under a system of private ownership of land must be rewarded for its contribution to production just as the owner of slave labour would be paid if slave labour were hired in the market. Almost all production is the result of land and labour combined; neither, at any rate not land, can wholly be dispensed with in production, but either can, at the margin of production, replace the other; and it is true of both that a one-sided increase of one, with an unchanged

quantity of the other, will lead to an ever smaller and smaller increase in the product.

With these reservations and limitations, this additional product will determine the magnitude of both wages and rent. The *total* contribution of labour or of land to the product cannot be ascertained. But this total contribution has no real importance, since, as has been said, neither of them, and certainly not labour, can be productive alone. Only at the margin of production, that is to say, at the point where equilibrium is reached, does the contribution of either assume an independent character, and it then determines not only the reward of those factors which begin to participate in production at that point, but also, owing to the law of indifference or competition, wages (and rent) as a whole.

It need only be said that the above applies, as will easily be seen, both individually and generally—according as we consider the additional product created by an individual productive enterprise when it employs one more worker or one more acre of land, or as we consider the addition to the whole social product when the total amount of labour or of cultivable land is increased by a small amount. Yet we must not forget that the law of "increasing returns" also applies to some extent to society as a whole. If a uniform increase both of the land and population of a country were to occur, say by a political union of two countries of much the same natural conditions, or simply by the removal of a tariff wall between them, then it is certainly not impossible, but even very probable, that the increased social division of labour would enlarge the combined product more than proportionately to the growth in the size of the society. Still more would this be the case, of course, if conditions had been different in the two areas; but that is, in part, a different question. With this last reservation, however, the diagrams and formulæ which we have used above apply, if the quantities taken represent the *whole* of the labour and land existing in the society. The importance of this observation will become clear in what follows.

C. The Influence of Technical Inventions on Rent and Wages.

We are now in a position to make a theoretical examination of a subject of the greatest practical importance—the influence

of technical and mechanical inventions on the distributive shares of the factors—especially wages. Naturally, we cannot give a complete answer to this question until we have discussed the rôle of capital in production. Machinery, however, in addition to having the quality of being, or representing, capital (which we shall define in greater detail later), also possesses the quality of modifying the conditions under which labour and land replace each other at the margin of production. In other words, it may alter their relative marginal productivities and thereby, according to our theory, their shares in the product. It is with this characteristic of machinery that we shall now concern ourselves. For the time being, we shall not permit this complex problem to be further complicated by allowing the third factor of production, capital proper, to enter. In other words, we shall regard machinery as *indirectly* employed (not as saved or " stored up ") labour and land.

The most striking feature of machinery is that it replaces human labour, i.e. allows us to produce the same quantity of goods as before with less labour ; and consequently, as a rule, more goods with the *same* labour. On the one hand, it may be thought that the greater productivity of labour ought to bring about, or at least render possible, the payment of higher wages ; on the other hand, it is commonly supposed to render a number of labourers superfluous, so that competition among the unemployed would depress wages. It would seem, therefore, that two opposing tendencies come into operation simultaneously, and that, according as one or the other predominates, the introduction of machinery will benefit labour or injure it. Opinions on this point have varied in the course of time. Formerly, under the influence of the mercantilist theory, no doubt at all was felt that labour-saving machinery took the bread from the mouths of the workers, and not only they, but also the authorities, stubbornly resisted the introduction of machinery in one or other branch of manufacture. The victory of the physiocratic school produced a sudden change, for according to its theory, especially as formulated by J. B. Say, goods must always ultimately exchange against, and therefore constitute a demand for, other goods ; an increased productivity of labour should of itself lead to an increased demand for goods hitherto not consumed, or consumed only on a small scale, and therefore

for labour to produce them. Hence, machinery would, at most, cause temporary unemployment and inconvenience to certain groups or labourers. In the long run it would be beneficial, would lead to increased opportunities for labour, and would raise and not lower wages. However, this optimistic view received a set-back when Ricardo, in a special chapter on "machinery", in the third edition of his *Principles,* proved irrefutably, as it was thought, that the introduction of machinery and other labour-saving methods may be economically advantageous to employers even when it does not involve an increase, but on the contrary involves a decrease, in the size of the product; provided that the net profit of the entrepreneur simultaneously becomes greater. In such a case the labourers could not be compensated by an increased demand for other commodities.

The question has remained in this somewhat unsatisfactory position until the present time. The theory of marginal productivity will enable us, I believe, to put it on a firmer foundation, and to substitute something better for this vague, and even in parts erroneous, analysis. Indeed, the expression "productivity of labour" has no comprehensible meaning when it is applied to production as a whole, for this is, as we have seen, always the combined result of labour and land. It is, therefore, the common productivity of labour and land which is increased by machinery. How much of the increase is to be ascribed to the action of one or the other factor cannot be ascertained, and is further of no importance in regard to their respective shares of the product. In this connection, marginal productivity alone is the determining factor. But an increase in the total product as a result of technical changes in the processes of production need not by any means lead to an increase—and certainly not to a uniform increase—in the marginal productivity of both factors of production. It may be that the marginal product of one of the factors decreases whilst the marginal product of the other increases all the more; either the marginal productivity of labour may increase at the expense of land, and consequently wages at the expense of rent, or conversely rent may increase at the expense of wages. Examples of the former kind are perhaps to be expected where, owing to some invention, the existing supply of natural energy is, as it were, increased; certain hitherto neglected sources of energy, such as coal or

water-power, find new uses; formerly useless land is rendered fertile, with or without preliminary treatment; forestry is replaced by market gardening, and so on. In such cases it is possible, or at any rate conceivable, that rents will fall both absolutely and relatively, so that the whole profit from increased production, and even more, will accrue to labour. It may, perhaps, be objected that the introduction of such changes, being contrary to the interests of the landowners, would never be allowed to take place; but this objection, as we shall soon see, cannot be maintained. The contrary result might be feared where an invention *prima facie* renders labour superfluous without calling into existence any new natural forces—as, for example, in the case of certain agricultural machinery for sowing, harvesting and threshing, etc., which replace human labour on a large scale by draught animals, or other non-human forces, without changing the actual method of tilling. Here, too, an increase in the total product is not excluded—we shall see later that in theory it must *always* occur. If, for example, the same product is obtained with a smaller number of labourers, then the displaced labourers must, nevertheless, always be able to produce something, so that the final result is an addition to production. But this result may none the less co-exist with a decrease, and even a considerable decrease, in the marginal productivity of labour, and consequently in wages.

The objection has been made, it is true, that under such circumstances the landowners neither would, nor could, consume their increased rents *directly, in kind.* They would therefore direct their consumption towards luxury articles and thereby increase their demand for human labour, so that wages would again rise. But this circumstance is, as will easily be seen, only of secondary importance. It may more or less modify the first probable result but can scarcely reverse it. And the objection clearly has no force if we maintain the assumption made above, of an economic society which, from its natural circumstances, only produces one or a few staple articles—and which must consequently procure all other commodities from other places or countries at exchange values which are determined in the world market, independently of anything they may do. If, for example, the landowners obtain, in exchange for their increased rent in corn, the most elaborate manufactures from other places

or countries, this will benefit their own labourers, more or less bound to the soil, just as little as if they had consumed it in kind—as fodder for racehorses, hounds, and so on. In neither case can there be any question of compensation to the workers in the form of another demand for labour.

On the other hand, it appears on closer examination—and the fact seems to me of great interest—that the objection raised by Ricardo is *theoretically untenable.* A diminution in the gross product, or in its value (assuming, as before, that prices of commodities are given and constant), is scarcely conceivable as a result of technical improvements—under free competition. This appears to be self-evident; for in that case anybody would be able, with the given means of production, to bring about at some point an increase of the product and thereby reap a profit as entrepreneur. Ricardo has here failed to draw the final conclusions from his own assumptions. It is true that in the passage referred to his starting-point is capital—which he divides into circulating capital (or wages-fund) and fixed capital. But his reasoning is, as he himself says, equally applicable under our simplifying assumption of production without capital, and in both cases it is open to the same objections.

Let us assume that the introduction of labour-saving agricultural machinery (haymaking machines, horse-harrows, etc.) has made a predominantly pastoral agriculture more profitable, other things being equal, than arable farming; so that the value of the product, though certainly less, produces a larger net yield, owing to the saving of labour. The direct consequence must then be that one or more farmers will go over to the more profitable form of production. If all were to follow their example, there would certainly be a more or less considerable diminution of the total product (or of its exchange value), *but this does not happen.* For as soon as a number of labourers have been made superfluous by these changes, and wages have accordingly fallen, then, as Ricardo failed to see, the old methods of production—in this case the old arable farming—will become more profitable; they will develop, using labour more intensively and absorb the surplus of idle labourers. It can be rigorously proved that equilibrium in this case necessarily presupposes a *division* of production between the old and the new methods so that the net profits of the entrepreneur

will be equally great in both branches of production and the total product, or its exchange value, will reach the maximum physically possible, and will thus finally increase, and not decrease.

We shall first show this by means of an example. Assume ten large estates, all of the same size and with the same natural advantages and each employing by the old methods 100 labourers. Wages are, say, 500 shillings, the gross product of each estate 100,000 shillings, and the net profit of each owner consequently 50,000 shillings.

Let us now assume that one of the landowners adopts the new method. He dismisses 50 labourers, but with the help of the remaining 50 he obtains a gross yield worth 77,500 shillings, so that his net profit is 77,500 — (50 × 500) = 52,500 shillings.

Of the 50 unemployed labourers, let us assume that 45 are absorbed into the nine other estates, or five in each, and that of these additional five workers :—

No. 1	produces an additional value of, say,	500 shillings.
No. 2	" " " " " "	490 "
No. 3	" " " " " "	480 "
No. 4	" " " " " "	470 "
No. 5	" " " " " "	460 "
	For five workers, total	2,400 "

At the same time, the consequence must be that wages will fall *all round*, let us say to 450 shillings, in which case the owner of the first estate may find it advantageous to re-employ, say, five of his previous employees. We will assume, for the sake of simplicity, that their additional product will be equal to the above, or 2,400 shillings. The final result will be :—

	Gross Product. Shillings.	*Wages. Shillings.*	*Net Profit. Shillings.*
In each of the nine old estates .	102,400	105 × 450 = 47,250	55,150
In the "new" estate after re-employing five labourers.	79,900	55 × 450 = 24,750	55,150

The total gross product, which was formerly exactly 1,000,000 shillings, will now be :—

$$(9 \times 102{,}400) + 79{,}900 = 1{,}001{,}500 \text{ shillings.}$$

Thus the result is that the total gross output has been *increased* and not diminished, and since the old estates, which employ more labourers, are more favoured by the fall in wages, they will

finally have the same profit as the "new" estate, so that there no longer remains any inducement to go over to the new methods.

In a more general form the proof is as follows: Let Fig. 9 represent the old method of cultivation and the Fig. 10 the new, in which a smaller number of labourers are employed on an equal area of land, and in which the gross product is also smaller; the net profit, however (the upper part of the area under

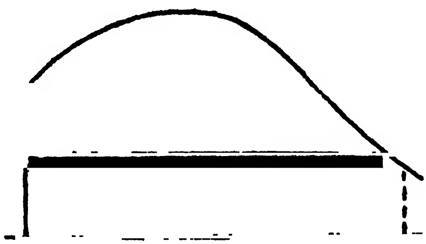


FIG. 9.

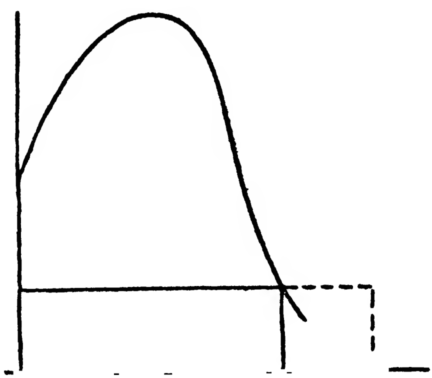


FIG. 10.

the curve), is greater. Let us suppose that one or more landowners go over to the new method of cultivation. A number of the dismissed labourers will then seek employment in the estates working on the old methods. As they are so few, they will produce on each of these estates an additional product *almost equally as great* as that of the last of the labourers previously employed, and since the net product of the estates adopting the new method is greater than previously, *the total gross product must consequently*

have increased. At the same time, marginal productivity and wages have fallen somewhat, so that the landowners' share, even in the old estates, becomes somewhat greater than before. The same process will repeat itself each time an estate goes over to the new method of cultivation, and since falling wages in themselves bring a *larger* profit to the owners of the *old* estates, as the number of labourers is greater in them than in the new, then sooner or later a point will be reached at which the net profit will be exactly the same in both, and every inducement to a further transition from the old to the new will therefore disappear. At this point, too, the total gross product will have reached the maximum.

This really follows directly from what has been said, but it can also be directly proved in algebraic form. If x and y are the number of labourers per acre on the first and second methods of cultivation respectively, and the productivity function in the one case is $f(x)$ and in the other $\phi(y)$; and if we assume that m acres are cultivated on the first method and n acres on the second, then we must look for the conditions under which the expression

$$mf(x) + n\phi(y)$$

reaches its maximum value if, at the same time,

$$m + n = B$$

and

$$mx + ny = A$$

where B is the number of acres and A the number of labourers available for the industry in question (here agriculture) as a whole. By differentiation and elimination (the partial derivatives of the first expression being put $= 0$) we can easily obtain the two equations

$$f'(x) = \phi'(y)$$

and

$$f(x) - xf'(x) = \phi(y) - y\phi'(y),$$

of which the former indicates that when the gross product is a maximum the marginal productivity of labour, and therefore wages, will be the same in both types of production, The second equation gives the same condition for rent per acre.

Thus, although at first sight the going-over of some firms to the new method of cultivation seems to diminish the total product, actually the total product is maximized; *but at the same time wages necessarily fall*, so long as we assume that the gross product is less in the estates cultivated by the new method than in those cultivated by the old.

Nor is the result any different if we assume that wages are already at the subsistence level (and cannot, according to the usual view, fall lower). In reality, wages can not only be forced below it for a little, but can remain below it indefinitely, if the labourers and their families can make up the difference by poor relief, as happened in England to a great extent at the end of the eighteenth and the beginning of the nineteenth centuries. If we assume that the available supply of labour must, under any circumstances, be somehow supported by the landowners, it would in fact be more advantageous for them to *reduce* wages to the point to which they would tend to fall as a result of free competition, and to add, by charity, enough to bring up their incomes to the necessary minimum ; it would be better to do this than to insist that every labourer employed should earn the subsistence wage. Especially after the discovery of a technical improvement of the kind in question, such minimum wage regulation might have the result that many labourers would be unemployed and, with their families, would become entirely dependent on poor relief.

Although we have so far only concerned ourselves with some of the forces at work, we may nevertheless proceed on the provisional conclusion that free competition is normally a sufficient condition to ensure maximization of production. But this maximization may very well be associated with, and even be conditional upon, a reduction in the distributive share of one of the factors of production—in this case, labour. This shows the serious error of those who see in free competition a sufficient means for the maximum satisfaction of the needs or desires of all members of society.

It might further be supposed that a result which led to a reduction in wages could not at any rate arise with the labourers as entrepreneurs ; and, on the other hand, a change in production that led to a reduction in rents would never be acceptable to landowners as entrepreneurs ; both of these results are, however, quite possible under free competition. To the individual entrepreneur who encounters a certain market rate of rent or wages, a technical improvement which increases his net return is in itself always economically advantageous. That it should have the contrary effect when all entrepreneurs follow suit does not, in general, affect the manner of procedure of the

individual, unless agreements, cartels, etc., take the place of free competition. In any case it is to be noticed that production (so far as our assumptions hold) reaches its maximum, from a technical point of view, with universal free competition. Co-operation between workers to raise wages and between employers and landowners to lower wages (in the course of which some land must remain uncultivated) would both lead to a *diminution of product*, and only if co-operation results in social collectivism could the maximum product, physically and technically possible, again be reached.

An interesting example of this is afforded, if I am not mistaken, by conditions in Swedish *forest districts*, for example, Norrland or Småland. If forest products rise in value, it may very well be that farming, which had previously been possible in such areas on occasion, will no longer be profitable, and from the point of view of the landowner it will be better to abandon farming and to plant trees on his fields. And this despite the fact that forestry obviously cannot support nearly so many men on a given area as even the poorest farming. That the owners of the land may acquire great and unearned wealth in this way, whilst wages are at the same time forced down by the superfluity of labour is a grievous wrong which should certainly be righted. But the supposed conflict between a private and public economic interest, which some people have found in these circumstances and which they have even sought to remedy by legislation, does not, if our observations are correct, exist. Indeed, the total national product will probably be greater if forestry is everywhere free to expand wherever—from the point of view of private economic interests—it is most profitable; and the superfluous labour (in so far as it cannot be absorbed into the *industries* based on forestry) seeks employment in those districts which continue, by reason of their natural advantages, to practise farming.

In other words, the evils here requiring a remedy relate exclusively to the problem of the social *distribution of income*, and *not* to that of the economically most advantageous *method of production.*

Exactly the same is true of the "parasitic" occupations much discussed in recent years, those in which the labourers, usually women and children, do not receive a living wage, but are partially supported by others (parents, relations, etc.). It is said that, in the interests of society, such occupations should be

forbidden where the employers will not, or cannot, offer full wages. Yet the only result of doing so would probably be that those now employed in them, far from having their position improved, would have to rely entirely on the support of others.

On the whole, it is a mistake to regard as obvious—as is so often done—that all healthy persons capable of work must be able to live by their labour *alone*, unless the country is (in the vulgar sense) overpopulated. On the contrary, it is quite conceivable that the total output of a society may be large enough for all, but that the *marginal* productivity of labour is none the less so small that labour has only a slight economic value. Even in a socialist state, under such conditions, the wages paid would only correspond to a part of necessary expenditure, whilst the rest would have to be found from the rent and interest of the society.

This, of course, does not exclude the possibility that the great majority of inventions and technical improvements may be beneficial in both directions; i.e. may in themselves tend to increase the marginal productivity of both labour and land, together with their share in the product. According to the ordinary rules of probability there is, indeed, an overwhelming probability that they will do so, as soon as the increase in total productivity becomes sufficiently general. If the colossal advance in all fields of production during, let us say, the last two centuries, has nevertheless brought only a relatively slight, and in many cases very doubtful, improvement in the conditions of labour, whilst rent has successively doubled and redoubled, the primary cause, as we have said, is to be found in the one-sided increase in one factor of production, namely labour, owing to the great increase in population during that period. Such an increase must, other things being equal, continually reduce the marginal productivity of labour and force down wages; or—what comes to the same thing, though the connection is easily overlooked on a superficial view—*prevent* the otherwise inevitable *rise* in wages due to technical progress. Unfortunately, collectivism cannot provide a remedy for this evil created by the labourers themselves—at any rate not in the long run.

It is scarcely possible to discover a simple and intelligible criterion which will indicate whether a change in the technique of production is in itself likely to raise or to lower wages. But in accordance with what we have said in our criticism of

Ricardo's theory, it may be asserted that, whenever the primary effect of a change in production is to cause employers to reduce the number of their employees without their having been compelled to do so by a rise in wages, it is a sign that the marginal productivity of labour has fallen and a larger or smaller ultimate reduction in wages will probably ensue. On the other hand, a technical improvement which favours labour must reveal itself from the beginning in an increased demand for labour and higher wages in much the same way as if, in the example on p. 137, technical improvements had tended to make arable farming more profitable than pastoral, instead of *vice versa*. But what we have said here applies mainly to wages and rent, in relation to each other. The appearance of capital in the field of production introduces, as we shall see below, certain modifications in our conclusions, without, however, rendering them invalid as a whole.

2. *Capitalistic Production*

A. The Concept of Capital.

We now come to the third group of factors of production —those which are commonly included in the term "capital". To give an account of the real nature of capital, its rôle in production and the grounds upon which its owners, like the owners of land and labour, claim a share in the product, is considerably more difficult than with the other two factors and has led to innumerable controversies among economists. One of the chief difficulties has been the varied and changing forms which productive capital assumes in reality. In the ordinary sense of the term, it includes all auxiliaries to production, with the exception of natural forces in their original form, and direct human labour. Thus, in the first place, it includes the houses and buildings in which work is carried on or which are otherwise necessary to business[1]; the implements, tools, and machinery with which it is conducted, and also a further very important group—livestock. Capital also includes the raw materials which are worked up, and finally—not the least important category—the provisions and other commodities

[1] To what extent dwelling-houses and durable objects of consumption are to be reckoned as capital is a disputed question to which we shall return.

which must be saved up or otherwise held ready, if labour is to be supported during the period while work is in progress. This, of course, is the commonly accepted sense of the term. Some writers, such as Stanley Jevons, go so far as to regard the last item as fundamentally including the whole of capital—that is to say, all capital in its form of free capital, before it is invested in production. This is, however, as we shall soon see, too one-sided a view of the matter.

At first sight all these requisites have only one quality in common, namely that they represent certain quantities of exchange value, so that collectively they may be regarded as a single sum of value, a certain amount of the medium of exchange, money. This also appears to be the reason for the name capital, for the word was originally understood to mean a sum of money lent, *capitalis pars debiti*—the principal of a loan as opposed to the interest. But, since the yield of production is also measured in value terms, capital, like loaned money, has the peculiarity that its share in the product—interest—is the *same kind* of thing as capital itself; interest is an organic growth out of capital, a certain *percentage* of capital, whereas wages as against labour, and rent as against land, are quite heterogeneous things. Land certainly has, especially in our day, a capital or money value, of which rent may be said to be a certain percentage, say 3, 4, 5, or more per cent, but this is, as we have already said, something derivative and secondary. Rent would remain essentially the same even if legislation forbade all purchase and sale of land, and land could consequently not acquire any exchange value; just as is nowadays the case with labour which, in contrast with earlier times, can no longer be bought or sold in the form of slave-labour.

In this connection, there is another peculiarity which is common to all, or at least to most, of what we call capital; namely, that it is itself a *product* ("produced means of production" is a common, and in a sense very good, definition of capital). Here again, it is contrasted with labour and land; or, at any rate, with unskilled labour and virgin soil. Man is born, but he is not produced—except in "slave breeding"—and the sum of natural energy, like the sum of matter, cannot be either increased or diminished by man.

The above circumstance, together with the indisputable

fact that capital greatly increases productivity, was long regarded as a sufficient explanation and defence of interest. Capital represents, it was said, "previously-done" labour (in fact, it represents, as we shall soon see, not only "previously-done" labour, but also the previously performed services of the land), and this, like all other labour, must have its reward; hence interest. Thus argued McCulloch, Bastiat, and others. In this simple manner they believed that they had discovered both a philosophical and an ethical foundation for the phenomenon of interest. The latter was especially necessary since, as is well known, all real interest, at least if it took the form of interest on borrowed money, was long forbidden in the Catholic, and to some extent in the Protestant world (though much less objection was raised, or none at all, to a landowner taking rent, even if he did not cultivate his land at all).

This explanation, however, is evidently very defective. The previously-done labour must, of course, have its wages; but these wages are not paid from interest, but from capital itself. If anybody makes a spade, a plane, or any other capital good, he obtains, by its use, compensation for his work—and he has no obvious claim to anything more. What is enigmatic is that the possession of capital, apparently at least, does procure something more, namely a permanent income in the form of interest, either without sacrifice of capital or while capital is constantly being replaced.

It is indeed true that, as a rule, the total product is increased by the employment of capital, by *more* (i.e. by a greater quantity —or value—of product) than corresponds to the capital used up in production. But this circumstance in itself requires an explanation. We may, with Böhm-Bawerk, ask why competition does not either reduce the value of the product or raise the value of capital goods [1] to such a point that the former exactly corresponds to the latter, without leaving anything over for interest. We must not simply take it for granted that capital can claim the whole of the surplus.

Strictly speaking, capital is necessary for *all* production;

[1] According to our previous assumption that prices of finished products are given in advance, i.e. determined by the world market, the former alternative should, of course, disappear; but certainly not the latter, since international capital transfers are excluded and the pricing of capital goods takes place in the home market, and must be investigated there.

in its absence the product would be more or less negligible. But can capital on that account claim the whole, or the greater part, of the product? This is impossible; for, with as much justification labour could demand the whole—and land also. There must be a division, but on what principle? The above argument gives no answer at all.

Among earlier writers von Thünen was certainly the most advanced in his conception of the nature and origin of interest. Just as he regarded the addition to the product made by the "last worker" as determining wages, so interest was determined by the "yield of the last increment of capital", but he did not follow out this thesis very far, and, indeed, it is not exactly correct. Still clearer was the light thrown on the subject by Jevons in his *Theory of Political Economy*, though unfortunately his theory of capital is still only a fragment of a complete theory. It was not until Böhm-Bawerk published his great work that we acquired a theory of the nature and functions of capital, and of the origin and determination of interest, which, in clearness and exhaustiveness, satisfies even the most exacting demands. But in spite of his brilliant style, Böhm-Bawerk's exposition is marred by a rather excessive diffuseness; its wealth of examples is sometimes confusing to the reader. On the other hand, in my opinion, his logical analysis of the subject was, in one important respect, not carried as far as would be desirable from an expository point of view. I propose, therefore, to present here Bohm-Bawerk's principal ideas in an abridged and, if possible, clearer and more comprehensible form.

B. *The Marginal Productivity of Capital. Investment for a Single Year.*

If for the moment we leave aside the question of the origin of the productivity (or value-creating power) of capital, and regard it as an empirical fact, we may readily apply to capital the theory developed above—that the share of the product going to any particular factor of production is determined by its marginal productivity. Actually this is what von Thünen attempted to do. Just as the additional product of the last worker regulates wages, so, according to von Thünen, the rate of interest on all capital is regulated by the yield of that portion of capital which

is last employed.[1] This may seem obvious, for so long as an entrepreneur obtains a larger return on the capital employed in his production than he need pay in interest for borrowed capital —or can himself obtain by lending his own—he will, of course, be inclined to increase his employment of capital. Conversely, if the interest on borrowed capital is higher than the return on the capital employed in production, or on the last portion employed, then he will, as far as possible, curtail his employment of capital to the most necessary purposes or to the more profitable branches of his production.

Further investigation, however, shows that this analogy between interest, on the one hand, and wages and rent, on the other, is incomplete. With labour and land, as we have already pointed out, the law of marginal productivity applies, with certain reservations, both to the economy as a whole and to every private undertaking. If there exists, in any place or country, a superfluous labourer or an acre of ground which are only capable of making an addition to production less than that which corresponds to the prevailing level of wages or rent, then wages and rent must tend to fall. (The fact that there may be a limit below which wages physically cannot fall, or on social grounds, cannot be allowed to fall, is a matter for separate consideration.) But this theory only applies to capital, as usually conceived, when we look at it from the point of view of the individual entrepreneur, to whom wages and rent are data, determined by the market. If we consider an increase (or perhaps a decrease) in the total capital of society, then it is by no means true that the consequent increase (or decrease) in the total social product would regulate the rate of interest. In the first instance, new capital competes with the old and thereby results, in the first place, in a rise of wages and rent, possibly without causing much change in the technical composition of the product or the magnitude of the return. For this reason, interest must certainly fall; but it need not fall to zero, or anything like it, even if the additional product of the new capital is almost nil. The increase in wages and rent may absorb the superfluous capital, so that the latter is now just sufficient for the needs of production, in spite

[1] [A mark against this passage in Wicksell's own copy of the second edition indicates that he wished to reconsider it.]

of the fact that production has in reality scarcely expanded at all.

The explanation of this curious divergence is quite simple. Whereas labour and land are measured each in terms of its own *technical* unit (e.g. working days or months, acre per annum) capital, on the other hand, as we have already shown, is reckoned, in common parlance, as a sum of *exchange value*—whether in money or as an average of products. In other words, each particular capital-good is measured by a unit extraneous to itself. However good the practical reasons for this may be, it is a theoretical anomaly which disturbs the correspondence which would otherwise exist between all the factors of production. The productive contribution of a piece of technical capital, such as a steam engine, is determined not by its cost but by the horse-power which it develops, and by the excess or scarcity of similar machines. If capital also were to be measured in technical units, the defect would be remedied and the correspondence would be complete. But, in that case, productive capital would have to be distributed into as many categories as there are kinds of tools, machinery, and materials, etc., and a unified treatment of the rôle of capital in production would be impossible. Even then we should only know the *yield* of the various objects at a particular moment, but nothing at all about the value of the goods themselves, which it is necessary to know in order to calculate the rate of interest, which in equilibrium is the same on all capital. Again, it is futile to attempt—with Walras and his followers—to derive the value of capital-goods from their own cost of production or reproduction ; for in fact these costs of production include *capital* and interest, whereas our analysis of the laws of the cost of production has hitherto proceeded on the assumption that production is non-capitalistic. We should, therefore, be arguing in a circle.

We can, however, escape from this difficulty if we refer to the common, or at least similar, origin of the various kinds of capital. We have already pointed out that capital itself is almost always a product, a fruit of the co-operation of the two original factors : labour and land. All capital-goods, however different they may appear, can always be ultimately resolved into labour and land ; and the only thing which distinguishes these quantities of labour and land from those which we have previously considered

is that they belong to *earlier years*, whilst we have previously been concerned only with current labour and land directly employed in the production of consumption-goods. But this difference is sufficient to justify the establishment of a special category of means of production, side by side with labour and land, under the name of capital; for, in the interval of time thus afforded, the accumulated labour and land have been able to assume forms denied to them in their crude state, by which they attain a much greater efficiency for a number of productive purposes—as Böhm-Bawerk, better than any other modern writer, has analysed and demonstrated in such a masterly manner.

In this circumstance is also to be found the whole explanation of the value-creating power of capital, or its so-called productivity. What emerges is simply the importance of the *time-element* in production. In the real sense, of course, only living human beings, and self-perpetuating natural forces, especially the sun and the earth's physical and chemical forces, are productive; only the original factors—man and nature. But the productivity of both becomes, or at any rate may become, greater if they are employed for more distant ends than if they are employed for the immediate production of commodities. As has been said, this increase in efficiency is a necessary condition of interest; it is the source from which it flows (just as the fruitfulness of the earth is the source of rent and the productivity of labour the source of wages); but it does not, on that account, regulate the *rate* of interest. Some part of this increase in productivity accrues, and must accrue, to the other factors of production, for their co-operation is essential and is indeed itself a part of the application of capital.

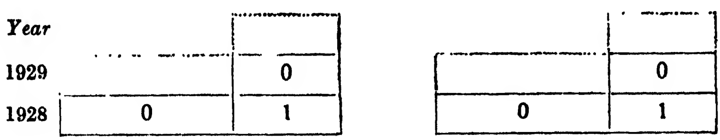
We may thus regard capital as a single coherent mass of saved-up labour and saved-up land, which is accumulated in the course of years. The addition of land is of importance; English political economy has suffered throughout from overlooking the fact that one part of capital consists of the saved-up services of land. John Stuart Mill flatly denied it. And yet this part of capital is without a doubt as important as the other. The more elaborate tools and machines may owe their existence principally to human labour; but domestic animals, raw materials, and so on, are types of capital-goods which come into

being mainly through the resources of the land incorporated in them. Trees, game, fish, and so on, when wild and uncultivated, are the sole product of natural forces (if, for a moment, we abandon the usual terminology and extend the term product to include also purely natural products). The great majority of capital-goods consist of saved-up labour and saved-up land in combination; but if these two elements are not separable in reality, we may separate them in theory, as we do in respect of labour and land as factors of production. In what follows we shall therefore speak of labour-capital and land-capital as conceptually distinct elements of the whole mass of physical capital and we shall mean by them labour and land already applied—if applied by others, bought and paid for: labour and land which have not yet ripened into finished products—*not* present or current labour and land *now* available.

A special position is occupied, as we have already remarked, by the stored-up energy derived from earlier periods of vegetation and found in coal and in ore deposits. They represent, if anything does, stored up resources of the land of much greater antiquity than any others employed in production. But since nobody has *owned* them from the beginning, they may be treated economically as stocks of raw material or semi-manufactures which are spontaneously available. In contrast to the fertility of the soil, it is largely true to say that these resources may be used up now, or left unused, according as we desire; but, on the other hand, they cannot be renewed. From the latter point of view they cannot, strictly speaking, be included in the scheme of a *stationary* economy.

We have now to consider the stratification of this volume of capital through time. Here also, we shall proceed gradually to our goal; we shall assume in the first place that, side by side with the resources of labour and land directly available for the current year's production, there exist, in the form of capital-goods, saved-up resources of the same kind from a *single* preceding year; and that these capital-goods are entirely consumed in the production of the current year. Naturally, this would bring about a considerable increase in the total product if the *whole* available supplies of current resources in labour and land were now used in the production of commodities intended for direct consumption. But, in that case, the advantage will obviously

be quite transitory and will be obtained only by the sacrifices of the preceding year and by leaving production in subsequent years in the same primitive non-capitalistic state as before. Consequently, we must suppose that a corresponding part of the resources of the current year is saved in the form of capital for next year's production, and so on. As has already been pointed out, we shall assume *stationary conditions* as the foundation of our observations. This will not prevent us from considering changes in the quantities concerned, provided that we do not take into account the actual transition stage, which is a much more complicated problem, but assume that these changes have already become final, so that "static equilibrium" (a stationary state) is again restored. We shall accordingly assume that the amount of labour and land, saved up in every year, is always the same. This presupposes a previous adjustment —which we assume to have been made—between these two



Resources of Labour. Resources of Land.

Fig. 11.

quantities; for—as we shall soon see—it may be advantageous, under given conditions, for the capitalist to save a larger amount of labour resources and a smaller amount of land resources; or *vice versa*. As soon as capital has once been formed, then *just as much labour and land will go to provide each year's production and consumption as was originally employed in the non-capitalistic state.* But since a part of these resources has been saved from the preceding year, in the form of capital, the total product will, as a rule, be considerably greater than before—at any rate up to a certain limit; and it will be greater in proportion as the part of the resources of labour and land thus employed in a saved-up form is increased.

This may be more easily understood by means of the above diagram, which represents production in the current year 1928. The amount of labour and land employed, either directly or in the form of capital, for the production of this year's supply of

commodities is represented by two rectangles, of which the left-hand divisions (0,0) represent the productive resources of the year itself, i.e. that portion which is directly employed in the course of the year. The right-hand divisions (1,1) represent the saved-up labour and land which are used in consumption this year, and the upper rectangles of the same size (0,0) that part of the current year's resources which are not employed in consumption till next year.

The dotted rectangles represent partly that portion of 1929 resources which, together with those saved up this year, will be used for the direct production of commodities next year, and partly those portions of the productive resources which will then be saved up and capitalized for the needs of the following year; and so on.

We shall—as before—assume free competition, at least in the main part of the field of production. In such circumstances, the problem of production will be essentially the same as before, except that the factors of production are now increased by two, namely the *saved-up* resources of labour and land. And it is still true that the total contribution of each particular factor of production cannot be ascertained *a priori* and does not even exist analytically. Its share in the product must therefore be determined by something else, and that something else is, for the same reason as before, *marginal productivity*. Now since experience shows that the replacement of a certain quantity of current labour and land by an equal quantity of stored-up resources of a similar kind tends in many cases to increase productivity, and since we assume that the quantity saved is only sufficient for use in these cases (and not even for all of them) it follows that the *marginal* productivity of the saved resources of labour and land is greater than that of the current resources—at any rate up to a certain point, not yet actually reached. This marginal productivity, and the share in the product which it determines, provides in the first place, a recompense for the actual capital used up in production, but it also provides something more. Under stationary conditions the exchange value of goods and services necessarily remains unchanged year after year, so that a person who, in one year, purchases labour and land in order to convert them into capital, intended for production in the following year, can always count upon obtaining more product, or value, than he has himself

paid out. This surplus is what is called interest. We thus arrive at the following definition :—

Capital is saved-up labour and saved-up land. Interest is the difference between the marginal productivity of saved-up labour and land and of current labour and land.

If conditions are *not* stationary, then of course we have to take into account changes in the value of *similar* commodities (even labour or goods of the same kind) which may occur in the course of production—and which may easily make the actual rate of interest earned negative rather than positive. That, however, is self-evident. Nothing is more common than for a large inflow of capital into a certain industry to cause so great a reduction in the price of the product that capital is employed for a while at a loss instead of a profit. The real theoretical difficulty is rather to explain how, under stationary conditions, the possession of capital can remain a permanent source of income. The application to *non-stationary* conditions offers no difficulty in principle.

So far as I can see, everything which can be said in explanation of this phenomenon is said in the italicized passage above. Of Böhm-Bawerk's *three main grounds* why "present" goods possess a higher value than future goods (or past goods higher than present goods), the first refers to the difference between wants and their satisfaction in the present and in the future ; the second to the subjective undervaluation of future needs and overvaluation of future supplies. These considerations, however, are only indirectly significant for the productive employment of capital. Those who borrow capital for the purpose of production will not, because of anticipated future supplies or of subjective overvaluation, pay more in interest than they actually obtain themselves by the technical employment of capital. (They may well be induced in this case to use some of the borrowed money unproductively for their own consumption and, to that extent, diminish the supply of capital and thus raise the rate of interest.)

On the other hand, these considerations play a very important rôle in the actual *accumulation of capital* ; and in its converse, the unproductive consumption of capital, as in loans for consumption purposes. Both logically and for purposes of exposition it would seem right to begin by examining the effects

of a given supply of capital already accumulated, and *then* to inquire the causes which influence, and eventually alter, this supply. Thus there remains only the third of Böhm-Bawerk's main reasons, namely the technical superiority of the commodities or means of production available from an earlier stage over those which will only become available at a later date. His reasoning in this connection essentially coincides with that which we have already advanced and which we shall develop further; but it is, as a comparison will show, considerably more complicated, and therefore probably not so intelligible as our own. This is mainly due to the fact that Böhm-Bawerk neglected to base his argument on the fundamental simplifying assumption of stationary economic conditions, though he did not really achieve any greater degree of generality. Moreover, he cannot be entirely absolved from the charge of trying to prove too much when he maintains that a "present" means of production, e.g. a month's labour available now, would be, *under all circumstances*, technically superior to one available in the future. That, of course, is not the case. There are a number of cases in which current labour and land must, from technical necessity, be employed in their original form and cannot in any way be replaced by stored-up productive power. But this is not the point; it is rather that the marginal productivity of the latter is greater, simply because current labour and land exist in relative abundance for the purposes for which they can be employed, whilst saved-up labour and land are not adequate in the same degree for the many purposes in which they have an advantage. This again is to be explained by the circumstances which limit the accumulation of capital.

It is also clear that interest, at any rate within the limits of the single year's investment here contemplated, must, according to our definition, be the same in all enterprises and all kinds of employment, and especially that the marginal productivity (and the share in the product) of saved-up land must stand in the same relation to that of current land as does saved-up labour to current labour. Otherwise it would be profitable to save more labour and less land on the next occasion, or *vice versa*. We may remind the reader, in passing, that the technical renewal of capital from year to year, which is here assumed, by no means excludes the accumulation and maintenance of capital by the

individual for possibly remote future use. Such an individual need only buy up labour and land in the market in one year in the form of implements, slaughter animals, etc., sell them in the following year, and thus repeat the same operation. In other words, the duration of "private capital", or, more correctly, of the ownership of "private capital", has nothing to do with the technical period of turnover of "social" capital.[1]

If we assume that the whole of the accumulated capital—in the form of tools and implements, domestic animals, raw materials, etc.—consists of A labour years and B acre years, i.e. of the total production in the last year of A labourers and B acres, and if l represents wages per labourer and r rent per acre then the value of capital in money or products will clearly be $A.l + B.r$. If, in the current year, there are employed in a particular business a workers and b acres of the current year, and a_1 labour years and b_1 acre years of the preceding year, turned into capital in one form or another, then the total product during the year may be regarded as a function of all these quantities, i.e. $F(a, b, a_1, b_1)$.

The partial derivatives of this function with respect to each of the variables will be on the one hand, $F_a = l$, $F_b = r$, i.e. wages and rent for current labour and land, and, on the other hand, $F_{a_1} = l_1(> l)$, $F_{b_1} = r_1(> r)$, or what may be called wages (including interest) for the saved-up labour and rent (including interest) for the saved-up land. Equilibrium clearly demands that $l_1 : l = r_1 : r$. The two equal quantities

$$\frac{l_1 - l}{l} = \frac{r_1 - r}{r} = i$$

will then each represent the rate of interest on the investment of capital for one year. Interest, or that part of the product which falls to capital, thus equals in the particular business $(a_1.l + b_1.r).i$; and the interest on the total accumulation of capital will equal $(Al + Br)i$—on the assumption that, under free competition, and in equilibrium, all capital will receive approximately the same return.

[1] A primitive form of the employment of capital mentioned by Aug. Bondeson in one of his rustic novels, is the communal use of sheep; i.e. sheep or other cattle are bought by small rural capitalists or old farm hands and let out for the summer, after which the profit is divided between the owners of the animals and of the land. In this case the life of the capital-good is, on the average, short, though it does not prevent the prolongation through decades of individual capitalistic holdings (and accumulations).

If we now compare two otherwise similar stationary states, both investing capital for a single year, but in one of which there is *more* capital employed, that is to say, in each year *more* labour and land are saved up for the following year than in the other case, a difficult, but extremely important, question will arise ; what influence will the increased employment of capital exercise on wages and rent or, in other words, on the share of the product accruing to labour and land in the current year ?

The fact that their marginal productivity is, normally (as we have seen) less than that of *saved-up* labour and land does not, indeed, prevent it from being increased by the increased use of capital. This may well appear obvious ; for, in any particular year, current labour and land participate in the direct production of commodities in smaller and smaller quantities, the more the capitalistic method of production is extended ; and it might be supposed that this would necessarily imply a relatively increased marginal productivity of those factors of production. But the matter is not quite so simple. Of course, *ceteris paribus,* a relative reduction in the supply of a factor should cause an increase in its marginal productivity ; and the increase in the product due to capital would thus accrue in part to capital, and in part to the other factors of production. But if the accumulation of capital coincides, as is usually the case, with technical discoveries and technical progress, it is quite conceivable that, despite increased employment of capital and increased production, the marginal productivity and the distributive share of current labour and land will be *less* instead of more. Only in so far as production in given technical conditions is *saturated with capital,* is it certain that wages and rent—usually both—will rise, whilst interest falls. Translated into our terminology, this means that the marginal productivity of labour and land in the last case gradually *increases* whilst the marginal productivity of saved-up labour and land decreases —so that the difference between them is successively reduced and may finally disappear altogether ; interest falling to nothing and the capitalists' share in the product consisting only of compensation for the saved-up labour and land employed, i.e. for the capital itself.

In the following section, we shall apply this conclusion to the more complex case of capital investment over a period of years.

C. Capital Investment over a Period of Years.

Before an excess of capital caused interest to fall to nothing, investment for a single year would in reality have given place, for the most part, to investment for a period of years. We shall now examine how this comes about. It is sufficient for our purposes to suppose labour and land to be saved up for no more than two years; investments is thus to be either for one year, or for two. What we have to say in this connection can easily be extended to processes of production and capital investment over any period whatever. We shall also ignore for the present the period of transitión, during which capital is accumulated for the first time and is suitably distributed over the period of production in question; we shall only concern ourselves with conditions as they are after full equilibrium has been restored.

Each particular year's production is now due (1) to current labour and land, (2) to resources which have been saved and capitalized during the *two* preceding years. But on the other hand, if conditions are to remain stationary, two quantities of labour and land (exactly corresponding to these) must be withdrawn from the production of consumption goods during the current year and devoted (1) to production of goods which will only be used in the following year, (2) to goods which will only be used in the year after that. Even this does not exhaust the list of capital goods existing at the moment; for there exists at the same time a group of services of labour and land saved up during the *immediately preceding* year and intended for employment only in the production of the *next* succeeding year. For this reason, they are to be regarded in the current year only as items to be carried forward—as it were, goods in transit. (Of course, in reality, the various annual groups of saved-up labour and land are not always so strictly separable, but are often combined in the same capital-goods—of which more later.) In the same way, if resources were saved up for three years, the labour-capital (and land-capital) available at any moment would fall not merely into 3, but into $3 + 2 + 1 = 6$ distinct groups (cf. the following paragraph); and so on, *mutatis mutandis*, for more extended capital investments. Thus the number of capital groups grows, as it were, both in height and

breadth, or as the *square* of the number of years. This, as we shall see, is a circumstance of great importance.

The following diagrams, which represent the supply of current and saved-up labour and land, at the present moment, (1) in capital investment for one and two years, (2) in capital investment for one, two, and three years, explain themselves. The figures 1, 2, 3 indicate that the capital groups concerned are 1, 2, or 3 years old, i.e. originate in 1927, 1926, or 1925. By 0 are

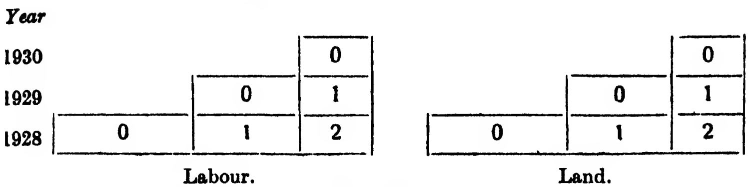


FIG. 12.

represented the current resources of labour and land, whether used in direct production for the year or saved and capitalized for the production of succeeding years. The years marked on the left are to be conceived as representing the year in which the existing capitalized productive forces on the same horizontal line are employed for the production of consumption-goods, and this naturally presupposes that they will co-operate partly with current labour and land of the same year, and partly with those

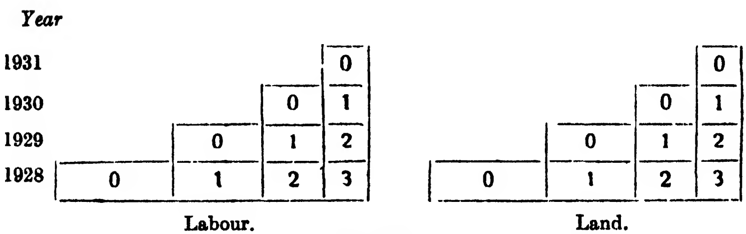


FIG. 13.

saved-up and capitalized during preceding years for use in a future year.

The sum of the rectangles indicated by 1, 1, and 2 (Fig. 12), or 1, 1, 1, 2, 2, and 3 (Fig. 13) represents the total supply of capital-goods in existence at the beginning of the present year, although only a part of them is employed—or, which amounts to the same thing, is consumed—during the course of the year. The rectangles one step higher up, identical in size and number,

indicated by 0, 0, and 1 (Fig. 12), or 0, 0, 0, 1, 1, and 2 (Fig. 13), represent the supply of capital at the end of the year.[1]

If we return to our *one–two year* capital investment, it is clear that the labour and capital saved-up for *two* years will be remunerated in accordance with its marginal productivity. If we consider the extremely primitive nature of the implements, domestic animals, etc., which are possible with investment for a single year, and the enormous improvement in the technique of production which would be possible in many fields with investment for two years, we shall easily see that the marginal productivity of two-year-old capital must, within very wide limits, be greater than that of one-year-old capital and *a fortiori* than that of current labour and land. But it should be carefully noted that this does not mean that, in all such cases, investment for two years would be *profitable.* For that to occur the three above-mentioned quantities must stand in a certain determinate relation to each other, corresponding to that which exists in a calculation with compound interest. In other words, if the marginal productivity of one-year-old capital (i.e. labour and land saved-up for one year) is related to that of current resources as, for example, 1·05 to 1, so that one-year-old capital yields 5 per cent interest, then the marginal productivity of two-year-old capital must necessarily be related to that of one-year-old capital at least as 1·05 to 1 ; and consequently to current resources of labour and land as $(1 \cdot 05)^2$ to 1, so that two-year-old capital will yield *at least* 10¼ per cent interest for its two years. This is obvious, for otherwise anybody who wished to save capital for two years or more would prefer to split up the hypothetical two-year capital investment into two successive one-year investments—so that the *technical* period of turnover of capital would still be only one year.

On the other hand, it may be asked whether the interest on two-year investments could not be permanently more than double, say three of four times, that of one-year investments. A levelling in the opposite direction cannot take place so directly, since those who desire the return of their capital after the lapse

[1] If all the rectangles were of the *same size*—and the co-operation of land in production were omitted—the above left-hand diagram might serve as an illustration of Böhm-Bawerk's famous example of a continuous "staggered" production. (*Positive Theorie des Kapitals,* 3rd edition, book iv, part ii D. In earlier editions, book iii, part v.)

of one year have no other choice, it might be supposed, than a one-year capital investment. But, in an advanced economic system, credit enters at this point as a levelling factor. So long as the total amount of social capital remains unchanged year after year (and of course still more if it continuously grows), the technical period of investment is a matter of indifference to the individual capitalist. As against those persons who, in the course of the year, desire to call in and consume all or some of their capital, there would (at least) be an equal number simultaneously desiring to build up new capital to the same amount. The transfer of capital from the former to the latter, and of the corresponding exchange values in money or consumption goods from the latter to the former, might be effected by a simple credit operation without the necessity for the simultaneous liberation of any real capital in the technical sense. Interest rates for long and short periods do, in reality, tend to be equal; the difference which actually exists should be regarded partly as an increased *risk premium* for long-term loans, partly as due to the fact that, under existing economic conditions, short-term debts on good security are largely used as cash (money substitutes), a fact with which we cannot here concern ourselves. Thus, in the supposed circumstances, one-year capital investments in the technical sense would be exchanged more and more for two-year investments until interest on the latter became slightly more than double, or, calculated per annum, *as great* as the former. If this levelling has been achieved and full equilibrium restored, it is easy to see that the surplus marginal productivity of all the groups of capital employed during the year, i.e. the total profit on capital of the year, constitutes *one year's interest* on the whole value of the total capital, each capital group being regarded as representing the value of the labour and land employed, *together with the accrued interest.* The same naturally applies to longer capital investments, so that there is complete agreement between theory and practice.

The whole available capital will now be distributed between one-year and two-year investment—since, for the moment, we ignore the possibility of longer dated investments—and in a definite proportion; so that the above relation between the marginal productivities will obtain. If capital *increases*, i.e. if

the accumulated quantities of labour or land, or of both, are increased, we may suppose that new capital, and consequently ultimately the whole volume of capital in existence, will also be distributed in the same proportion as the old capital between these two periods of investment. Yet this does not usually happen. Such an increase must of itself, in view of what we have said, and apart from simultaneous technical inventions, etc., reduce the marginal productivity of saved-up resources and, at the same time, increase the marginal productivity of current resources. Excepting for the case where a uniform increase of both has a specially marked tendency to reduce the marginal product of resources invested for two years, so that we may suppose the marginal product of each to fall in about the same proportion, then it may easily be seen that the relation between the yields of the two forms of capital will be necessarily disturbed to the advantage of the *longer-term* investment; the interest on both one-year capital and two-year capital has fallen, but that on two-year capital is now somewhat more than double that on one-year (perhaps two and a half to three times as high). Investment for two years is thus relatively more profitable than before and extends to fields which it had previously not entered; whilst one-year investment expands relatively little, or may even contract. Thus, in the end, the relative marginal products of both are brought back to the right relation. In addition to this, investments for three, four, or five years, etc., which have previously been unremunerative, in spite of their higher marginal productivity, now yield a profit and will therefore be made.

If we represent the marginal productivity of two-year labour and land by l_2 and r_2, respectively; then, in equilibrium, we must have

$$l_2 : l_1 = l_1 : l = r_2 : r_1 = r_1 : r.$$

If we represent this common ratio by $1 + i$, then

$$l_1 = l(1 + i),\ l_2 = l(1 + i)^2 = \text{about } l(1 + 2i),$$

and similarly for r_1 and r_2. Now if l_2 and l_1 are reduced in the same proportion relatively to l (for example in the ratio $1 : 1 - \epsilon$ where ϵ is a proper fraction which is not too small) we obtain

$$l_1 = l(1 + i)(1 - \epsilon) \text{ or, approximately, } = l(1 + i - \epsilon).$$

But, on the other hand,

$$l_2 = l(1 + 2i - \epsilon) > l(1 + i - \epsilon)^2.$$

If, in this case, $\epsilon > i$ then one-year capital investment would show a loss and would certainly be contracted; if $\epsilon > 2i$, the two-year investments must also contract and the centre of gravity of capital investment would shift to longer investments; and so on. If, as in the above example, the rate of interest is 5 per cent per annum, and if, owing to the accumulation of new capital, the marginal productivity of one- and two-year capital goods is reduced relatively to that of current labour and land by, say, 1 per cent, then one-year interest will consequently fall to 4 per cent, but two-year interest to only about 9 per cent instead of what it should be in equilibrium—namely $(1{\cdot}04)^2 - 1$, or rather more than 8 per cent. Two-year capital investment thus becomes (absolutely less but) relatively more profitable than before. Under certain simplifying assumptions, such as those made by Böhm-Bawerk and by ourselves in the next chapter of this work, this fact, which is of fundamental importance for the whole of the theory of capital, and whose significance was already recognized by Ricardo, can be proved mathematically as a universal principle.

This has important consequences for the remuneration of current labour and land, i.e. wages and rent. An increased investment of capital itself tends, as we have seen, to reduce the quantities of current labour and land available for each year's direct production, and consequently to raise their marginal productivity. If, however, a relatively larger share of this capital than before is placed in two-year investments, and the capital is thus *divided into two different parts*, one of which is only used in the next year, then clearly there will be a reduction, at any rate relatively, in the quantities of accumulated labour and land employed each year; but, at the same time, there will also be a reduction in that part of the current labour and land which must be saved and capitalized each year to renew that which is consumed. A larger part will remain over for the current year's direct production of consumption goods, whilst, at the same time, its marginal productivity will fall. It is the peculiarity of capital that, when it grows, it grows in height as well as in breadth, and in this there is a counter-weight to the tendency for an increase of capital to raise wages and rents.

Other things being equal, however, this last tendency can

never be entirely overcome. Inevitably wages and rents (or at any rate one of them)[1] will finally rise—though not so much as one might at first suppose—as a consequence of the increase of capital as such. But the position is different where, as may easily happen, some *technical invention* renders long-term investment, even without a simultaneous growth of capital, more profitable (absolutely) than previously. The consequence must necessarily be—so long as no further capital is saved—a diminution in the "horizontal-dimension" and an increase in the "vertical-dimension", so that the quantity of capital used in the course of a year will be reduced; an increased quantity of current labour and land will consequently become available for each year's direct production; and, although this need not necessarily cause their marginal productivity and share in the product to be reduced —since the total product has simultaneously been increased by the technical discovery, yet a reduction may clearly result. The capitalist saver is thus, fundamentally, the friend of labour, though the technical inventor is not infrequently its enemy. The great inventions by which industry has from time to time been revolutionized, at first reduced a number of workers to beggary, as experience shows, whilst causing the profits of the capitalists to soar. There is no need to explain away this circumstance by invoking "economic friction", and so on, for it is in full accord with a rational and consistent theory. But it is really not capital which should bear the blame; in proportion as accumulation continues, these evils must disappear, interest on capital will fall and wages will rise—unless the labourers on their part simultaneously counteract this result by a large *increase in their numbers.*

That the transformation of circulating into fixed capital, i.e. the change from short-term to long-term capital investments, may frequently injure labour is beyond doubt. But Ricardo was mistaken in his belief that this consequence was due to the fact that the gross product is simultaneously reduced. This is, as may easily be proved, theoretically inconceivable. The gross product under free competition (where such is at all possible)

[1] This observation must be made, for capital investment undoubtedly tends to disturb the conditious under which labour and land are able to replace each other at the margin of production. It may therefore happen in exceptional cases that wages alone reap the benefit of a growth of capital, whilst rents fall; or *vice versa*. (Cf. also p. 215.)

always tends in the main towards the maximum which it is physically possible to obtain with the existing means of production.

In my work, *Über Wert, Kapital und Rente* (Jena, 1893), p. 104, I pointed out the easily-intelligible fact that, if capitalist employers by common agreement extend the period of production, and thereby the period of capital investment, beyond the point consistent with their interests under free competition, their profits will rise, because, with an unchanged quantity of capital, wages and rents calculated in money or goods must necessarily fall.

But, at the same time, the *annual product* would, up to a certain point, *increase*—a fact which may appear to conflict with the general principle that free competition brings about the maximum return from production.

If, however, we regard capital, as we should do, *genetically* (i.e. as the total of a number of years' accumulation of labour and land) then it is clear that, in this case, there has actually been an increase in the volume of social capital—that is, an accumulation of real capital—at the expense of labourers and landowners, who do not receive its fruits unless, by co-operation, they succeed in obtaining better conditions in the future, by profit sharing, and so on. A somewhat similar phenomenon may occur as a result of the operations of entrepreneurs in the money and credit markets—as we shall see in the next volume.

But the assumption underlying the principle outlined above was that all the factors of production had a given and constant magnitude and, to this extent, it holds good, even though it may be difficult—if not impossible—to define this concept of social capital with absolute precision, as a definite quantity. In reality, it is rather a *complex* of quantities.

We have now completed the foundation of our static theory of capital. The complications which we must still take into account in passing from abstract theory to the concrete phenomena of reality are not questions of principle, and present only difficulties of detail in mathematical treatment. The most important among them is that, on the one hand, labour and land *of different years* are incorporated in one and the same capital-good; and, on the other, that a capital-good is not, as we have hitherto assumed, consumed in one year's (direct) production,

but often serves for many, sometimes for a long succession of years—so that the productive forces embodied in that good only come into employment successively. What exactly is consumed in each particular year cannot, as a rule, be determined. But even in this case, the law of marginal productivity must be fully satisfied in equilibrium, for otherwise it would undoubtedly be profitable, at some point in production, to transfer resources, either by simultaneously decreasing—or increasing—the factors employed at some other point in the period of production, or by increasing or diminishing the value of the capital-good. For example, suppose that a machine has been constructed in the course of three years and is afterwards used for twelve years before it becomes necessary to scrap it. If, in the construction of the machine, an additional quantity of labour, say one day's labour, had been added in the first year of production, then the utility of the machine might possibly have been increased by, let us say, the value of three consecutive days' work during the last year of its use. This day's labour would yield an interest of about 8 per cent; for $(1 \cdot 08)^{14} = 3$ approximately.

This rate of interest must agree with the rate prevailing elsewhere, for, if it were higher, it would be profitable (in future production) to employ more labour on this kind of machinery; if it were lower it would be advantageous, in the future, to content oneself with machines of inferior quality and utility, which cost less in labour or land for their production.

It is, of course, another matter that some forms of capital (such as houses, railways, certain forms of improvements of land, etc.) normally last so long that the quantitative and qualitative adjustments, theoretically necessary for attaining equilibrium, become impossible in practice. Unless we wish to extend our observations over periods of time in which centuries are mere episodes, we must content ourselves with noting that there is always a *tendency*, perhaps very incompletely realized, working in the direction indicated above. Of especial importance is the reservation regarding periods of great industrial development, in which equilibrium is usually conspicuous by its absence. We shall consider certain questions of this kind in greater detail in a later section.

Note on Böhm-Bawerk's Theory of Interest

What has been said above modifies and completes Böhm-Bawerk's theory—a theory which has been the object of more or less acute criticism by numerous economists. The great majority of the objections raised are, in my opinion, based entirely upon misunderstanding or on an inadequate appreciation of his reasoning. But some, or rather *one*, of them does not entirely lack justification, although, as far as I can see, it by no means destroys the foundations of his theory. I shall, therefore, give a brief résumé and criticism of Böhm-Bawerk's theory of interest as he presented it.[1]

The first part of his main work, *Geschichte und Kritik der Kapitalzins—Theorien* (Capital and Interest), I must omit. In my opinion, Böhm-Bawerk was entirely successful in showing how untenable are all the earlier attempts at explanation which emphasize inadequately, or not at all, the importance of the *time-element* in the phenomena of production and value.[2] With earlier writers, such as von Thünen, Senior, and others, who really do consider this element, it seems to me that Böhm-Bawerk's criticism is carried much too far and is sometimes merely hair-splitting. In particular, I agree with Cassel [3] (while profoundly disagreeing with his general opinion of Böhm-Bawerk) that he scarcely did full justice to Ricardo. However fragmentary Ricardo's theory of interest may be, it appears to be quite correct so far as it goes. Among other things, it contains, in a somewhat different form, one of the corner stones of Böhm-Bawerk's own theory. The passage in Ricardo to which I refer is to be found in chapter i, part v, of his *Principles*. Ricardo there raises the question why the employment of labour-saving machinery is always more profitable with high than with low wages, although at first sight it might appear as if machinery, being itself a product of labour, would rise in price with a rise in wages. With great acumen Ricardo shows that this cannot be the case: the price of machinery includes interest as well as wages, and if wages as a whole have risen, then, other things being equal, interest must fall. (The purchaser who uses the machinery must, for the same reason, reckon a lower interest on the purchase price of the

[1] I have treated this subject in greater detail in an essay in *Ekonomisk Tidskrift*, 13 (1911), p. 39 *et seq.* [Cf. also vol. 16 (1914), p. 322 *et seq.*]

[2] Before Böhm-Bawerk wrote, Professor Davidson had, in his early and valuable essay on "De Ekonomiska lagarna för Kapitalbildringen" ("The Economic Laws of Capital Accumulation"), subjected the so-called "use theory" of Hermann to a criticism, which though brief, essentially corresponds with that of Böhm-Bawerk, whose fundamental ideas he often anticipates.

[3] *Nature and Necessity of Interest*, 1903.

machinery.) This is fundamentally the same reasoning as that with which Böhm-Bawerk proves (as we have done above) that a rise in wages must lead to a lengthening of the period of production or of capital investment.

It also follows from what has been said, that a rise in wages may lead to increased use of machinery for another reason: machinery is used as a means of replacing labour by land, if rent has not risen to the same extent as wages.

The second part of Böhm-Bawerk's work, his *Positive Theorie des Kapitals*, will always retain its place as one of the finest achievements of economic theory; but even there he did not succeed in unifying his theory completely. It seems to rest on two (or even three) different and imperfectly co-ordinated foundations.

Already in his Introduction we find the brilliant suggestion that we should regard the *capitalistic process of production* ("the adoption of wisely-chosen round-about methods") as the *primary* concept and capital itself as the *secondary*—"the complex of intermediate products emerging at the various stages of the round-about process of production taking time". This idea, which renders all further discussion of the nature and content of the capital concept unnecessary, is subsequently developed in the masterly book ii, "On the rôle of capital in production and on the accumulation of capital." The theory is only finally completed, however, in the chapters on the origin of interest and the height of the rate of interest [1]—particularly in the second section of the latter chapter, on the determination of the rate of interest on the market. In these, for the first time in the literature of economics, a proper account is given of the relation between wages and interest and, to that extent, a solution is advanced to the problem of distribution under free competition, albeit on greatly simplified assumptions and with the deliberate exclusion of land as a factor of production. These parts of his work may be read by themselves, and constitute a complete whole of the very greatest scientific importance and value. And yet, here also, Böhm-Bawerk was not entirely consistent, for in his account of the quantitative factors determining interest he reverts, probably for reasons of exposition, to the earlier Jevonian conception of capital as a *subsistence fund*, a sum of (potential) wages; so that capital again becomes the primary, and the capitalistic process of production the derivative, concept.

[1] [Book iii, chs. 4 and 5 (first edition); Book iv, chs. 2 and 3 (3rd and 4th editions).]

The long section of the book which lies between these two portions is of an essentially different character; and it is this section which has received by far the most attention from his critics. After an exhaustive account—excellent for the purpose—of modern theories of value and prices (in their "Austrian" form) he proceeds (under the heading "Present and Future in Economic Life") to his well-known theory of interest in its widest sense. He here puts forward the doctrine that interest is originally an exchange phenomenon (and thus no longer exclusively the result of production and distribution)—it is the *agio* which arises in the exchange of present against future goods. This treatment may be justified, in so far as interest is undoubtedly a broader concept than productive capital itself. It can arise in a mere exchange of present against future goods or services *without* any intervening production and thus without any real accumulation or employment of capital. But the proof is not quite convincing. In Böhm-Bawerk's opinion, the difference in value between present and future goods which comprises this *agio*, originates, like all other exchange values, in their different *marginal utilities*. But at an earlier stage, Böhm-Bawerk himself had defined marginal utility as "the significance of the least significant of the concrete needs or partial needs which are satisfied by the available supplies of the commodities of the kind in question", and we may add, in full agreement with the whole trend of his reasoning, "during a given consumption period." But if we seek to apply this directly to present and future goods, the difficulty clearly arises that both the *supply* (of future goods) and the *period of consumption* are quite indeterminate. This difficulty is not overcome by comparing, as Böhm-Bawerk sometimes does, present and *past* goods. In that case, of course, the supply of the latter is known (it is the quantity of available capital-goods), but the period of consumption remains indeterminate; for it is far from true that all existing present and past goods are to be employed in the consumption of the current year.

Böhm-Bawerk endeavours to *circumvent* this serious difficulty, for he clearly asserts that, in all possible cases—or, at any rate, in the great majority ("in aller Regel")—the utility of present goods is greater *absolutely* than that of future goods (and less than that of past goods) of the same kind and quantity; from which it must follow that their marginal utility, and consequently their value and price, must also be greater. But this position is evidently untenable. His argument is relatively most successful

when applied to the *second* of the three grounds cited as causing the superiority of present goods, namely the subjective undervaluation of future needs and the overvaluation of future resources —due to lack of imagination or weak will. This phenomenon is undoubtedly general, and so long as it exists it creates a (subjective) over-stress on present goods. But even the *first* of the main grounds—the existence of an, objectively, more abundant future satisfaction of needs—is evidently not general in its application. The circumstance adduced by Böhm-Bawerk that those who expect a *less* abundant satisfaction of their needs can always hoard existing commodities (especially the precious metals and other durable goods) cannot, in itself, be a guarantee of a positive rate of interest, but only implies that interest cannot fall lower in a negative direction than would correspond to the risks and costs associated with the storing of these objects.

Equally unsatisfactory is the treatment of the *third* main ground; the technical superiority of present goods—including present agents of production—over future goods. This part of Böhm-Bawerk's exposition is, indeed, the one which is most open to criticism. Proceeding from his general theory of the profitability of round-about methods of production, he argues that a certain quantity of present factors of production—for example, a labour-month—must inevitably have a greater value than an equal amount which is available at a future date, say next year; the former can be employed as a link in a longer process of production than the latter and must consequently be more fruitful, *whatever point in the future is regarded as the final point of production.* This is undoubtedly wrong, for the principle of the advantage of round-about methods of production by no means implies that the productive process might be successfully prolonged for an *indefinite* time. In order to avoid the absurd argument that, in such a case, all production might be infinitely prolonged, Böhm-Bawerk refers to the "first and second main ground", as bringing the "economic centre of gravity" to a nearer date; but this is merely a last resort, not to be taken too seriously. What really limits the length of productive processes —as Böhm-Bawerk himself quite clearly points out later, in book iii, chapter 5 [1]—is not this, but simply the circumstance that a longer period of production, even if technically more productive, would yield to entrepreneurs (whether capitalists, labourers, or a third party), with the available supplies of labour and capital,

[1] [In later editions, bk. iv, ch. 3.]

a smaller *profit* than the productive processes actually begun. This has already been shown in the foregoing.

Böhm-Bawerk's real error—his cardinal error, as Bortkiewicz calls it—is that at this point in his exposition he seeks to solve the problem of the *existence* of interest—as distinct from its actual rate—without referring to the market for capital and labour. This error had already been pointed out by Walras and is, indeed, the only one of major importance which can be attributed to Böhm-Bawerk.[1]

In a subsequent part of his work, Böhm-Bawerk himself completely rectified this error. It may therefore justly be said that the work contains, albeit in a somewhat imperfect form, the real and definitive theory of capital, whereas Walras and his successors (Pareto, Barone, and others) still continued to hold a theory of interest which contains both formal and material defects and which is seriously incomplete. Walras' formula for interest, as may easily be seen (cf. the preface to the second and subsequent editions of his *Éléments d'économie politique pure*) reduces itself, on the assumption of *stationary* conditions, simply to the equation $F(i) = 0$, in which $F(i)$ is the amount of annual savings conceived as a function of the rate of interest i. In other words, it expresses the truism that, in the stationary state, the inducement to new savings must have ceased; but it affords no answer to the question why a given amount of existing social capital gives rise to a certain rate of interest, neither higher nor lower. The importance of the time-element in production was never properly appreciated by Walras and his school. The idea of a *period* of production or of capital-investment does not, as we have said, exist in the Walras-Pareto theory; in it capital and interest rank equally with land and rent; in other words, it remains a theory of production under essentially non-capitalistic conditions, even though the existence of durable, but apparently indestructible instruments, is taken into account. In the same way, Barone, who, in the essays in the *Giornale degli Economisti* cited above, approached the views of Böhm-Bawerk, appears, from a later essay in the same journal, to have reverted to the earlier unsatisfactory point of view.[2]

[1] There is, however, no question of an error in Böhm-Bawerk's theory of capital, but in my opinion only of a lack of clearness in exposition, for which reason I do not think it necessary to examine his reply to Bortkiewicz, to whose criticism as a whole I cannot subscribe.

[2] [Cf. Wicksell's article *Zur Zinstheorie* (*Böhm-Bawerks Dritter Grund*) in *Die Wirtschaftstheorie der Gegenwart, herausgegeben von Hans Mayer*, iii, 199–209 (1928). The manuscript was copied and despatched soon after the author's death.]

D. *An Alternative Treatment of the Problems of Interest and Distribution.*

The following method of considering interest is designed to bring out the importance of the time-element, which is the real kernel of the capital concept.

Let us begin with the simplest conceivable case of the employment of capital; this undoubtedly occurs in that form of production where the original factors, land or labour (or both), are *used only once*, as it were in an indivisible moment of time, after which their fruits are spontaneously matured by *free* natural forces. A concrete example of this kind (at any rate approximately) is to be found in the laying down of wine for consumption — a copybook example rightly favoured by economists; or alternatively in the planting of trees on barren land (where no question of rent need enter during the period of growth) and so on. In such cases, the function of capital is merely to preserve, for a longer or shorter period, the services of the labour and land in question; or, where hired labour or land is used, to advance wages or rent for the corresponding period. If the total supply of labour and land is given, the *length of time* will thus be *the only variable dimension of capital.* If, in such a simple case, we are able to deduce the general laws of capital and interest, this deduction may be regarded as an essential ingredient in the explanation of all the more complex phenomena of actual employment of capital.

Let us imagine a country or district which, as far as its land, labour, and capital are concerned, is a *closed* economy and which by reason of the nature of the land and climatic conditions, produces only a single commodity—let us say a certain kind of wine—in exchange for which it obtains all other commodities from neighbouring countries or districts.

Let us further suppose the price of the matured wine to be determined in advance on the market in such a way that, within certain limits (not reached in practice) it *increases* continuously with the *age* of the wine. The annual vintage, say one million hectolitres, we regard as the product of land and labour only; and for the sake of simplicity we ignore the capital employed in the viniculture itself—though in practice this is very important. The price of the grape juice V_0 (per hl.) may thus be entirely

resolved into wages and rent. How it will be divided between them (since we ignore the labour required in later stages) is a problem of exactly the same kind as we have considered in the previous section (ii, 1) and with which we need not further concern ourselves. We might even assume, without violence to the general applicability of our principle, that the whole value of the raw product consists of wages only, by assuming that the use of the land is *free*.

The price V_0 is still an unknown quantity and must be carefully distinguished from the price W_0 which the new wine would command if it were *now* offered for consumption. But we shall assume that the latter alternative is not in question, as it would be too uneconomical. Rather the whole vintage will be stored, either by the producers or by other entrepreneurs, for a number of years—in order that it may be sold to greater advantage. How *long* it will be stored depends, as we shall soon see, exclusively upon the amount of the existing capital, which, on our assumption of a closed economy, can neither be increased by additions from outside nor diminished by export. The whole of the circulating capital of that society will consist of stored wine, though it can at any time be wholly or partially converted into money; we still make no definite assumption about the *value* of this capital in terms of money, but we assume that it just suffices for each year's vintage to be stored for a particular period (say four years).

In that case, *as a rule,* the 4-year storage period must be the one which is the most profitable from the point of view of the individual vine growers. For if, at the current price of grape juice, or, in other words, at the current rate of wages (or wages and rent combined), a 5-year storage period would be more profitable (would yield a higher rate of interest) it would be preferred by some or all owners of the wine; but since the total capital is not sufficient for that, the consequence would be that at subsequent harvests a smaller amount of money would be available for the purchase of grape juice, so that the price of grape juice, and consequently wages and rents, would fall. If, however, the price of the new wine was lower (as our arithmetical example below will show) it can easily be proved that *a shorter* storage period would be more profitable than the one which had previously yielded the best return.

Again, if the price of new wine (on the home market) were so low that a storage period of only three years was the most profitable from the individual point of view, then, on our assumption, there would now be an excess of capital, so that *more* than the sum previously available each year from sales would be devoted to the purchase of new wine. The price of new wine would thus rise, and the storage period most profitable from the individual point of view would become longer. Equilibrium therefore requires an equal storage period for all —and a period of such length that the whole of the capital in existence finds employment in the only productive use which is open to it on our assumption—the storing of wine. All this is true *as a general rule.* We shall later consider a not unimportant exception (though it is more apparent than real).

We now further assume that the price of the matured wine, which is definitely fixed in the world market, is such that, when sold for consumption abroad, 3-year wine commands a wholesale price of 90*s*. per hl., 4-year wine 100*s*., and 5-year wine 110*s*.

We have now all the data which are necessary to determine (approximately) the unknowns of the problem, which are :—

(1) The equilibrium *rate of interest* in the community.

(2) The price of grape juice, or what comes to the same thing, the sum of *wages* plus *rent* (the division between these two, as we have said, being each determined by the law of marginal productivity in the non-capitalistic production of new wine, which we have postulated).

(3) The amount of *capital* in the community, reckoned in terms of money.

First of all, it is evident that the equilibrium rate of interest must be *greater* than 10 per cent, since 5-year storing would otherwise be at least as profitable as 4-year—if not more so ; for the conversion of 4-year wine, with a selling value of 100*s*., into 5-year, with a selling value of 110*s*., would yield interest at exactly 10 per cent per annum.

In the same way, the prevailing rate must necessarily be less than 11 per cent (or to be exact, less than 11·11 per cent), for it would otherwise be equally profitable, or more profitable, to sell out the wine after three years ; for the maximum rate which can be obtained by leaving the wine for another year is

about 11 per cent on its price at that time of 90*s*. (its price after four years being 100*s*.). The actual rate of interest must, therefore, lie between these two limits—say at 10½ per cent; for a more exact determination we should have to know the selling value of the wine when it was between three and four and between four and five years old.

The rate of interest being known, it is easy to solve the rest of the problem. It is clear, for example, that the price of the 3-year wine *in transactions between holders themselves* (which we may call V_3) must be such that, when capitalized for one year at the current rate, it equals the selling price of the 4-year wine. In other words, we obtain the following equation :—

$$V_3 = (1 \cdot 105)^{-1} \times 100s. \text{ (per hl.)}.$$

This price, which we may call the capital value of the 3-year wine, is, as calculation shows, a little *more* than the 90*s*. which the wine would have fetched if sold for consumption, which agrees with the fact that, in those circumstances, such a sale would *not* be profitable. In the same way, the capital value of the 2-year wine must be $(1 \cdot 105)^{-2} \times 100s.$, and that of 1-year wine $(1 \cdot 105)^{-3} \times 100s.$, and, finally, the 0-year wine or new wine in the home market must fetch an amount represented by the equation :—

$$V_0 = (1 \cdot 105)^{-4} \times 100 = 67s. \text{ (per hl.)}$$

This will therefore be the sum paid out in *wages* (and rent) for the production of 1 hl. of new wine. The total wages and rent per annum will consequently be 67,000,000*s*.

Apart from the supply of cash to effect transactions and certain other requisites, the circulating capital of the community —as we have already said—consists entirely of the stored wine of four successive vintages. Consequently, its money value at the *beginning* of each year of account, when the mature wine has been sold, or exchanged for commodities from abroad, and a new vintage has just been laid down is :—

$$K^4 = [(1 \cdot 105)^{-4} + (1 \cdot 105)^{-3} + (1 \cdot 105)^{-2} + (1 \cdot 105)^{-1}]$$

$\times$ 100 million shillings, or, what amounts to the same thing :—

$$67 \text{ million shillings} \times [1 + 1 \cdot 105 + (1 \cdot 105)^2 + (1 \cdot 105)^3] =$$

$$67 \, \frac{(1 \cdot 105)^4 - 1}{0 \cdot 105} \text{ million shillings} = 314 \text{ million shillings}.$$

At the end of a year of account, shortly before the next sale, the whole stock of wine has become a year older. Its value has thus increased to

$$67\ \frac{(1\cdot105)^5 - 1\cdot105}{0\cdot105} = 347 \text{ million shillings.}$$

The difference between these amounts, 33 million shillings, is the remuneration of capital for the year, and may clearly be regarded either as four years' interest on the purchase price of new wine, i.e.

$$67\ [(1\cdot105)^4 - 1] = 100 - 67 = 33 \text{ million shillings,}$$

or as *one* year's interest on the *whole* of the capital existing at the beginning of the year, i.e.

$$314 \times 10\tfrac{1}{2}\% = 33 \text{ approximately.}$$

Now if, by continued saving, the capital of the community is *increased* so that it just suffices for 5-year storing, then (with the same reservations which we shall discuss in detail later) this storage period must necessarily be the most profitable from the individual point of view. In order to calculate the approximate rate of interest under such conditions we must also know the selling price of 6-year wine, which we will assume to be 120*s*. per hl. In equilibrium the rate of interest must then be less than 10 per cent, but more than $\frac{10}{110}$ (about 9 per cent). We will assume it to be exactly $9\frac{1}{2}$ per cent. The price of new wine must consequently be $V_0 = 110 \times (1\cdot095)^{-5} = 69\cdot88$ or 70*s*. nearly. Thus wages and rent will now amount to nearly 70 million shillings. The remuneration of capital will thus be just over 40 million shillings per annum and the community's total capital at the beginning of each year of account :—

$$69\cdot88\ \frac{(1\cdot095)^5 - 1}{0\cdot095} = \frac{40\cdot12}{0\cdot095} = 422 \text{ million shillings.}$$

This considerable increase in capital has thus somewhat increased wages plus rent, whilst at the same time lowering the rate of interest. Nevertheless, the share of capital in the annual product has *increased*, since 40 : 70 > 33 : 67—a relation which, with a continued increase of capital, must finally be reversed, so that the relative, and ultimately the absolute, share

of capital in the product will be *decreased* when capital has increased sufficiently.

The rate of interest here appears clearly in its simplest form as *the marginal productivity of "waiting"*. By prolonging the period of storage (i.e. the period of production or capital investment, which here coincide) by one year—from four to five years—the annual product has been increased from 100 to 110 million shillings, or 10 per cent; if it were prolonged yet another year it would increase from 110 to 120 million shillings, or about 9 per cent. *Between* these two lies the real rate of interest for exactly five years' storage.

On the other hand, we find from this reasoning that von Thünen's doctrine of the determination of the rate of interest by the yield of the last portion of capital applied, gives, when taken with reference to the whole capital of the community—reckoned in money (or consumption goods)—too *low* a value. Capital was increased by $422 - 314 = 108$ million shillings and gave rise to an increased annual yield of 10 million shillings, which, on that basis of calculation, would correspond to a rate of not quite $9\frac{1}{4}$ per cent. A further increase of capital, bringing the period of production up to six years, would in the nature of things produce a still smaller increase in the relative yield; and between these two lies the yield of the last portion of capital when the period of production is exactly five years. Thus it is in any case *less* than $9\frac{1}{2}$ per cent, on which basis we have calculated the money value of capital. This relation appears to be general, and the difference may be of any magnitude whatever.

In the example here selected it may, of course, easily happen that the capital of the community may become too great for 4-year storage and yet not great enough for 5-year. In that case, wages (the price of new wine) will simply rise until 4-year and 5-year storing are equally profitable, and capital is distributed between them. But it might also happen that one or more vintages (e.g. 5- or 6-year wine), although more valuable than newer wine, may fetch a market price relatively so low that it does not pay to sell for consumption wines of these ages. As capital increases, the storage period will then rise in discontinuous jumps from four to seven years. This is the *exception* to the rule, which we have already mentioned.

In fact, such cases are not infrequent. In the same industry

(it happens in shoe manufacture in Sweden) there may exist side by side two or more methods of manufacture, perhaps requiring entirely different amounts of capital and with different production periods (e.g. hand-made and machine-made shoes). Only in proportion as capital (and with it wages) increases will long-period capital investment finally supplant short-period investment (except possibly for certain specialities).

We refer the reader to the following pages for a more exact deduction of the above principles, as well as for a treatment of the more general case in which the application of labour and land is not (as we have here assumed) simultaneous, but made at different times.

In an algebraical treatment it is simplest to start with a *continuous* production and sale; that is, the production of so many hectolitres of grape juice per *day* and the sale of an equal amount of matured wine every day, on the assumption that these two operations are separated in time by a period of t (years).

If we again represent the price of one hectolitre of grape juice as V_0 and the price of the mature wine, treated as a function of its age, as W_t or W (as distinct from V_t by which, as before, we represent the capital value in the home market of wine t years old), we shall clearly have

$$W = f(t) = V_0(1 + i)^t,$$

in which i is the rate of interest; or, as it is more convenient to write it

$$W = V_0 e^{\rho t} \tag{1}$$

in which e ($= 2{\cdot}718$) is the base of natural logarithms and ρ the rate of interest at a moment of time (*Verzinsungsenergie*). The individual capitalist cultivator has now, with a given value of V_0, to maximize i or, what comes to the same thing, ρ. This requires that

$$\rho = \frac{W'}{W} \tag{2}$$

where W' represents the first derivative of W with respect to t. This is Jevons' well-known formula for interest: "the rate of increase of the produce divided by the whole produce."

The further condition for a maximization of ρ can be written :—

$$\begin{vmatrix} W, & W' \\ W', & W'' \end{vmatrix} < 0 \tag{3}$$

where W'' is the second derivative of W with respect to t. This may also be written :—

$$W' : W > W'' : W',$$

and it is consequently always satisfied if W increases less than geometrically when t increases arithmetically ; this must always be the case *in the long run*, since the contrary assumption would lead to absurd consequences, though it need not, of course, hold for *every* value of t.

By the elimination of ρ between (1) and (2), we obtain the value of t which maximizes ρ for the given value V_0. If, instead, we had assumed the value of ρ to be known, then the *same* formulæ would have given the value of t which maximizes V_0, i.e. the storage period which the cultivators themselves would adopt, if they could borrow money at the rate of interest ρ for their current expenses.

Let us now assume that the capital of the community is just sufficient for a storage period of t years—t being assumed to be known. The equations then give us the values of V_0 and ρ, which correspond, when the community is in equilibrium, to wages (or wages plus rent) and the rate of interest.

If the grape harvest comes in once a year and if V_0 is the total value of this annual harvest, W_t having a corresponding significance, then the money value of the social capital will clearly be :—

$$V_0 . \sum_{x=0}^{x=t-1} (1+i)^x = \frac{W_t - V_0}{i}$$

On the other hand, with production, storage and sale, all going on continuously, the result will be :—

$$K = V_0 . \int_0^t e^{\rho x} dx = \frac{W_t - V_0}{\rho} \qquad (4)$$

If the social capital is exactly equal to this there will be equilibrium. If it is greater or less, the equilibrium will be disturbed ; the value of V_0 will rise or fall and the storage period most advantageous from the individual point of view will be altered, until a new equilibrium is reached. It is clear that, with an increase in K, there must be an increase in V_0, in t, and in W, but a fall in ρ. By logarithmic differentiation of (1) and applying (2) we obtain :—

$$\frac{\delta V_0}{V_0} = - t\delta\rho = - \frac{\begin{vmatrix} W, W' \\ W, W'' \end{vmatrix}}{W^2} t\delta t \qquad (5)$$

and since the determinant in the last expression is assumed to be *negative* δV_0 and δt will clearly have the same sign, while δV_0 and $\delta\rho$, as well as δt and $\delta\rho$, will have opposite signs. That δK and δt must have the same signs, is inherent in the nature of the case, but can easily be directly proved. By differentiating (4) with the help of (5) we obtain:—

$$\delta K = \frac{\rho W' - \rho'[W - V_0(1 + \rho t)]}{\rho^2}\delta t$$

Since, in accordance with the above, ρ' is always negative and $W = V_0 e^{\rho t} > V_0(1 + \rho t)$, the coefficient of δt clearly > 0 so long as W increases with t.

In the same way we obtain:—

$$\frac{dW}{dK} = \rho + K\frac{d\rho}{dK} + \frac{dV_0}{dK} = \rho + (K - V_0 t)\frac{d\rho}{dK}.$$

Now since $d\rho : dK$ is always negative and K is always $> V_0 t$ (from (4) since the function under the integral sign is always > 1 so long as $\rho > 0$), clearly $dW : dK$ is always *less* than ρ. This proves that the above-mentioned theorem of von Thünen is *not* correct, if by "the last portion of capital" is meant an increase in the *social* capital. The divergence may in point of fact be of any magnitude whatever, since $K - V_0 t$, and also $d\rho : dK$ may have any values whatever.

If we desire to represent these conclusions graphically, it is simplest to take the natural logarithm of the productivity function, $y = \phi(t) = \log_e (W_t)$ as the ordinate of a curve whose abscissa is the time t. Similarly we take W_0, i.e. the fixed price of new wine on the world market (as distinct from the variable V_0) as a unit for measuring W_t, so that $\log W_0 = 0$.

The curve must then pass through the origin.

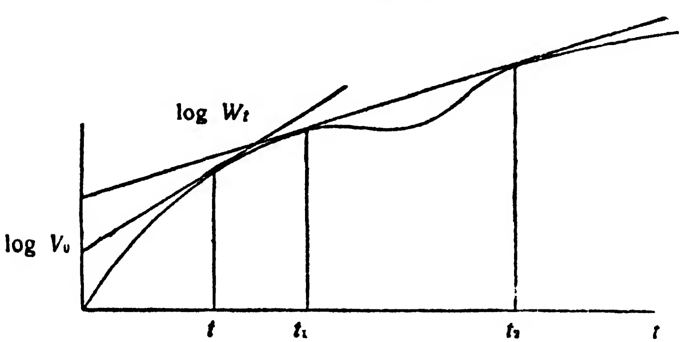


FIG. 14.

If $\log_e (V_0)$ is called y_0, then for any value of t, $\rho = \frac{y - y_0}{t}$, so that ρ becomes the trigonometric tangent of the angle of inclination of a straight line connecting the point y_0 on the y-axis with the corresponding point on the curve $y = \phi(t) = \log_e (W_t)$ and ρ will be a maximum when this line becomes a *tangent* to the curve. In accordance with what has been said, the curve must be roughly parabolic—i.e. it must be concave to the t-axis, since a rise in y_0 and t always leads to a fall in ρ. If, in exceptional cases, the curve should at some point bend downwards, then this point will be bridged over by a double tangent to the curve; capital will be divided between two equally profitable periods of investment (or production) t_1 and t_2, different in length; while ρ and V_0 remain unchanged until the community's capital increases to such an extent that it more than suffices for the whole of investment to be made for a period t_2, after which V_0 will again begin to rise and ρ to fall.

We may consider briefly the somewhat commoner case in which labour and land are still employed, once and for all, in what is practically an indivisible moment of time, but when they are employed at different points of time, during the period before the completion of the commodity; as for example it would happen if the grapes themselves were a spontaneous gift of nature, for which no appreciable wages, though some *rent*, need be paid, and the actual labour is employed in the making of the wine at a later time not definitely predetermined. In an *individual* firm, the value W of the finished product available during a given unit of time (say one year) would clearly be a function of the quantities of labour and land employed (a and b) and also of the periods of time (t and τ) for which each was invested in production:—

$$W = f(a, b, t, \tau).$$

Out of this value W must be paid wages, rent, and accumulated interest. If l represents wages and r rent we thus obtain:—

$$W = f(a, b, t, \tau) = a.l.e^{\rho t} + b.r.e^{\rho\tau} \qquad (1)$$

where e and ρ have the same significance as before. If ρ is to be maximized, we can differentiate partially (1) keeping ρ constant. By partial differentiation of (1) we then obtain:—

$$f_a = le^{\rho t} \qquad (2)$$
$$f_b = re^{\rho\tau} \qquad (3)$$
$$f_t = \rho ale^{\rho t} \qquad (4)$$
$$f_\tau = \rho bre^{\rho\tau} \qquad (5)$$

From these five equations the unknowns a, b, t, τ, and ρ can generally be determined. From (2) and (3) we readily obtain :—

$$af_a + bf_b = f(\quad) = W.$$

This equation, however, is an identity so long as $W = f(\quad)$ is a homogeneous and linear function in a and b and is thus of the form $b . F\left(\frac{a}{b}, t, \tau\right)$; in other words, if large and small-scale production (at any rate after a productive capacity, which is not too great, has been reached) are equally profitable.[1] In that case the number of independent equations is reduced to four, but we can still determine t, τ, and ρ as well as the ratios between a and b since (1), when divided by b, gives :—

$$F\left(\frac{a}{b}, t, \tau\right) = \frac{a}{b}\, le^{\rho t} + re^{\rho\tau}.$$

If the whole production of the community is of one and the same kind we may, on the above assumption, simply replace a and b by the total annual services of labour and land (A and B). These, however, are to be regarded as given and constant; but the above five equations (1) . . . (5) can, after this substitution, serve for the determination of l and r (as well as t, τ, and ρ). Since, however, only four of them are independent, a further equation is required, which may be obtained either by supposing t or τ (or some particular relation between them) to be given, or else by some supposition as to the money value of the social *capital*, which in this case will be equal to the sum of t years' wages and τ years' rent plus interest accruing at the rate of ρ (or i).

From (4) and (5) we clearly obtain, by addition,

$$\rho = \frac{f_t + f_\tau}{f(\quad)},$$

which corresponds with the above-mentioned formula of Jevons and, on special assumptions, coincides with it.[2]

In the same way it is easy to see the significance of equations (2) and (3). The partial derivatives with respect to a and b (or A and B) no longer correspond (as in the case of non-capitalistic production) to the actual wages and rent paid, but rather to the amount which labourers and landowners would receive, if

[1] [Wicksell's notes indicate an intention to re-write this passage.]

[2] It does so if $W = f(\ )$ is a function of $\frac{t + \tau}{2}$ only (as well as of a and b) and if t and τ should happen to be equal.

they could *wait* until their product was finished; which must otherwise be discounted at the rate ρ for the period t or τ.

At this point we cannot enter into a detailed discussion of these formulæ. We have already remarked that an increase in capital need not in this case necessarily lead to an increase in *both* wages and rent; one may sometimes remain stationary, or even decline, whilst the other correspondingly increases when capital is increased, and *vice versa.* On the other hand, it appears inconceivable *a priori* that an increase of capital could, *ceteris paribus,* coincide with a decrease of both wages and rent—though the question should perhaps be further investigated.

We must now try to solve the problem of production and distribution in the general case, where the original factors are employed not merely at one or more discreet points of time, but are distributed over the whole period of production. This distribution—which varies within very wide limits—is only partly determined by the technique of the different industries—and is actually modified in the effort to maximize profit.

It is evident that the solution would be impossible, even from a purely mathematical point of view, if it necessitated a precise treatment of the production and distribution of the community as a whole. But the only questions of practical importance which economists have to answer relate rather to the recurrent, relatively small, *changes* in a scheme of production, the elements of which are known from experience; and of foreseeing the probable effects of such changes on production and distribution, within the community. (Even the revolution which would follow the introduction of the socialist state would probably only be relevant to the question of the ownership of the means of production, with which we are not concerned here; it might affect the technico-economic side of production and distribution to a much smaller degree.) [1]

Even with this reservation, the problem must probably be regarded as incapable of solution at present—chiefly owing to the lack of reliable industrial statistics. On the other hand, the mathematical aspect should not present any insurmountable difficulties once the principle is established.

The problem is considerably simplified if the period of production, or the rate of interest, or both, are so small that we

[1] [A pencil mark against the last lines indicates that this observation is made with reservations.]

can use simple interest without risk of serious error (as Böhm-Bawerk does in his illustrations). In such circumstances the *average* investment-period of both labour- and land-capital will be *independent* of the rate of interest and will simply be equal to the (weighted) arithmetic mean of the individual periods of investment. We may then regard the productivity function $f(\ )$ as merely a function of these two average investment-periods t and τ (as well as of a and b or A and B) and everything can be reduced to the simple formula on p. 181, in which the *exponential functions* on the right-hand side of the equation are replaced by the expressions $1 + i.t$ and $1 + i.\tau$.

This is not without practical importance, since in a more or less stationary society—as we shall proceed to show—one can completely ignore the longer periods of investment; for capital-goods already in existence (such as houses, railways, etc.) will stand in a similar relation to circulating capital and labour as land itself. The investment period of circulating capital, therefore, is reduced to a few years, and it will thus be sufficient to employ simple interest in its capitalization. The line of demarcation between fixed and circulating capital must, of course, be drawn more or less arbitrarily, but in such questions we can never achieve more than approximately valid conclusions.

It should perhaps be pointed out here that the assumption that the average period of investment is independent of the rate of interest (i.e. of simple interest) only applies, strictly speaking, where several different capital investments relate to one and the same future act of consumption (as in Böhm-Bawerk's example). In the opposite case, where one (or more) factors of production are invested in a single capital-good or durable consumption-good, it may easily be seen that the average investment-period will be dependent on the rate of interest, even with simple interest.

On the whole, the theory of the coincidence of the rate of interest and the "marginal productivity of waiting" is only applicable as an exact mathematical formula on certain abstract assumptions. This is quite natural, for waiting on the part of society as a whole—and frequently also on the part of the individual—is not a simple quantity, but is, as we have just pointed out, a complex; "average waiting" as a rule exists only as a mathematical concept, without direct physical or psychic significance. But it should, nevertheless, be retained as a concise general principle, reflecting the essence of productive capital.

E. Controversies Concerning the Theory of Capital.

Before proceeding, we may turn to consider, in the light of the theory we have now developed, some of the controversies concerning capital which have for a long time past engaged, and are still engaging, the attention of economists. If we succeed in throwing new and clearer light on these questions, this will be the best proof that the new theory really makes some scientific progress. In this case—as in so many others—a closer examination will show that the difficulty is, to a large extent, purely formal, and is due only to an imperfect formulation of the point at issue.

(1) This is probably true of most questions concerning the content of the capital concept itself, and especially of the question whether or not *land* should be included under the designation of capital. There is no doubt that we can give to the word *capital* a meaning wide enough to include land also. Here, as in practically all economic definitions, we are concerned with a more or less conscious extension of a concept whose meaning was originally more restricted. Such an extension can be taken as far as we like in view of the question in hand—nothing in principle need be excluded. If we contrast capital (as being equivalent to material means of production) with labour, then of course it also includes land. One might, though the practice is unusual, go further and, with Walras and Pareto, consider man himself (human skill and ability) as capital. The latter concept will then be equivalent to the *sources* of productive power in general, or, from another point of view, to the concept of a *source of income*, of any kind, in contrast with income itself. There is nothing to prevent us from speaking of "capital in the wider sense" as well as of "capital in the narrower sense", so long as no misconception arises. We believe, however, that we have already given good reasons for the tripartite division of the factors of production into land, labour, and capital, which is commonest among economists. The almost complete analogy between land and labour, from an economic point of view—which has so long been overlooked by economic science—appears very clearly from the modern theory of marginal productivity; in contrast with these two original, current, present or direct productive forces, capital appears as a combination of accumulated labour and land.

It is admittedly difficult to determine where the line is to be drawn between capital and non-capital, indirect and direct productive forces. The human labour employed on land, and the resources of the land accumulated from earlier ages and employed for the same purpose (e.g. the work of beasts of burden in improving the land ; manure ; timber for roads ; agricultural and other buildings, etc.) are undoubtedly to be regarded as capital, when the measures and expenditures in question are taken in order to yield interest at a future date—as in the case of all other capital. Such improvements to the land often leave a permanent residual benefit. This happens, for example, in the case of major blasting operations to secure water in mountain regions, the building of roads, protective afforestation, etc. These new qualities which, once acquired, the land retains for all posterity, cannot of course be distinguished either physically or economically from the original powers of the soil ; in the future they are to be regarded not as capital, but as *land.* Very much the same applies, moreover, to human skill : a manufacturer who enlists skilled foreign labour in order to introduce a new industry into the country makes a capital investment which may perhaps repay him to the full in a few years. But the skill in this industry, which perpetuates itself within the country, will be a future asset for labour and not for capital.

It may be further pointed out that nearly all long-term capital investments, nearly all so-called fixed capital (houses, buildings, durable machinery, etc.) are, economically speaking, on the border line between capital in the strict sense and land. We have already said that the operation of the laws of capital depends upon the assumption of a constant adjustment of concrete capital goods in an endless repetition of the same process of investment and production. But this is only of practical importance in capital investments of relatively short duration.

If, therefore, our analysis is only applicable within a fairly short period, then, strictly speaking, only short period capital-goods (in other words, circulating capital) can be regarded as capital proper. The volume of fixed capital, on the other hand, can, in the long run, be *increased* by the conversion of circulating into fixed capital—in so far as this is generally profitable—but it cannot be appreciably diminished—the reverse operation being usually impossible. Hence it is, in most respects, on the

same level as the unchanging original productive factors, labour and land. This circumstance is sometimes in evidence during booms, when large quantities of circulating capital are converted into fixed capital, and it is not possible to replace the former quickly enough. In the subsequent depression the conditions are usually reversed: there is plenty of circulating capital, but it is no longer *profitable* to convert it into fixed capital.

(2) Similarly, the question of the inclusion of necessities of life for the labourers within productive capital is—at least in part—of merely formal importance. They have long been considered as a part of circulating capital; while Jevons considered that, fundamentally, all capital—especially in its original free form—consisted of the means of subsistence. In apparent opposition stands Böhm-Bawerk, who would entirely *exclude* such commodities from productive or social capital; for, in his view, the latter consists rather of the sum of the *intermediate products* appearing in the course of production and right up to the final stages—whereas the labourers' means of subsistence are finished products and direct objects of consumption. It might be thought that this almost direct contradiction indicated a deep-seated difference in the capital concepts of the two writers. Yet they are fundamentally in agreement and both may be described as thorough-going representatives of the modern theory of capital. The whole controversy is, in reality, merely formal; if we regard the selling process as a stage in production, the finished products may also be regarded as intermediate products, in the technical sense, until they pass into the hands of the consumer. Since, in our day, almost all labour—at any rate in industry—is hired labour, the means of subsistence, in proportion as they are consumed by the labourers (in other words *real* wages) may be regarded as the *price* of the labour which the capitalist acquires in their stead, and which he adds to his stock of capital-goods, in the form of saved-up labour of one kind or another. The cases in which the labourers themselves are entrepreneurs may be regarded in a similar way—the labourer's *wages* being considered as a quantity of goods equal to that which he would obtain in the market if he hired out his labour. If we look at the problem in this way, there is no real difference between the views of Jevons and Böhm-Bawerk.

The fact that Jevons' definition of capital is too *narrow*, since he proposes to reduce it merely to labour and its means of subsistence, is quite another matter. In so doing, he takes account of only one part—though usually the larger part—of capital; whereas in reality another part, and certainly a very important part, consists not of saved-up labour but of saved-up land—not of wages advanced but of rent advanced. But this part—which cannot be physically separated from the other—permits, as we have seen, of exactly the same treatment.

Hence, when Böhm-Bawerk observes, in support of his case, that if labour's means of subsistence are reckoned as capital the consumption-goods of landowners and capitalists must also be so reckoned, the first part of this observation (concerning landowners) is undoubtedly true. The capitalists' means of subsistence evidently constitute a part, not of capital, but of the *interest* on capital. Nor are they advanced—for who could advance to the capitalist? On the contrary, they are obtained subsequently, when the production of commodities, with the help of capital, is concluded.[1]

(3) Of more real substance is the dispute, which still continues, whether *capital* is really the *source of wages* or whether the source is not rather to be found in the annual *product*—in the results of production. The former is the classical view, to which Böhm-Bawerk subscribes, and, in reality, also Jevons—although he appears to be in opposition to it. The latter view has been zealously advocated by Socialist writers—also by the American, F. A. Walker and, even more pointedly, by his fellow-countryman, Henry George. Among noted European economists, Charles Gide tends more or less to this point of view Those who hold it point to the obvious fact that finished products are consumed by the workers, and by everybody else, *in proportion to their production*, and that there exists beforehand no fixed and insuperable barrier between those which are consumed by the labourers (and therefore should, in accordance

[1] When, in the third edition of his *Positive Theorie des Kapitals*, p. 632, Böhm-Bawerk asserts, and even expressly emphasizes the fact that the capitalist also obtains his income in advance, I cannot understand his reasoning. If this happened, it would indicate, in my opinion, that the capitalists consumed part of their capital—which Böhm-Bawerk certainly cannot have meant. His further simile of induction currents is much too vague to support his argument in any way.

with the classical theory, be regarded as capital) and those which are consumed by the other classes of society.

The foregoing observations concerning this keenly-contested dispute should show that the truth is not to be found entirely on either side, though it is nearer to the classical view. In so far as the product of labour is consumed *directly*, no capital is required for the payment of labour—and this is largely true of labour even in the most capitalistic societies, especially of all personal services and of labour engaged in the final phase of actual production—e.g. of the baker, and still more of the shopkeeper who sells his bread. Wages may be said to arise here by a simple, though indirect, exchange of the commodities consumed by the worker for the product of his work, which is more or less simultaneously consumed by the employer or his customers. Indirectly, it is true, these labourers benefit by the existence of capital, for when the marginal productivity of labour is raised, as happens almost invariably with the advent of capital, this applies, owing to the operation of competition, to all work performed—even to that for which wages need not be advanced by capital for any appreciable period of time. There is, however, no *division* of the product between labourer and capitalist—i.e. the owner of the circulating capital from which wages are paid—but the labourer enjoys his product undiminished. Or, if it be preferred, he has to share it only with the landowner and the owner of fixed capital. (The baking of bread requires, *inter alia*, an oven; the sale of bread, a specially equipped shop, and so on.) It is, of course, not always so easy to determine the value of a piece of work which is the last of a long series in production; we must have recourse to the same criterion which has guided us throughout, namely marginal productivity. By the exercise of greater care in the baking of bread—for example, by the employment of one more labourer in the bakery in question—the daily selling value of the product may, *ceteris paribus*, be increased by, let us say, five shillings. After making deductions for increased wear and tear of implements and machinery, cost of increased space, etc., this will constitute the marginal productivity of the labour concerned and will determine, in equilibrium, the wages of this labour—and of all labour of a similar kind.

In most phases of production, however, there is a longer

or shorter interval between the employment of labour and the final production of an article for sale. Since the labourer does not usually wait for his wages for the whole of this period, but more usually obtains them soon after he has performed his work, it must be evident that he does not obtain them *from the product of his labour*, either directly or by the exchange of the product for other products. Strictly speaking, moreover, the time must be reckoned from the performance of the labour to the moment when a finished product, ready for *consumption*, is brought into being. If, for example, a labourer is employed in the manufacture of a harvesting machine, the product of his labour is not really finished when the machine is ready for sale, but only when the crop harvested with the help of the machine has been sold and converted into bread. And it should also be remembered that the same machine will be used for several harvests and consequently for several years' baking. Some other person or persons must thus advance the wages—and this, as the above example shows, for a much longer time than is generally supposed. It should also be observed that the advancing may, in the interval, be transferred from one capitalist to another, as when the harvesting machine leaves the possession of the manufacturer and passes into the hands of the agricultural capitalist. That wages (real wages) are paid in products more or less *simultaneously produced* signifies nothing from an economic point of view. The modern labourer has, as a rule, nothing to do with manufacturing these products ; they are the final result of a series of processes whose various phases of labour have, as a rule, been paid for. The fruits of these productive processes belong—with a right which may be disputed by other labourers, but not specially by the labourer at present engaged—to the capitalist entrepreneur, and may be employed as he chooses, either for *new production*—in which case he maintains, or even increases, his capital—or for *his own consumption*. If this consumption, either of his own products or of products obtained in exchange, is direct, then, of course, the labourers (i.e. those seeking work in the market this year) will be deprived in a corresponding degree of an opportunity for consumption. If it is *indirect*—by exchange for a new, directly consumable service of labour, e.g. personal services—the labourer will, it is true, still receive his wages, and it may accordingly appear

indifferent to him whether capital is accumulated and maintained or not—provided that there are sufficient products in the market to pay his wages. But this is a great mistake and to act upon it would be fatal. For if capital is not maintained by renewal, then, as it is consumed, the longer processes, which are characteristic of the present technique of production, must be curtailed or interrupted one by one; thus the whole of production, including the marginal productivity of labour and wages, would return to the small dimensions of primitive times. Or, more correctly, the working population—which could not possibly support itself in its present numbers, if we returned to primitive conditions—would, for the greater part, starve to death.

We do not wish to deny that consumers as such can, to some extent, influence rates of wages by a suitable selection of articles of consumption. This appears from what has already been said, as well as from what follows. But their power in this direction is certainly more strictly limited than is commonly supposed. Broadly speaking, even if not in detail, we must recognize the truth of Mill's well-known principle that demand for commodities is not the same as demand for labour—unless it results in the accumulation of new capital.

In conclusion, it may be observed that what has been said concerning the relation of labour to capital applies in exactly the same way to land. Rent also is advanced by the capitalist (who may often be the landowner himself) in so far as the final product—the product ready for *consumption*—is brought into being at a later date than that of the use of the land—as is usually the case. This is evident from what has been said, but it is almost always overlooked in economic reasoning—an error which has contributed in no small degree to a lack of clearness as to the place of the factors, especially that of capital, in production.

Such an oversight may easily lead to paradoxical results—as in the following example, which, for the sake of simplicity, has been based upon Ricardo's theory of rent and capital.[1]

[1] The result will, however, be the same if we assume with Böhm-Bawerk that production takes several years and is continuously progressive. If we conceive this production as divided into annual parts, stepwise, it will easily be seen that capital need only amount to a half of the total sum paid out in wages during the whole period of production.

A capital of 1,000,000*s*. gives employment in one-year production to 1,000 labourers on land for which no rent need yet be paid. Wages would thus be 1,000*s*., and if the returns per labourer are 1,100*s*. there remains interest for the capitalists at a rate of 10 per cent per annum. Assume now, however, that the number of labourers is *increased*—capital remaining unchanged—to 1,111 men. Wages consequently fall to about 900*s*.—whereupon one-tenth of the old capital employed on the land becomes superfluous and must seek investment on new land. But there only remains (we assume) " worse land ", from which the yield per labourer is only 900*s*. We should then obtain the remarkable result that interest, despite reduced wages, *would fall to zero,* not only on the worse land, but all along the line, in consequence of the competition of capitalists. The whole of the gain would accrue to the owners of the better land, which would now receive the difference in the yield between the better and the worse land 200*s*. per labourer, or 200,000*s*. in all.

If, however, we consider that rent is also advanced from capital, the result will be quite different. Wages and rent *together* will then correspond to the existing capital, or 1,000,000*s*., and since the value of the whole return is $1{,}100{,}000 + (111 \times 900)$, or 1,200,000*s*., interest will really have risen to nearly 20 per cent. Rent will continue, in this case also, to be the difference between the returns from the better and the worse land, but *discounted* by one year's interest (i.e. $200 \div 1{\cdot}2 = 167$) for the area employing one man; wages, however, will fall to about 750*s*. Of course, this example is too simple to have any counterpart in reality and is only intended to emphasize the principle set forth above.

On the other hand, Böhm-Bawerk is probably mistaken in the assertion which he makes in the third edition [1] in reply to an objection of mine, that the advance of rent from capital tends to raise interest—in the sense that interest would be lower if land were obtained gratis. The exact opposite would happen. Both rent and wages—or their equivalents in land and labour—constitute a part of the productive capital on which interest is paid from the surplus yielded by production. If it were at all conceivable that all land were free, then all capital would be paid out in wages and they would thus rise. If in the process there were no change in the period of production, the surplus product, and consequently the rate of interest, would be exactly the same as before. In reality, however, a lengthening of the period of

[1] See *Positive Theorie des Kapitals*, p. 630, note 2.

production would prove economically profitable, and such a lengthening would, according to Böhm-Bawerk's own argument, lead to a larger surplus product and a *higher* rate of interest. If, on the other hand, the landowners did not receive their rent in advance, but only when production was completed, the rate of interest would certainly fall, but such a change in the rent demanded would be equivalent to new capital accumulation by the landowners, concerning which we refer the reader to the conclusion of the next section, IV, and especially to p. 213 *et seq*.

(4) Our present analysis may also serve to guide us to a true view of the famous *wage-fund theory*—once so highly esteemed, later denied even by its former advocates, then interred but not yet quite defunct. We have already indicated that we cannot, strictly speaking, refer to a fund for *wages* alone, but only to a *wage-and-rent* fund. Capital in its free form is employed to advance both wages and rent; how much falls to wages and how much to rent depends upon the circumstances which determine the present marginal productivities of labour and land—which, in equilibrium, correspond to wages and rent and therefore absorb without any residue, the capital which is for the moment *free*—i.e. the wage-fund. But does such a fund really exist? That it does not exist in reality, as a fixed and unchanging quantity, follows from the fact that capital in all its parts may either increase or decrease, to a larger or smaller degree, at any given moment. This, however, has not escaped the defenders of the wage-fund theory. If we imagine a society under more or less stationary conditions, in which a given capital in the possession of the propertied classes is employed year after year without appreciable increase or decrease, then each year about an equal part of that capital will be set free. That part (together with the consumable direct products of labour and land) constitutes the whole production of finished commodities and services of the year. When the capitalist class has taken the surplus, corresponding to interest on its capital, it must, in order to maintain its capital, reinvest the remainder—which it does by hiring labour and land for new production. This part, therefore, is what might be called the annual wage-fund (more correctly, wage-and-rent fund).

But there can be no doubt that little is gained in the explanation of economic phenomena by the introduction of this term; and the simple process by which it was attempted to determine wages (dividing the wage-fund by the number of workers) was certainly too elementary. In the first place, as we have said, the proportion in which the common fund is divided into remunerations for the services of labour and land is by no means given and determined *a priori*; and, moreover, with a change in the amount of capital, the wage-fund may undergo considerable changes, in so far as the average *period of turnover* of capital is lengthened or shortened. As we have already shown, there would inevitably be a shortening if by reason of a diminished supply of labour (due perhaps to emigration on a large scale) wages rose, other things remaining the same. In other words, a reduction in the divisor would itself bring about a reduction in the dividend, though not quite in the same proportion. But, on the other hand, a reduction in the number of labourers would increase the distributive share of labour, not only at the expense of the capitalists, but also—perhaps to a greater extent—at the expense of the landowners. Hence the advice which the advocates of the wage-fund theory gave to the labourers, namely to limit the supply of labour in the market in their own interests, was in itself, *good* advice, even though based upon inadequate reasoning.

It would also be possible to regard *all* capital, as Böhm-Bawerk does, as wage-fund. But this amounts to the same thing; for in any case it is only the part annually set free which can purchase labour (or land).

The real error in the classical wage-fund theory was, as Böhm-Bawerk pointed out, that it frequently identified the wage-fund with capital as a whole, although it conceived the wage-fund to be invested for only one year. A very striking example of this is Senior's "last hour", immortalized by Karl Marx.[1] Senior thought he could prove that a shortening of working hours per day by about one-eleventh would reduce the profits of capital from 10 per cent to nothing. He based this conclusion on the absurd assumption that *all* capital, including that invested in factories and machinery, has a one-year turnover,

[1] *Das Kapital*, i. Third edition, p. 206 et seq.

which did not prevent him from calculating, *in addition*, annual depreciation for wear and tear on buildings and machinery. If we calculate *correctly*, with the figures advanced by Senior, we shall obtain for fixed capital a period of turnover of about 8 years (sixteen depreciation allowances) and, for capital as a whole, 7 years. *Ceteris paribus*, a reduction in the hours of labour would certainly reduce the profits of capital, but only from 10 to about 8 per cent, and with somewhat greater intensity of work not even by so much as this.

It is curious that Marx himself does not seem to have observed the yawning gap in Senior's argument, to which he devotes a prolix refutation. Or perhaps he hesitated to point out an omission the revelation of which would inevitably have exposed the weakness of his own "exploitation theory".

Another criticism which has been made against the wage-fund theory is that it is correct only on the assumption that the labourers take their wages in kind at the same time as they render their services. If, on the other hand, they wish to take their wages partly or wholly "in capital"—in other words, to wait for their wages until their own product is ready for market—then wages may, within the limits of what is produced, rise to any height whatever and be independent of the size of the wage-fund or capital. This is of course quite correct, but it is scarcely a proper objection to the wage-fund theory, except in its most rigid form; for, by such a procedure, the workers would themselves become capitalists and would build up capital, so that the fruits of their labour which were not exchanged for products, i.e. for a part of the existing capital, would constitute a real addition to it.

This method of paying wages is the essence of the profit-sharing system, and if it has occasionally had beneficial results this may perhaps be most simply explained by the fact that the system stimulates the workers to accumulate capital, whose future fruits are usually sweet, even if its roots in the present are bitter.

Later on, we shall discuss the accumulation of capital—which is an important element in the theory of capital. But we will first return to the theory of *exchange* and see how this appears when it is linked up as it ought to be with the theory of production outlined above.

3. *The Interdependence of Production and Exchange. The Theory of Exchange Value in its Final Form*

Hitherto we have been reasoning on the assumption that production is carried on at given prices for all products. We must now drop this assumption and approach the real world—in which production and exchange mutually affect each other. Whilst we thus obtain a more complete theory of distribution, modified in some respects from that set out above, we shall also have an opportunity of resuming and completing our discussion of the theory of exchange value, which we were compelled to interrupt at the point at which its dependence upon, and connection with, the theory of production and distribution became clear. We shall, however, restrict our observations to the problem of the production and exchange of only *two* articles; the argument is much facilitated by such a simplification and there is no theoretical difficulty in subsequently extending it to all the infinitely varied products which are actually exchanged. In spite of this simplification, however, the problem resolves itself into two essentially different questions, which are best surveyed and treated separately. On the one hand, we may assume that the two articles exchanged are produced in different countries or districts, between which *there is no transfer of labour or capital*, so that all the resources available in each community are engaged in the production of one article. On the other hand, we may assume that the production of both articles takes place in a closed economy in such a way that land, labour, and capital can be transferred from one industry to the other. The former case is typical of what is usually called in economics the theory of *international* trade and international values; the latter of the theory of *internal* exchange under free competition. It is unnecessary to add that neither of these abstract assumptions corresponds to the phenomena of the real world. Perfect mobility of labour and capital within one country is just as improbable as is the complete absence of such mobility between countries.

Let us first assume that each country, owing to natural conditions, is compelled to produce one commodity only. It is then clear that under free competition every producer will endeavour with the available means to obtain the maximum

net profit, which, in equilibrium, must cause the whole production of the country to reach its maximum. It is true that we have only proved this on the assumption of production without capital, but it will easily be seen that its essence remains unchanged, like the objection of Ricardo, which it was our purpose to refute, even if the argument is applied to capitalistic production.[1] What has been said will by no means apply if production and exchange are effected co-operatively, or if the producers are otherwise associated in trusts or cartels; the country would then have to be rega ded more or less as a monopolist with respect to the commodity in the production of which it has greater natural advantages than other countries. Production would therefore be carried on with reference to the most advantageous monopoly price; a contraction of production might be to the advantage of the country even if all the available factors of production were not employed. If each of the two countries monopolizes the production of one commodity, then pricing is theoretically *indeterminate*; we have in fact reverted to isolated exchange, with the further complication that not even the quantities available are given beforehand, since they are the objects of production. If there is free competition, then, in accordance with the law of production and exchange, each country will produce as much as possible of its own commodity and exchange will be effected at the price which will normally equate supply and demand. It might well be that a restriction of the production of one commodity would, *if simultaneously undertaken by all*, be to the advantage of all producers of this commodity[2]; but restriction by an individual producer must, *ceteris paribus*, do him harm, since his supply does not appreciably affect prices. This would also be the case if the country manufactured several commodities, whose relative exchange values must be taken as given for the individual producers.

We have, therefore, simply to combine the foregoing laws of production for a single commodity (or for several commodities whose relative prices are given) and the laws of market value of a given stock of goods. The former determines the quantity of goods which accrues to each individual in each country in the

[1] [A mark against this passage in the author's copy of the second edition indicates that he wished to recast it.]

[2] Hours of labour may be influenced by the possibility of exchange.

form of wages, rent, or interest; the latter then determines the quantities of goods which will be mutually exchanged, and the relation between them—which constitutes international exchange value. The theory of international trade—or, more correctly, the abstraction so called—is therefore much simpler in principle than the problem of exchange in the internal market, in which the free transfer of the factors of production from one commodity to another must be presupposed. That the earlier economists thought otherwise was due to their erroneous idea that *costs of production,* which were assumed to regulate exchange value in the home market, could be determined on grounds *independent* of exchange value itself.

If l, r, and i represent the rate of wages, rent, and interest in one country, and A, B, and C the available quantities of labour, land, and capital, then $A.l$, $B.r$, and $C.i$ are the total quantities of wages, rent, and interest in the country, expressed (like capital itself) in terms of the one commodity produced in the country (or in one of them if there are several). Personal distribution will depend on the labour performed, or upon the land or capital owned by each individual. In the other country, the annual supply of the product of each person will be determined in the same way, and since the personal dispositions of all the individuals as regards consumption must be taken as given, we thus possess all the necessary determining factors for establishing the price and the quantities exchanged.

A close comparison between the above theory and Mill's treatment of the theory of international trade [1] is of great interest and affords, at the same time, a striking proof of the need for a more carefully-developed theory. In the first two editions of the *Principles*, as in an earlier treatise on the same subject, Mill set up a theory which, so far as it goes, fully accords with the assumptions made above. The various factors of production cannot, on these assumptions, pass from one process of production to another; and consequently, says Mill, the necessary prerequisite for determining the relative prices of goods by their relative costs of production is absent and we must fall back on the more general law of supply and demand. If there is equilibrium between supply and demand under such conditions that the supply of each commodity always increases when its price rises (and *vice versa*) then equilibrium will be stable. A relative increase in the price of one commodity would lead to an increased

[1] J. S. Mill, *Principles*, book iii, chap. xviii.

supply but, on the other hand, to a decreased demand[1] for it; a lower price would similarly lead to diminished supply and increased demand, so that, in both cases, prices would tend to revert to the original level. So far so good. But in this connection, Mill considered the case in which an increase in the relative price of one commodity (*A*), and consequently a decrease in the relative price of the other commodity (*B*), does indeed lead holders of (*A*) to increase their demand for (*B*), but at the same time it causes them to *decrease* their offers of (*A*) because their need for (*B*) now rapidly approaches satiation; thus equilibrium between marginal utilities is achieved before the offers of (*A*) have reached the same level as before. One of his critics, W. Thornton (who by his later criticism induced Mill to abandon, somewhat too hastily, his wage-fund theory), pointed out that, under such circumstances, equilibrium between supply and demand would, even when other things were equal, be possible at *more than one price*. If, *at first*, 17 units of (*B*) exchange for 10 units of (*A*), but the price of (*B*) happens to fall, so that 18 units of (*B*) must be given in exchange for 10 units of (*A*), then, on Mill's assumption, it might happen that holders of (*A*) would *reduce* their offers of (*A*), though at the same time holders of (*B*) would certainly diminish their demand for (*A*); and it is quite conceivable that equilibrium between demand for and supply of (*A*)—and *eo ipso* of (*B*)—*would also occur at this new price*. To us there is nothing remarkable in this. The case considered by Mill is, in fact, exactly the same as the one we have considered above, in which the supply and demand curves intersect when the former begins to fall; and we know that when this happens it is quite possible that the curves will intersect at more than one point. Mill, however, without further examination, derived from Thornton's remark the unfortunate conclusion that equilibrium between supply and demand would occur under such circumstances *at any price*—which can only be so in quite exceptional cases. In other words, he assumed that the problem is essentially *indeterminate*, so that *more than the above data* would be required to determine international exchange values.

He therefore undertook to complete his theory in this direction, but without success. It has justly been remarked that the latter part of Mill's chapter "On International Values", which he added to the third and subsequent editions of his *Principles*, really contains only a repetition, in a new form, of

[1] Strictly speaking, this applies only if the two commodities cannot substitute each other in consumption.

what he had already said elsewhere. Besides reciprocal demand, there is, in his opinion, another relevant factor—*the means of satisfying this demand*, set free in each country by the re-orientation of its industry. What he really adds, however, *is only a particular arbitrary assumption* as to the relation between the price of a commodity and its supply and demand. He assumes that the *supply* of each commodity is entirely *independent of its price* and that *demand* is in inverse proportion to the price of the commodity; as though each economy first satisfied its need for the commodity which it manufactured itself and then disposed of the surplus *at any price*.

Graphically represented, this would mean that the supply curve of each commodity would be a line parallel to the price axis and the demand curve a rectangular hyperbola. On this assumption, it is clear that the two curves can only intersect at one point and that the price equilibrium is stable. But in that case we should find no expression for the fact that a rising price of either commodity might lead its owners to *reduce*, instead of increasing, their supply. In reality, Mill neglects the whole of this question, which was, after all, the very starting-point of his investigation, and begins instead to inquire which of the two countries would *profit* most by a change in price caused by different conditions of production for one of the commodities. But in this way he finds no use for the new determining factor which he wishes to introduce, and he is finally forced to the almost pathetic confession that "the new element, which for the sake of scientific correctness we have introduced into the theory of international values, does not seem to make any very material difference in the practical result". But, as we have said, he has not really introduced any new element at all; not only the practical results of his inquiry, but the theoretical results too, are entirely unchanged.

On our assumptions of free competition and immobility of the factors of production there are, indeed, no determinants of price except equilibrium between supply and demand. This is sufficient for a theoretical solution of the problem, although the possibility of several solutions, usually only a finite number, is not excluded.

Somewhat more complicated, at least at first sight, is the other problem, of ascertaining the relation between production and exchange in the "home market", i.e. on the assumption that the available factors may be freely transferred from the production of one commodity to that of the other. And yet

the main lines of the solution are simple enough even here, although—as the history of the science shows—they are not so easy to discover. If we suppose, for a moment, that a given proportion of the available labour, land, and capital—i.e., in the last resort, given quantities of original factors of different years—is always used in the production of the one commodity and the remainder in the production of the other commodity, then the problem of equilibrium price and the quantities exchanged would be exactly the same as in the preceding case. In other words, for every such hypothetical distribution of factors of production we should have one or more possible solutions. Now, in this case, the distribution of the factors is precisely one of the quantities required for the solution of the problem, though we find instead three new conditions, or logical relations, which must be satisfied: namely, the requirement that rent and interest shall be the same in both branches of production, which *cannot* be assumed where two countries are concerned.[1] Every conceivable distribution of the factors gives rise, in each branch of production, to certain rates of interest, wages, and rent—expressed, in the first instance, in terms of one of the goods produced but also expressible in terms of the other since there is an exchange relation between the commodities, which follows from the same assumption; it is clear, therefore, that the problem is completely determinate by the equation of these three quantities individually. It should be capable of mathematical solution as soon as all the other data (the total productivity of land, labour, and capital, their distribution among individuals, and personal preferences in consumption) are exactly known. In reality, this problem of equilibrium may also be solved by trial and error; so long as wages, rent, and interest are greater in one branch of production than in the other, labour, land, and capital will flow into the channel where they reap the higher reward and there will be a simultaneous adjustment of relative exchange values, so that equilibrium will finally be achieved as far as is generally possible.

In order to avoid any misconception, one more observation should be made. The fact that the form of capital may change,

[1] In the article "Handel", in *Schönberg's Handbuch* (cf. *Ekonomiska Samhällslivet*, ii, p. 478), W. Lexis has been guilty of a serious omission in relation to this point, which makes his argument deceptive.

that labour-capital (i.e. saved-up labour) may be, to a certain extent, replaced by land-capital (i.e. saved-up natural resources) and, *vice versa*, that capital investments (or capital-goods) of shorter duration may be exchanged for those of longer duration —these do not introduce any element of indeterminateness into the problem ; for, in each particular branch of production, they are all governed by the general economic principle which we have already developed in the treatment of production. It may well be questioned what importance we are to attach to the claim that, under stationary conditions, the amount of capital must remain constant from year to year. But here we must distinguish two different things. In equilibrium, the capital employed in production has already assumed a certain technical dimension and composition, as well as a certain exchange value (expressed in terms of one of the commodities). It can now be asserted that, so long as capital of this magnitude and composition, or even of this exchange value, is maintained and utilized from year to year, equilibrium cannot be disturbed if, from the beginning, the other conditions of stability are fulfilled. But it would clearly be meaningless—if not altogether inconceivable—to maintain that the amount of capital is already fixed *before* equilibrium between production and consumption has been achieved. Whether expressed in terms of one or the other, a change in the relative exchange value of two commodities would give rise to a change in the value of capital, unless its component parts simultaneously underwent a more or less considerable change. But even if we conceive capital genetically, as being a certain quantity of labour and land accumulated in different years, a change in the value of commodities would also alter the conditions of their production and thus necessitate a larger or smaller change in the composition of capital.

This indeterminateness—which was inherent in our first main example,[1] and even in the pure problem of production—is, of course, primarily due to the fact that capital, unlike labour and land, is not an *original* factor of production which can exist (even hypothetically) independently of, or antecedently to, production. Its origin and maintenance inevitably presuppose that production is taking place. But it also has another, more deep-seated, cause. In reality, the amount of capital is not

[1] That of International Trade.

determined by physical conditions, but by the equilibrium between psychical forces which, on the one hand, drive us to save and accumulate capital and, on the other, to consume already existing capital. In other words, *the accumulation of capital* is itself, even under stationary conditions, a necessary element in the problem of production and exchange. We have now reached a point in our exposition at which this new factor forces itself upon our attention. We shall, therefore, consider this subject in our next chapter—though the laws of capital formation have been too little studied for a treatment of the subject in its entirety to be of much real use.

We consider the total amount of a commodity produced as a function (homogeneous and linear) of all the quantities of labour and land employed (i.e. annually consumed) both *current* and *saved up*. We then obtain for one commodity

$$P = \phi(A_0, A_1, A_2 \ . \ . \ . \ B_0, B_1, B_2 \ . \ . \ . \ )$$

in which A_0 and B_0 indicate current services of labour and land, A_1 and B_1 services one year old, etc. The partial derivatives of this function with respect to each of the quantities included gives us the wage (l), and the rent per unit of land (r), payable in this industry, expressed in units of the product, and also the marginal productivities of all the constituents of capital. From these we can deduce the rate of interest which is payable (i). With the relation which must exist in equilibrium between the yields of capital-goods of different duration and between the yield of land-capital and of labour-capital, we are now in a position to express all the above quantities in terms of three of them (e.g. A_0, B_1, and A_1). In the same way, we obtain for the other commodity :—

$$P^1 = \psi(A^1_0, A^1_1, A^1_2 \ . \ . \ . \ B^1_0, B^1_1, B^1_2 \ . \ . \ . \ ),$$

from which we can determine the values of l^1, r^1 and i^1, for this industry, l^1 and r^1 being expressed in units of the second commodity ; and can similarly express all the quantities included in terms of three only—A^1_0, B^1_0, A^1_1.

The number of unknowns is thus reduced to six only. To determine them we have the following additional relations. In the first place, under stationary conditions, the sum total of the quantities of labour annually consumed—current or saved up —must be equal to the supply of labour annually available in the country ; and the same applies to the land which is employed

either in its original or capitalized form. If the country has at its disposal A units of labour and B acres of land, we therefore obtain :—

$$A_0 + A_1 + A_2 + \ldots + A^1{}_0 + A^1{}_1 + A^1{}_2 + \ldots = A$$

and

$$B_0 + B_1 + B_2 + \ldots + B^1{}_0 + B^1{}_1 + B^1{}_2 + \ldots = B.$$

By means of the other data we can also express the exchange value of the two commodities as a function of the above quantities and therefore finally in terms of our six unknowns. If we represent this exchange value (e.g. the price of the latter commodity, expressed in terms of the former) by p, and if wages and rent are identical in both industries, then :—

$$l = p.l^1 \text{ and } r = p.r^1.$$

The rate of interest must also be the same in both ; thus $i = i^1$.

We have thus obtained *five* independent relations, but we still require a sixth. This can be obtained from our assumption concerning the amount of *capital*. The quantities A_1, A_2 . . . B_1, B_2 . . ., etc., are only those parts of capital which are annually consumed. Corresponding to them, under stationary conditions, there must exist other parts of the total social capital, whose amounts can be exactly determined. There must be *one* more element corresponding to A_2, *two* more elements corresponding to A_3, three to A_4, etc., and similarly as regards B_2, B_3, B_4, etc. (cf. Fig. 12). In equilibrium, the composition of the sum total of capital is thus definitely fixed. All its parts can be expressed separately either in the first three or in the last three of our six unknowns. If, for example, we now wish to impose the condition that in equilibrium the sum total of capital shall have a certain *exchange value*, measured in terms of one of the products, we need only calculate the exchange values of all parts and add them. These exchange values are (in accordance with the above) the original exchange values of the portions of capital concerned, *plus accumulated interest.* Thus, for example, the present portion of capital indicated by A_3 has the exchange value $A_3.l.(1 + i)^3$. The two identical portions also represented quantitatively by A_3, since they represent equal quantities of saved-up labour, have, on the other hand, the values $A_3.l.(1 + i)^2$ and $A_3.l.(1 + i)$, respectively. The portion of capital represented by B_3 has the exchange value $B_3.l^1.p.(1 + i)^3 = B_3.l.(1 + i)^3$, etc.

If these values are summed and are put equal to a certain given quantity—the total exchange value of the capital employed

in *the two industries together*, expressed in terms of the first commodity, we shall then obtain the necessary *sixth relation*, and the problem will at last be completely determinate.

If it were permissible to calculate with simple interest, the problem would be simplified in so far as the accumulation of capital through time need not be taken into consideration—though its distribution as labour-capital and land-capital, advanced wages and advanced rent, must; we should then only have to deal with the average period of investment.

It may perhaps be asked whether, in a case such as this (in which both commodities are manufactured in the same country), *more than one* relative equilibrium price is possible. This is quite conceivable if—as is usually the case—wages, rent, and interest enter into the manufacture of the two commodities in different proportions. If the prevailing equilibrium persists, and a higher relative price is paid for one commodity, then, obviously, that factor (or factors) which enters into the production of the commodity in relatively large amounts is favoured at the expense of the others.

As will easily be seen, there is no difficulty in extending the above reasoning to any number of commodities. Under the designation of commodity we may also include the factors of production themselves when they are directly employed by their owners. We can therefore abandon the simplifying assumption hitherto made—viz. that all factors of production on the market are available in given determinate quantities, which are offered in their totality by their owners, irrespective of the price they will fetch. This is very important, especially for labour, for we can now consider the case in which the hours of labour are variable and determined by the workers themselves, on the basis of the equality of the indirect marginal utility of work and the direct marginal utility of leisure.

Just as exchange and exchange value thus assume their final form by their connection with production, so, of course, exchange for its part considerably modifies the production and distribution of the product. Each producer—labourer, landlord, and capitalist—receives a substantial increase in utility from the possibility of exchanging the commodities, in the production of which he participates, for others (production in the modern sense would indeed be inconceivable without this possibility,

for nowadays production is carried on almost solely for exchange). And further, the relative distribution of the product between the three classes of producers becomes quite different, when there is a possibility of exchange with other districts or countries. A well-known example of this is the fall in rents, to the advantage of the landless classes, which has occurred in parts of Europe, as a result of the importation of foodstuffs from extra-European countries. Another is the more doubtful, but perhaps equally real, case in which the workers, or the great masses of the population in the latter countries, have suffered from the supply of cheap manufactured goods from Europe, to the advantage of the landowners.[1]

[1] See my *Finanztheoretische Untersuchungen*, pp. 63 ff. (Jena, 1896).

PART III

ON THE ACCUMULATION OF CAPITAL

BIBLIOGRAPHY.—The literature on this subject is very meagre. Among earlier writers there is virtually only H. von Mangoldt (*Volkswirtschaftslehre*), and among recent writers, Böhm-Bawerk (*Positive Theorie des Kapitals*), who have devoted detailed attention to the accumulation of capital. Karl Marx, *Das Kapital*, vol. i, section 7, *Der Akkumulationsprocess des Kapitals*, also deserves attention, despite his bias and exaggeration. Compare also Wagner, *Grundlegung*, part ii, vol. iii. In Schönberg's *Handbuch* the whole theory of the accumulation of capital is despatched in a single page, and in Conrad's *Handwörterbuch der Staatswissenschaften* in a single column. Cassel's *The Nature and Necessity of Interest* contains a noteworthy attempt to carry discussion on some points further than had previously been done. The best material for an examination of the problem is probably to be found in the statistics of banks, and especially savings banks, as well as in statistics of capital wealth, though the latter are unfortunately extremely sparse and rudimentary.

So far, our discussion has been based on the assumption that productive capital, like the two other factors, is constant. In reality, however, capital is not, like land—and, for shorter periods, labour—physically limited. It can be increased at any moment by saving ; it can be reduced by unproductive consumption. Neither is the supply of capital renewed in the same way as the supply of labour, by the work of nature—although it is natural to accumulate capital at certain periods of life (particularly middle age) and to consume it at others (early youth and old age). A rational theory of saving is thus necessary before we can clearly understand the conditions of a stationary society, with a constant supply of capital ; and still more, of course, before we can understand and foresee the gradual changes in the amount of social capital.

Unfortunately, such a theory has not been worked out, and the phenomena which it should explain depend on a number

of motives—partly selfish, partly altruistic, but in any case very complex. People *save* for themselves, but also for their successors. Some people often save merely for the pleasure of saving. Exceptional people may save and accumulate capital simply because they cannot help themselves—e.g. certain multi-millionaires whose capacity for consumption even the ingenuity of the luxury industries cannot stimulate. Large families encourage thrift, because a source of income, say a landed estate, which has hitherto supported the family, may now be inadequate for that purpose. But, at the same time, a large family frequently constitutes an insuperable obstacle to saving, since every available source of income is urgently and immediately needed. On the other hand, if the capital in an individual's possession is already so great that only a small portion of its yield is required for the maintenance and expenses of the family, then it will grow of itself—at least at present rates of interest—at such a pace that even great fecundity in the family cannot keep pace with it. The ever-growing wealth of certain multi-millionaires is therefore, from a *social* point of view, a not inconsiderable danger to society.

Among the many influences affecting the accumulation of capital, the rate of interest is undoubtedly one—although even its influence is uncertain and ambiguous. Theoretically, the individual should always carry his accumulation of capital (or it may be his consumption of capital) to the point at which the present and future marginal utilities of the goods saved is equal. By sacrificing one shilling this year he can, for example, count upon obtaining two shillings in ten or fifteen years. The question then becomes whether, *at that time*, two shillings will have more or less subjective value for him than *one shilling* now. The answer to this question naturally depends on a number of circumstances over which he himself can exercise some influence —such as the savings which he is likely to make *during the immediate period*. Here the rate of interest has a two-fold influence ; a high rate *increases* the yield of present saving and consequently its future marginal utility, i.e. the future utility of the last unit of capital *now* saved [1] ; but, on the other hand,

[1] Cassel is not quite correct when he says : " A man who attaches the same importance to future needs as to present ones, if he expects to be able to provide for his needs in the future just as easily as he does now, has no reason for setting aside anything of his present income " (*op. cit.*, p. 141). This argument actually presupposes the absence of any rate of interest.

at a given rate of saving, it makes provision for the future more ample and thus *reduces* the marginal utility of future goods for that individual. The latter tendency may even outweigh the former, so that, for certain individuals, a low rather than a high rate of interest may act as a spur to the accumulation of savings.

Individual saving is therefore a very complicated phenomenon. But if we consider society as a whole, and regard its average economic conditions as *approximately* stationary, the progressive accumulation of capital must be regarded as economical so long as any rate of interest, however low, exists. For the average individual, or rather for society as a whole (regarded as an individual who never dies), the accumulation of capital presupposes the exchange of a lower marginal utility for a higher—provided that it is not too rapid and does not absorb too much of the present means of consumption. Under such conditions, we should therefore expect a continual accumulation of capital—though at a diminishing rate—and, at the same time, a continual fall in the rate of interest.

In *The Nature and Necessity of Interest*, Cassel adduces certain apparently very striking reasons why a heavy fall of interest rates is *not* to be expected in the future. He rightly points out, in the first place, that every fall in the rate of interest causes a number of long-term investments which were previously unremunerative to become profitable ; and every such large-scale absorption of free capital naturally acts as a brake on a further fall in interest rates. He especially observes that a general demand for *larger houses*, entailing extensive building operations, would arise if, as a result of a heavy fall in interest rates, expenditure on houses is practically restricted to the mere costs of maintenance—and site rent. To this it may be objected that larger premises, at least in our climate, involve various *other* outlays, especially for fuel and light, which are often as considerable as the rent itself. Increased housing accommodation for the poorer classes, however desirable it may be in itself, is therefore scarcely to be expected, unless their level of income can be raised. With certain reservations, however, this part of Cassel's reasoning is undoubtedly correct—though it evidently sets no *limit* to the downward trend of the rate of interest, but only relates to the rather slow *tempo* at which the movement may be expected to occur.

The latter part of Cassel's argument would be of much greater importance here—if it could be regarded as correct. He considers (in agreement with the classical economists) that, with a certain rate of interest which is not too low, the very desire or ability to accumulate capital practically disappears, so that the rate of interest could not fall lower.

The case which Cassel exclusively considers is that of a business man who in his prime has accumulated a fortune, upon the yield of which he lives after he has retired from business. If the rate of interest is sufficiently high, he can do this without in any way encroaching on his capital. He may therefore have the satisfaction, or indulge in the vanity, of leaving it undiminished, or perhaps even augmented, to his heirs : the interest alone is quite sufficient for his needs. If, on the other hand, the rate of interest should materially fall, say to 2 per cent or 1½ per cent, then, says Cassel, such conduct would usually become impossible. Either the capital must be so great that the efforts or good fortune of a single individual would seldom suffice for its accumulation ; or else the mere yield in interest will be so small that he could no longer live on it without a serious change in his habits of life. He will therefore live on his capital, e.g. by the purchase of an annuity—Cassel shows, by detailed figures, how strong the temptation would be, since at so low a rate of interest he could multiply his annual income. And, says Cassel, he has a perfect moral right to do so. As a rule, he has already provided for the education of his children and perhaps for establishing them in life. He does not owe them more than that. On the contrary he may justly expect that they, in their turn, will act in the same way as he : work and accumulate a fortune during their youth and middle age, and consume it in their old age after they have provided for *their* children's education.

Cassel's argument may roughly be presented in this form. That it is correct in some cases cannot be denied, but as a *general* argument it can scarcely be accepted, for it is evidently based on the assumption that most fortunes are the fruit of the work of a single generation. But this is not the case even nowadays, and it evidently becomes less and less conceivable in proportion as the rate of interest falls. If we assume that the capitalist has *inherited* the whole or the greater part of his capital, the conclusion will be quite different. By consuming it, or even by failing to increase it, he would usually put his children in a more unfavourable position than he had himself occupied. This, however, conflicts with such an elementary impulse in human nature that

we can safely assume that it will not usually occur. It is, therefore, rather difficult to imagine, even in a society based on private property, any limit below which the rate of interest could not fall, because the accumulation of capital would come to an end. We shall endeavour to show that the degree or rapidity of its fall depends mainly on an entirely different circumstance, which is scarcely mentioned by Cassel; namely the degree of probability with which we may expect the future *growth of population* to be on the same or a similar scale to the present.

If, however, the facts are not quite in accordance with theoretical speculations (such as those on p. 208) and if, in particular, the long-prophesied ideal of economists, in which interest will have fallen to a minimum, is tardy in its realization, the cause is presumably to be found in the following circumstances. In the first place, there is the effect of the *subjective undervaluation* of future needs and overvaluation of future resources, which was observed by Böhm-Bawerk. This, in turn, is primarily due to the fact that, to the individual, the future is always in a high degree uncertain. He does not know whether he himself, or those in whose well being he is most interested, will really profit by his sacrifices. Moreover, even if capital accumulation as a whole increases production, the return on individual capital accumulation, even the technical return, is uncertain. The enterprises in which capital is invested may perhaps yield large profits if they are very successful; but the chances of such success are not very great. And since, in accordance with the general law of marginal utility, the possibility of a loss of wealth outweighs, for the individual, the prospect of an equal gain, such an enterprise, from the point of view of individual business, must always be regarded as unprofitable unless the chances of gain considerably exceed those of loss. This is probably the general rule. The special inducement which hazardous enterprises offer to gambling or adventurous spirits is a compensation, but operates perhaps more in the destruction than in the accumulation of capital. In this connection, we need only call attention to the large extent to which the modern concentration of capital and the credit and insurance system stimulate and facilitate saving by levelling out and reducing these risks to a minimum.

In these respects, however, a collectivist society would

afford a much better guarantee for the rapid accumulation of capital than does the existing individualistic society. The capital saved by united efforts would equally benefit all individuals and the whole of society in the future ; and the failure of some enterprises would be of little importance, if those which succeeded yielded a correspondingly greater return. Though this is opposed to current opinion, it is precisely in a collectivist society that we should expect a progressive accumulation of capital until production was fully supplied with new capital and the national dividend reached its technical maximum—assuming that interest in the well-being of future generations was not less than in existing society.

Another reason why interest is still comparatively high is the fact that states destroy capital (especially in war and armaments) at the same time as it is being privately accumulated. The enormous national debts contracted by European and extra-European states in the course of years (especially for purposes of war) naturally presuppose a more or less corresponding amount of savings on the part of subscribers (though it is true that war-loan is generally issued below par) ; but they do not represent any really productive capital, only a claim by certain citizens on present and future generations of taxpayers. In this connection it may be asked, at least when the rate of interest begins to decline more rapidly than capital increases, and the earnings of capitalists consequently decline absolutely, whether this must not act as a brake on further capital accumulation. In purely abstract theory this would not be the case in an individualistic society in which each individual manages and saves on his own account. If a particular individual increases his capital, the effect on the rate of interest is not appreciable. The result of his saving will therefore be an unconditional gain for him. On the other hand, it cannot be denied that capitalists as a class will gladly welcome all measures destructive of capital, such as armaments and war—for which they will largely be compensated by the State's contractual obligations, and which will help to raise the rate of interest. This constitutes a not inconsiderable political danger, as Adolf Wagner pointed out. But the collectivist state will be quite unaffected by a lowering of the rate of interest as such, since all sources of income would be more or less common to the whole community,

and, in such a case, the other sources would necessarily increase in a more than corresponding degree.

But the most important reason why the rate of interest has not fallen is probably that our modern societies differ in a high degree from the stationary type. Hitherto, we have only considered capital accumulation on the assumption of completely stationary conditions; if we abandon this assumption the problem becomes essentially different. For example, if a country for some reason, such as the successive exhaustion of the land, passes from a *higher* to a *lower* degree of productivity and prosperity, then the same quantity of commodities will have, on the average, a higher marginal utility, and consequently a higher subjective value, in the future than in the present. The mere retention of consumption goods for future use thus becomes advantageous, although it cannot, of course, give rise to increased productivity and therefore cannot, in the usual sense, yield any interest. Even in our day, people always save stocks for the lean season, and it was formerly very common to save grain for bad years—a custom which in countries with bad communications, such as India and Russia, may still be necessary. If, on the other hand, a country passes from a *lower to a higher degree* of prosperity *independently of the growth of capital* (as a result of technical discoveries, etc., or when a colony is first peopled) capital accumulation may be uneconomical, even though technically it might give rise to an increased productivity of labour and land. A larger quantity of products might then represent a lower marginal utility, since prosperity as a whole had increased.

Again, if the growth of population is accompanied by an increased demand for all kinds of products, on the one hand, and by an increased supply of labour available in the future, on the other, then a capital accumulation which might have brought down the rate of interest to practically nothing under stationary conditions will not now be sufficient to do so; or will only just suffice to maintain capital at about the same *relative* level, for which reason it will continue to possess a high marginal productivity and to yield a high rate of interest. In addition, capital accumulation is here impeded by the number of unproductive consumers, large families, etc. If *both* these causes operate (increased productivity and great increase of

population) as often happens in flourishing colonial lands, since, up to a certain point, the increase of population in itself brings improved technical conditions of production, the rate of interest may be incredibly high for a long period—as high as 50 per cent or more—as Adam Smith observed in the North American colonies. The marginal productivity of capital here is extremely high, yet capital is not rapidly accumulated, but remains just as inadequate in relation to demand. Everybody rightly expects that his own, or his children's, economic condition will automatically improve in the future, and nobody therefore considers it desirable to sacrifice the moderate provision which he is able to make at present for himself and for them. Capital loans and investments from older countries with a lower rate of interest soon flow in, moreover, and counteract, in a greater or lesser degree, the conditions which we have just described.

But it is clear that these cases are all only exceptions to the rule. The unprecedented growth of population recently witnessed in Europe, and still more in certain extra-European countries, will certainly, sooner or later—probably in the course of the present century—prepare the way for much slower progress and possibly for completely stationary conditions. Then interest will also fall, and the capitalist will have to be content with quite a small share in the product—both absolutely and relatively—and perhaps (though, for the reasons given, this is somewhat improbable) with nothing at all. But this, of course, would not render capital unnecessary for production. On the contrary, it would then have attained its maximum importance; for just as land, when it is in excess, yields its products gratis or for a very low compensation, so a perfected capitalistic system of production, though in many respects very different from a primitive system without capital, nevertheless resembles the latter in that labour and land alone (or practically alone) will share the product.

Such a state, however, would be far from desirable in an individualistic society based on private property. So far from disappearing, the gulf between the propertied and the propertyless classes would be well-nigh impassable if land, capitalized at an extremely low rate of interest, possessed almost infinite exchange value. Even now, a very large part of what is commonly called capital and interest is, in reality, land and rent. Think,

for example, of the colossal increase in site values, especially in the large towns. Even capital goods proper have their value increased in so far as the land incorporated in them is now re-assessed according to a higher standard of value; or, as it is said, because the cost of reproduction has increased. A large part of apparent annual savings is accounted for by this increase in the capital value of land and is thus not a real increase in wealth at all. Monopolies are another source of income of a similar kind which is not exhausted by increased capital accumulation, but rather becomes more abundant.

In his work, *Om den ekonomiska fördelning och kriserna* [1] (1909), Brock (like Cassel) is sceptical of the possibility of a fall in the rate of interest, but nevertheless criticizes our analysis of the consequences of such a fall. According to him, it would occasion a fall in rents also, since a sufficiently low rate of interest would render practicable a number of substantial improvements to land which are now not profitable owing to the lack of cheap capital, and the supply of land for all productive purposes would become excessive; so that the fall in interest would benefit labour exclusively.

The abstract possibility of this cannot, as we have already said (see p. 164, n.), be denied; just as, on the other hand, it is not entirely inconceivable that a fall in interest might benefit *landowners* exclusively—in so far as the low rate of interest would mainly lead to the introduction of fixed automatic, or semi-automatic, machinery, so that human labour would become superfluous. To what extent the conditions observed by Brock are of practical importance, however, depends on circumstances which it is difficult to survey. There is no doubt that many swamps and much poor soil, not least in Sweden, could, with an unlimited supply of cheap capital, be converted into fertile fields. And if the crowding of human beings in the cities could, with the help of capital, be counteracted (by rapid and cheap communications by land, water, and air), then site values, which in certain countries already greatly exceed agricultural rents, might be lowered—though only on the assumption that the population was reduced or ceased to grow; otherwise a continued rise in rents is practically certain—and capital might grow, even relatively to population, to any extent.

Another related question which was much discussed in the past is the extent to which the unchecked progress of capital

[1] [" Economic Distribution and Crises."]

accumulation is of advantage to those who only indirectly profit by it, and especially to the labourers. The older economists usually had very exaggerated views on this point, because they supposed—on the basis of the wage-fund theory—that an increase or decrease in capital would produce a *proportionate* increase or decrease in wages. This, of course, is not the case. A great increase (or decrease) in capital may doubtless be associated with an insignificant change in the rates of wages, less in proportion as there exist opportunities for long-term investment. And since, in our day, the labourers often do some saving themselves, their position will, of course, be much better if somewhat higher wages enable them to save something on their own account than if the capitalist employers, by paying lower wages, were enabled to save a corresponding (or even larger) amount on *their* account. In the former case they are enabled to reap both the direct and the indirect profits of capital accumulation; in the latter case they have only the indirect profit, which may be very small.

In this connection, we may refer to a celebrated and very peculiar speculation of the famous German economist, von Thünen. He remarks that if the labourers themselves are willing to save and accumulate capital, then they are best served if wages are neither too high nor too low; for if they are too low, their savings will be insignificant, and if they are too high (in relation to the output of labour) the profits of capital and consequently the interest on their own savings will be so small that there will be no inducement to save.

If we call the product of labour p and wages l, then $p - l$ will be the employer's surplus, and interest (for as many years as capital remains, on the average, engaged in production) will be measured by $\frac{p-l}{l}$. The labourer also must be able to count upon the same interest on his savings. If he consumes the quantity a only and saves the rest of his wages, then his income from interest on these savings will clearly be proportionate to :—

$$\frac{(l-a)(p-l)}{l} = p + a - l - \frac{ap}{l}.$$

Since p and a are to be regarded as given, this equation will reach its maximum when the sum of the two negative terms (on the right-hand side) is as *small* as possible. But these terms have for every value of l a constant product ap; their sum therefore will be least when they are equal. Thus we obtain :—

$$l = \frac{ap}{l} \quad \therefore l^2 = ap \quad \therefore l = \sqrt{ap}.$$

This last expression—the geometric mean of the workers' minimum standard of life (or usual standard) and the total value of the product of labour—is therefore regarded by von Thünen as the "natural wage"—and he wished to have this formula engraved on his tombstone. We will not pause to criticize it thoroughly. In any case, the formula must be considerably modified if it is to correspond with reality. For, in the first place, the rate of interest is not reduced proportionally to the expression $\frac{p-l}{l}$ when l increases (which would, as will easily be seen, presuppose a constant period of production), but, as a rule, much more slowly, owing to the fact that employers react to every increase in wages by lengthening the period of production (introducing labour-saving machinery). In the second place, the interest of the labourer in his savings is not limited to the mere income which they yield, but includes the saved capital itself; he saves for furnishing his house, for his children's education, for his old age, and so on. The most advantageous value of l is therefore probably much nearer p than von Thünen supposed.

What has been said may suffice to indicate, rather than to solve, the many problems associated with the question of capital accumulation—which has been so little investigated. The subject has, however, several further important and interesting phases which are related to the fact that, in our day, capital is almost always accumulated in the form of money. We shall revert to these phases when we deal with the theory of money.

On the other hand, we must be careful not to forget that money or credit is only one guise, one form, of capital accumulation. The amount of hard cash in a country can be neither increased nor decreased by saving, but remains, on the whole, constant; and credit documents of various kinds are at most only titles to material property, except in so far as they presuppose a destruction of real capital, as in the case of war-loans, etc. Real, productive, saving therefore always assumes the form of *real capital*. In the normal course of business this process is clearly visible. The commodities which a person foregoes by saving, and by restricting or postponing his consumption—or rather the labour and land which would otherwise have gone to the production of those commodities—he places directly (or by means of money, credit or credit-institutions) at the disposal of an entrepreneur who converts them gradually, as the savings are effected, into more or less fixed

capital-goods, i.e. real capital. At the close of a boom, paper credit often seems to make up, in part (though actually it does not), for the shortage of real capital—and still more in a period of depression when investment in fixed capital hardly pays, but savings continue, though perhaps at a slower pace. The process of capital accumulation is here not a little enigmatic. It *must* continue in some real form, since there is no other ; but in what ? Further investigation of this question is highly desirable and would probably throw much light on a field which is still the darkest in the whole province of economics, namely the theory of the trade cycle (and of *crises*). But we cannot consider that subject here since we have, throughout, restricted our observations to the economic phenomena of equilibrium in the ordinary sense—to static analysis as distinct from dynamic.

APPENDICES[1]

1. Professor Cassel's System of Economics[2]

(i) Cassel's refutation of the theory of value, his theory of exchange, and his views on the pricing mechanism.
(ii) The theory of interest, the theory of rent of land and mines, the theory of wages.
(iii) The nature of money and international payments.
(iv) The theory of trade cycles.

I

Professor Cassel, like so many others, has felt a call to present his scientific system to a wider public than that which could follow his lectures. He has for this purpose secured the collaboration of Professor L. Pohle, of Leipzig, who is eventually to publish the preliminary part, dealing with historical and sociological developments, of their joint work, *Lehrbuch der Allgemeinen Volkswirtschaftslehre.* Professor Cassel is the author of the second and purely theoretical part, which is now published in a large volume.

To review this book one must sit in judgment on the whole of the author's lifework in the sphere of theory. Professor Cassel expressly desires that all his writings—even the earliest and least mature of them—should be regarded as indispensable foundations for the theoretical edifice, which now appears in its completed form. The wisdom of refraining from a fundamental revision of his earlier, and in my opinion, less completely developed views, while not letting them fall into oblivion, may perhaps be questioned. But naturally this is his own concern. For my part I also have felt the need of arriving at an understanding of his whole approach to theory. On various grounds, mostly personal, I have never undertaken a public criticism of any of his work,

[1] The translation of the appendices which follow is the work of Mr. Solomon Adler.

[2] [This review of Cassel's *Theoretische Sozialökonomie*, Leipzig, C. F. Winter, 1918 (viii, 582 s.), first appeared in the *Ekonomisk Tidskrift*, 1919, No. 9, and was published in German in Schmoller's *Jahrbuch*, 1928, vol. lii-2, No. 5. Unless otherwise stated, all page references are to the 2nd edition of *The Theory of Social Economy*.]

with the exception of his very first essay in the *Tübinger Zeitschrift*.[1] If I delayed much longer, it might be too late for either or both of us. This may excuse the unusual length of the following essay.

The many excellent qualities distinguishing Professor Cassel's earlier writings and also—I believe—his direct teaching activities, are to be found in abundance in this work. I envy him his ability to present generally accepted economic doctrines concisely and comprehensively, and to throw light on them with well-chosen examples from the world of affairs, with which he appears to have acquired a practical acquaintance. Last and not least there is his laudable attempt at a description of concrete economic phenomena, based on statistical material—this being especially evident in the fourth section of the book, on cyclical fluctuations, which I hold to be the best.

With these merits, however, Professor Cassel possesses the defect of desiring at all costs to be esteemed an original and even path-breaking theorist, and this in every branch of economics. It remains a riddle how with his diligent activities as a publicist and with his numerous public duties, he can have found time for these inquiries, for nothing is so consuming of time as scientific thought. I am afraid that his claim is based on an illusion. His originality does not extend in most cases beyond an exposition of the ideas of others in a new, if not always improved, version. Innovations are generally the mark of an indomitable desire to penetrate the obscurer regions of theory; but this end is not often achieved by those who have too cursory an acquaintance with the terrain. The reader finally ends in a bog of mental confusion, which a facile style can no longer conceal, and from which the only escape is to revert to precisely that literature which is here so contemptuously dismissed as "unnecessary" and "scholastic".

The first and most striking of these *tours de force* is the wholesale rejection—already appearing on the first page of his introduction—of "all the old so-called theory of value". Of course, he means the modern theory of value. He has always been more amiably disposed towards the older theories of value; this, together with his charm of exposition, was what recommended him to the aged Schäffle.

[1] *Grundriss einer elementaren Preislehre* (1899).

On the other hand, he wants to extirpate the modern subjective theory of value; but he substitutes for the concept of marginal utility either nothing at all or the "principle of scarcity". He asserts that the psychological phenomena lying behind price do not belong to the economist's domain. This idea reminds us of the English stockbroker, who earned his income, year in year out, by buying and selling railway stock without knowing where the railway was. He also repeats his old objection about the impossibility of "measuring utility", as though exchange and economic activity in general—even in a primitive economy—would be conceivable, if we could not estimate the utility of different objects to us. Similarly, the deliberations of members of Parliament on problems of taxation would be meaningless, if it were impossible to compare the utility of the same good to different persons. (It is characteristic of Professor Cassel that when he has to talk about so-called collective wants, he dismisses the whole thing as a manifestation of force "*Zwang, Zwangswirtschaft, Zwangorganisation*," *et praeterea nihil.*) He himself is of the opinion that the "economist" must adhere exclusively to *money* prices as being a "precise magnitude"—and this was written or printed in the last year of the Great War, when money as a measure of value became completely bankrupt.

He also maintains that marginal utility as the basis of exchange value presents the disadvantage that it is neither given nor determinate, but is itself variable with and dependent on the prices which it is intended to explain. But how does this apply to "scarcity?" A commodity is not scarce because it is present in small quantities, but, as Professor Cassel himself states in the Introduction, it is scarce only in relation to wants, or to the extent that it becomes an object of demand. And the degree of scarcity is measured in exactly the same way as marginal utility, by the strength of the next unsatisfied need, which first causes the commodity to be recognized as "scarce". In other words, scarcity and marginal utility are fundamentally one and the same thing; Walras already recognized this, for the word "rareté" (which he used as an alternative to "utilité finale") signifies scarcity as well as rareness.

Such, however, is not Professor Cassel's opinion. Strangely enough, he himself—apart from the above introductory passage—

gives no description of the concept of scarcity, otherwise the relationship might have become clear to him. To compensate for this omission, he prints in italics the following definition of the principle of scarcity. "In the exchange economy," he says, "the principle of scarcity signifies the necessity, by the pressure of prices, to adjust consumption to a relatively scarce supply of goods" (p. 74). What is meant by a "scarce supply" remains, as we have said, unexplained. But even so, this definition is absurd, for there is no need to be afraid of consumption exceeding supply. In the later course of the work the word "consumption" is consistently replaced by "demand" in this connection. And in this formulation one can indeed recognize the principle, for price undeniably has the "task" of equating supply and demand, so that all the supply is sold and no effective demand remains unsatisfied.

But if the Principle of Scarcity did not contain anything *else*, it would be absolutely identical with the ancient principle of equilibrium between supply and demand, of which Cassel does not suppose himself to be the discoverer. The doctrine of marginal utility goes beyond this by stipulating equality or proportionately between commodity prices and their "scarcity" (= marginal utility) for each exchanging individual. That this principle is by no means so easily established, ought to be proved by the fact that, even Gossen, the first expounder of the theory of marginal utility, did not reach it. That it was not "unnecessary", seems to follow from the fact that not less than *three* genuinely path-breaking scientists advanced it as an important discovery at roughly the same time. Certainly it contains nothing absolutely new, but neither did the discovery of the differential calculus; the service of both consists in their having replaced diffuse or unsystematic ideas by a clear general concept and, what is no less important, by an adequate formulation of it.

Why has Professor Cassel absolved himself from this task? Why, apart from an otherwise perfectly correct résumé in a few lines, has he withheld from his readers this serviceable guide through the labyrinth of the theory of exchange? For example, the concept of *elasticity* of demand with falling and rising prices becomes much clearer, if it is based on the elementary principle that marginal utility always remains proportional to price. Here

also Professor Cassel advances without closer examination a series of statements, of which a few are correct, but generally require an explanation, while others are doubtful or completely wrong; such as the statement (p. 80) that demand must invariably rise with a fall in price, and *vice versa*. This is not certain in the case of goods which are partly substitutable in consumption, and in the case of the reservation demand of the holders of the goods themselves. The effect on the latter of a rise in the price of their goods may quite conceivably be such that they retain a greater proportion for their own use. This must have been the case to a great extent, if I am not mistaken, with those who produced for their own needs during the war. Professor Cassel himself gives an extremely brief account of the fundamental concept of marginal utility, and it would not surprise me, if his readers, so far from thinking this account "unnecessary", were, like Oliver Twist, to ask for more.

Another peculiarity of Professor Cassel's to which we have elsewhere drawn attention, and which, remarkably enough, he regards as a step forward, is his use of money as a "scale of reckoning".[1] He boldly maintains that when the classical economists attempted, wherever possible, to abstract from the use of money in their inquiries into economic phenomena, this was due to their preconception (which according to Professor Cassel is false) that in primitive society no money was used. This statement is characterized by a naïveté which one would hardly have attributed to Professor Cassel. On the contrary, this conscious abstraction from the functions of money—the conception of trade, external as well as internal, as consisting in the last analysis in the exchange of commodities, of capital as *real* capital instead of as a sum of money,[2] of wages as *real* wages—was the decisive step which first gave economics a truly scientific character, and first raised it above the hazy and incoherent ideas of Mercantilism.

For the rest, Professor Cassel does not succeed in carrying his method through consistently. In his treatment of the "pricing mechanism", to which we shall later return, he first

[1] Cf. his "Grundriss", where he tries to base the whole of economics on the useful "fiction" that a shilling has the same economic significance for all men, whatever their economic position.

[2] Later on, Professor Cassel has to "warn" his readers against confusing real capital and money.

assumes that each consumer has a certain purchasing power (expressed in money), and he naturally arrives at the result that the prices of commodities will be completely determinate. I became vastly interested in reading this, for I thought that the next step would be an attempt to demonstrate how this monetary purchasing power arose and was maintained. But nothing comes of it. The hypothesis is merely advanced only to be dropped later, and quite unexpectedly the explanation follows that the phenomena of exchange and production only suffice to account for relative commodity prices, but *not* absolute money prices, a task which must be kept in reserve for the theory of money as such. Now it should be noticed that Cassel explicitly says that at this stage he will consider money *exclusively* as a "scale of reckoning", and will thus provisionally abstract from its function as a medium of exchange, which perhaps may even be taken over by some other commodity—as in Homer's times, when "oxen" were used as a measure of value, although they hardly constituted a general medium of exchange. In this case, however, money would retain *its character as a commodity* perfectly intact, and the use of money as a measure of value would not prejudice it in the least. In other words, the exchange value of money would be determined by the phenomena of production and exchange in exactly the same way as that of other commodities, and *ex hypothesi* the prices of commodities expressed in money would be uniquely determined precisely by the "pricing mechanism", and *not* merely, as Professor Cassel states, prices multiplied "by any factor whatsoever". Further, we should get the same result, if money were, strictly speaking, nothing more than a medium of exchange, i.e. the means of presenting goods on the market. For an amount of money, however small, can, as we know from experience, effect the exchange of any quantity of commodities whatsoever. Now money has also a *third* function, which is in practice the most important, i.e. as a "store of value" a reserve or cash balance. It is through this characteristic that with given commodity prices the need for a certain amount of money obtains, and it is here that the *amount* of money becomes a factor of the first order for commodity prices. Further, it is through this property that the character of money as a commodity recedes into the background, becomes secondary, or even vanishes

altogether. This was perfectly clear to Walras ; he first treated money as "numéraire" (unit of account) and only later as "monnaie" (medium of exchange) ; as far as its first property is concerned, it is for him only one commodity among many. Professor Cassel, on the other hand, argues as though the "oxen" of Homer were not generally the object of consumption and exchange, but only served as a "scale of reckoning". Here at least the premature introduction of money has contributed not to increased lucidity but rather the reverse.

A more valuable element in Professor Cassel's account is the emphasis laid on the reciprocal relationship between products and the factors of production in the pricing process. In those of his earlier writings which I regard as his best,[1] Professor Cassel has shown with a masterly clarity that as soon as we have more than one factor of production (e.g. simple manual labour), and in fact we have hundreds of different kinds, the principle that costs of production determine the exchange value of a product can no longer be maintained. These costs become quite simply the *prices* of the factors of production, which are necessarily determined in combination with the prices of commodities in a single system of simultaneous equations. This idea, however, belongs to Walras ; it is his powerful synthesis which in the last analysis lies at the basis of Professor Cassel's "pricing mechanism". Professor Cassel's indebtedness to him is obviously very great, but instead of showing the gratitude he ought to have expressed, he does not mention Walras' name once in the whole book.[2] He adheres, though not altogether consistently, to the principle of never quoting anybody but himself. He has not, however, accomplished any improvement in Walras' exposition —apart from a certain simplification in the formulæ. On the contrary, he breaks off in the middle, with a resulting loss in coherence. Following Walras, he describes how the total rewards of the factors of production are in the main identical with total (real) incomes, and are at the same time the source of the demand for goods and services ; he adds that these incomes are not all

[1] *Die Produktionskostentheorie Ricardos*, etc. (Tubinger Zeitschrift, 1901). In this well-written essay he also pays to the theory of marginal utility a tribute for which one looks in vain in his other writings.

[2] In his "Grundriss", he explicitly bases himself on Walras ; but only a couple of years later he describes his own essay as "the first attempt" of its kind ! (*Der Ausgangspunkt der theoretischen Oekonomie*, Tübinger Zeitschrift, 1902, p. 697 f.)

consumed, but are partly saved. But at this point the equality (which we had previously accepted) between the sum of the factors of production now available, and that part of them which enters into the various goods demanded for consumption, ceases to obtain, and Professor Cassel's system of equations (7) (p. 144) is no longer valid. For the whole system to function, it is not only necessary that the savers, as Professor Cassel assumes, should decide how much to save on the basis of ruling prices or in relation to the determination of prices, but also that they, or the entrepreneurs in their stead, should be clearly aware what factors of production to demand in order to invest their savings most profitably. Of this, Professor Cassel says not a word. But even if the reader in distress could fill this gap unaided, he would in any case begin to have doubts, when, in the course of the book he meets Professor Cassel's factor of production "capital-disposal" and its "price", interest, and tries to accommodate these magnitudes with the other factors of production and the prices in the formulæ previously given. It would have been of great interest if Professor Cassel had indicated how this could be managed without double reckoning.

Walras proceeds in an entirely different manner. For him the capital-*goods* themselves are factors of production just as much as labour and the forces of nature ; and the rate of interest "le taux du revenu net" is considered as the ratio between the expected yield of the capital-goods now being made (= the price for their factors of production minus the necessary amortization costs) and their own cost of production according to *present* prices. Thus it here represents a "parameter" in the "functions" which determine saving. Savers and entrepreneurs strive to maximize this ratio, and equilibrium is reached when it is the same for all alike. In this way Walras constructs an extraordinarily coherent and rigorous system, which, when it is combined with the systems of Jevons and Böhm-Bawerk, both completes and is completed by them.[1] Professor Cassel simply omits the whole

[1] Clearly Walras' method does not yield the *actual* rate of interest which the future reveals, but the anticipated interest on which the level of the loan rate is directly dependent at any moment of time. At this point I must withdraw an objection which I previously made against Walras—i.e. that his theory of interest necessarily presupposes a *progressive* type of society. Walras indeed said so himself, but the truth of the matter is that it is just as applicable

of this vital section of Walras' system, and defers the theory of capital to the next book, when he moves more freely without having to trouble himself about algebraical formulæ. But more of this later.

We are now confronted with the difficult task of giving an account of another peculiarity in Professor Cassel's presentation of fundamentals. It is well known that the classical economists were often inclined to adopt a method of approach in which they regarded free competition or a free pricing mechanism as a kind of *moral* factor—an economic providence, so to speak, which gave each participant in total production his allotted and just share of the product and, at the same time, gave the maximum sum of satisfactions to all. Among contemporary economists, Professor Cassel should be one of the last to find it easy to escape from this approach. It is true he does not go so far as to make the existing state of society, based in principle as it is on free competition, the ideal of social justice ; his own parable "of the bread of the poor which is sometimes thrown to the dogs of the rich" bears evidence of this. But essentially he stands by the classical system. He emphasizes as often as possible its economic superiority, and if he can do nothing else he praises "the free choice of consumption goods" which it provides in contrast with, for example, a similar socialist state. He is so little afraid of provoking laughter that in another context he adduces *salt* and *ink* as proof of the fact that even the poorest can almost completely satisfy some of their needs.

Actually the lower classes in present-day society do not in the least possess free choice in consumption ; as far as means of subsistence proper are concerned, they are allotted all the cheapest brands, and their remaining consumption is similarly organized. A compulsory rationing of the most important commodities, as in war time, would certainly give them greater freedom in their "choice of consumption-goods".

When we are dealing with production apart from distribution, we can then say that, in a certain sense, economic freedom

to the stationary state, and in fact gains thereby in rigour. The underlying assumption is that the factors of production will have the same relative values or prices in the future as they have at the present moment. Actually this is true for the stationary state, but it does not hold for the progressive economy, unless we postulate a *uniform* increase in production, which is strictly speaking inconceivable, as the sum of natural forces cannot be increased.

promotes "economy", for as soon as a surplus of exchange value can be obtained at any point with the available factors of production, under free competition they are necessarily transferred to that point. Yet we must of course remember that the *kind* of production is determined by effective demand, and not by the socially desirable demand for products—two concepts which Professor Cassel is only too inclined unconsciously to confuse. The problem of the share of the factors of production in the resulting total product is, in the last analysis, identical with that of social distribution, and no eloquence can conceal the fact that "the Principle of Scarcity" only produces a *bare* mechanical levelling, which *faute de mieux* may perhaps be preferred to any other, but which is not based on any ethical or sociological principle. The "simultaneous equations" are no guarantee that any "variable" cannot assume the value nil, even if we are discussing so important a social factor as wages, or so questionable—not to say odious—a social factor as the rent of land, site-rent, or certain monopoly revenue, etc.[1]

The situation is worsened if free competition is abolished by agreements, and the contracting parties are ranged against each other like two opposing armies, for here in most cases the result is at least as uncertain as that of war in general, and mutual destruction is the only outcome of which we can be sure.

In an extremely well-written section, xiv, which is, however, too optimistic and "apologetic" in tone, Professor Cassel himself gives a vivid description of the tendency towards an ever-stronger limitation of freedom of production which characterizes modern economic development. But even here he seeks, wherever possible, to defend his *suum cuique.* He wishes us to believe that when large-scale enterprises agree to form trusts, it is because they would otherwise have been forced by internecine competition to produce *at a loss.* And he points out that when the State is compelled to grant monopolistic powers to certain corporations such as railway companies, it seeks to limit the pernicious utilization of this monopoly by

[1] In "Der Ausgangspunkt, etc.," Cassel is so radical that he wishes to confiscate all ground-rent proper (p. 686); in the present work he arrives at the surprising conclusion that urban site rents are "essentially the result of man's productive activity". It is not stated how this should be taken, and what conclusion he infers.

maximum rates and the like. Moreover the monopolist is sooner or later threatened by latent competition, for example, from abroad.

Here it should be noted that a monopolistic system of prices is by no means the worst; if we assume that there is only *one* or at most a few monopolies in the whole market. If, however, many branches of production become monopolized or trustified, it would ultimately become aimless for them to raise their prices against each other; they would have to seek their profit—apart from a certain technical advantage lying in combination—in more or less common action by which the prices of those factors of production not in their own possession (and especially the wages of labour) would be forced down, or at least be prevented from rising. The State has not, at least up to the present, been in a position to react against this procedure.

This vagueness in Professor Cassel's social views is analogous to a similar vagueness in his theory. Although in his own system there are no independent costs of production but only *prices* for the different factors of production (which equal their share in the total product), he considers it important to mention that the price of each good must coincide with its costs of production—which on his assumption becomes merely a platitude, a self-evident fact (in so far as the costs of production can be imputed). In other words, "each demand must carry the costs bound up with it," or, as Professor Cassel sometimes expresses it, without further classification of the concept, it must carry "the necessary costs". All this is hopelessly obscure; perhaps it is Walras' principle of the tendency of entrepreneurial profits to zero, of which originally he had a glimmering. But later it seems to have escaped his attention, that it holds only under perfect competition, and that certain "costs", which even in perfect competition are economically necessary, are not therefore socially necessary. A division of the yield of land among the consumers of food or the yield of forests among the consumers of timber on a *pro rata* or some other basis would not indeed lead to a fall in the price of this commodity, but it would lead to a fall in the actual expenditure on it.[1]

[1] Apart from its application to the rent of land, the first statement of this view of Cassel's is to be found in his essay "Der Ausgangspunkt, etc.," an essay which in my judgment is extremely obscure and completely mistaken in its results, and with the shortcomings of which Cassel is not yet sufficiently acquainted. In the present work Cassel has somewhat modified the

In order to "stàte" or "produce"—whichever word we prefer—this equality between costs and the prices of commodities, the "Principle of Scarcity", according to Professor Cassel, is no longer sufficient. In many places there is some indeterminateness in costs of production, to overcome which no less than *four* extra supplementary principles are in his opinion necessary. These he calls the *Differential Principle*, the *Principle of Decreasing Average Costs*, the *Principle of Substitution*, and finally the *Principle of Joint-products.* They are four too many. The last may have some significance, but rather as an exception to the "Principle of Costs" than as a means of establishing it. If two or more products are technically combined in production in a constant technical proportion, then an imputation of their costs is out of the question, whereas naturally each has its particular market price determined by supply and demand. If, however, which is more usually the case, the technical proportion varies (sheep for mutton or wool respectively, etc.) particular costs exist at the *margin of production*, and these must coincide with their price in the usual way. The same holds for the Principle of Substitution. On the whole the different factors of production are not wholly substitutable, but are simultaneously applied. At the margin of production, however, a contemplated (virtual) increase or decrease in any one of them can be regarded as its economic contribution, and this is necessarily proportional to its price. But this substitution value, or, what comes to the same thing, this marginal productivity is measured in the same way as the "scarcity" of the good, with which it is thus identical if it is correctly defined. Professor Cassel's reiterations to the contrary are, in my opinion, merely evidence of an incomplete analysis.

Neither is the *Differential Principle* an extraneous addition to the Principle of Scarcity; here it is essentially a question of different factors of production, each of which in spite of an external similarity has its own scarcity and price. Two pieces of land of different fertility or at different distances from the market are not the same thing, even though they may appear to be.

statements he there advanced; for example, he no longer e.g. calls for a *fee-principle*, where it can be generally applied, which shall be the only right one for the financing of public enterprises. But he still strongly inclines to this opinion, and in a discussion in the *Nationalekonomisk Förening* (Economic Club) he curtly opposed every other kind of "cover" for State railways, for no reason whatsoever.

But most suspicious and to me most incomprehensible is the " Principle of Decreasing Costs ". Professor Cassel, in his introductory remarks on this subject extending over several pages, maintains that as he has already treated the position of different firms in relation to each other (under the " Differential Principle "), he will now assume for the sake of simplicity that each commodity is produced by only one large firm. Even so, costs of production can in his opinion be indeterminate, in so far as they vary with the size of the firm. If costs increase as the firm increases, the case is simple (he says)—it is the highest costs, i.e. the marginal costs, which determine price. He says nothing about the destination of the profit in such a case, but suddenly abandons the whole of this interesting special question. And it is just as well, for it is difficult to imagine a large firm with increasing costs of production. If production on a small scale is more remunerative than on a large, factory work gives way to domestic work, large property is parcelled out into smallholdings, etc.

On the contrary, the large firm with decreasing costs (as the firm's scope extends) is an actual fact. Here, says Professor Cassel, the highest costs cannot be price-determining, as " they are at the bottom and not at the peak of production ". But neither can the price of the good be equal to its marginal costs, for the firm could not then maintain itself. Consequently, he concludes that we must choose the *via media*, where the price is determined in such a way that it just covers the average costs, so that there is no profit for the entrepreneur—in contradistinction to the previous case ! Professor Cassel gives no clue to the entrepreneur's reasons for such benevolent behaviour. He goes on to show—and it is not difficult—that in this case " at least two " prices must exist, a higher and a lower, with each of which the firm's expenses would be covered. From these he decides that it is the lower which is chosen ! He first takes the case where the relation of sales to price varies in such a way that, within limits, production just pays its way with any price—presumably in order to make this conclusion more palatable to the sceptical reader. Here we must admit the producer has no incentive to fix the price higher than is reconcilable with the consumer's interest. But what of all the other cases ? Between the maximum and minimum prices there lies a whole series of prices which would yield a

surplus profit for the producer. Then why does he not choose one of them? If we assume that he alone is master of the situation, he will certainly fix on that price which will yield the *maximum* profit; if, on the other hand, he has competitors, even though they be smaller and weaker, he presumably chooses a somewhat lower price in order to ruin them, after which he can again raise his price. In other words, when the law of "increasing returns" holds for a firm, and holds for any expansion whatsoever, then free competition is impossible, and the profits of the entrepreneur, which finally become a monopoly gain, have no tendency to disappear.

The astonishing thing is that Professor Cassel is actually very well aware of this, and mentions it in the very next paragraph (p. 129). Nevertheless, he later appeals without further ado to this peculiar "supplementary principle" in his chapter on "the pricing-mechanism" (pp. 161 *ff*.). It remains a puzzle how all this can be understood.

The confusion increases when the author, with reference to these "principles" (p. 111), applies the expression "increasing" and "diminishing returns" in an entirely different sense, i.e. as the result occurring when one factor of production is combined in increasing quantities with another which remains constant—for example, when a fertilizer or an increased amount of labour is applied to land of a given quality. Here there is no question of an increase in the scale of production! The principle remains the same, whatever the area of the land employed. It is true, as he observes, that an increase in the scale of production often occurs together with a change in the proportion of factors employed or is even conditioned by it. This naturally complicates the problem, but should not lead to a confusion of fundamentally different concepts.

The whole of this farrago—I can scarcely call it anything else—is largely to be attributed to the fact that Professor Cassel stubbornly passes over the earlier specialist literature on this subject, which he ostensibly finds "superfluous" since the appearance of his own book.

Amongst minor points in the first book we may only mention that he (p. 52) includes "trade marks and patent rights" as part of the "total capital" of a "closed exchange economy" (expressly as "real capital"); and if I understand him aright,

he includes the increase in the value of land and sites occurring during the year in "total income" (p. 57). Neither can be right. An invention, i.e. a certain method of work, has nothing in common with real capital (though it may well have cost a large sum of capital), and when a patent expires, society is none the poorer, if anything it is richer—otherwise why should legislation restrict patent rights? Again, mere increase in land values may certainly be included in the national income from a fiscal point of view, but hardly from any other. Professor Cassel merely says that the national income suffices to pay for the rise in land values (which he calls an important principle), but vague terminology does not improve the matter, nor does it render it more intelligible. Why not clarify important social relationships instead of obscuring them?

II

In the second book, the chapter on interest should arouse mixed feelings in most readers acquainted with the subject, and this as much for its critical as for its constructive contributions. The wage-fund theory is categorically described as "sterile dogmatism"—it was at least of some use, and the error in the older version consisted above all in regarding the fund without further proof as a fund stored up for a single year. It was this error which led even Ricardo to certain fallacious conclusions. But this defect has been remedied in more recent times by the analysis of Jevons and still more by that of Böhm-Bawerk.

I doubt whether Professor Cassel has the support of any serious economist when he describes the work of Böhm-Bawerk (and Menger) as a "definite retrogression"(p. 191)—except in the sense that they actually "went back" to the original ground of the whole phenomenon of interest (i.e. the exchange of present against future advantages). It is in this way that their theory can embrace all kinds of interest, even the case in which no capital is accumulated in the physical sense, as in consumption-loans [1]; most other theories of interest are narrower in this

[1] Professor Cassel "prefers" to regard consumption-loans as "negative capital accumulation" from the borrowers' point of view. Socially or physically, however, there is no negative accumulation of capital, only an uncompleted positive accumulation, by which some existing capital goods are destroyed of themselves, and the stock of social capital is thus diminished.

respect. The discourtesy of Professor Cassel's judgment [1] is even more offensive than it is absurd. Böhm-Bawerk, in his *Geschichte und Kritik,* without altogether approving of Senior's theory of interest (which stands in the closest agreement with Professor Cassel's) declares it to be "incomparably superior to his predecessors' theories in its profundity, its system, and scientific seriousness", and defends it against unjustified attacks. One has only to compare this treatment of so distinguished an economist as Senior with Professor Cassel's remark on Böhm-Bawerk in order to appreciate on which side "sober scholarship" is to be found. And since it is clear that Professor Cassel, like others, takes most of what he really knows about the functions of capital and interest from Böhm-Bawerk, one involuntarily recalls the words with which Dr. J. Bonar, for the most part in good will, concluded his review of the "Nature and Necessity of Interest": "*Maledicti, qui ante nos nostra dixerunt!*" [2]

Jevons' theory of interest, which is essentially identical with Böhm-Bawerk's, is nevertheless called an "important advance". Professor Cassel's first objection against it is that capitalistic production does not require "an accumulated stock of foodstuffs" Did Jevons make any such assertion? Jevons says that capital in its "free" form, i.e. at the beginning as well as at the end of its existence as (invested) capital, assumes the form of means of subsistence; but that is not to say that this disinvestment must occur *en masse* and at one blow in any particular enterprise. I shall return when reviewing Professor Cassel's own construction to another objection he makes against Jevons. A third objection is that Jevons "wishes to determine interest" exclusively "by means of the marginal productivity of the extension of the period of production", which "completely loses sight of the Principle of Scarcity". As we have shown, scarcity and marginal productivity, correctly understood, are one and the same thing. If we consider capitalistic production in society as a whole, it consists in the application of the annual "endowment" (to use a felicitous term of Böhm-Bawerk's) to preparing for a consumption, which, on the average, lies at some point in the future, and at a point more remote, the more

[1] His objection that "some saving would take place even with a zero rate of interest" is ludicrous in this context; if there is no rate of interest, there is no need for any explanation.

[2] *Economic Journal,* 1904.

intensively capitalistic production is. Here the duration of the capital-investment is the only variable dimension, and an increase in the social capital is thus *ipso facto* equivalent to a lengthening of the average investment-period. It is of course assumed that the original factors of production, land and labour, remain constant, or, which amounts to the same thing, that capital increases relatively to them. Professor Cassel's reference to "conservative agriculture", where an increase in capital need not bring about any change in the period of production but at most an extension of the area under cultivation, is therefore only an *argumentum ad ignoriantiam*—how far it is *ex ignoriantia* it is for the reader to say.

We return once more to Böhm-Bawerk. Of his *magnum opus* "Kapital und Kapitalzins", Professor Cassel says that "in spite of the solid and extraordinarily careful work put into it, it is in the main misdirected, both in its critical and historical and in its constructive parts". Böhm-Bawerk's critical monograph, a work without peer in economic literature, which clearly and decisively demonstrates [1] the obtuseness, superficiality, and error so characteristic of most of the older attempts to explain interest—can it be "in the main misdirected"? Perhaps for a change, Professor Cassel will enlighten us as to why his own loud praises of Turgot's theory of interest (in his "Nature and Necessity of Interest") are now suddenly silenced.[2] But he may well rejoice that his own youthful *jeu d'esprit*, the idea of identifying interest and the quota of capital accumulation for the splendid reason that they are both proportional to capital as well as to time,[3] escaped Böhm-Bawerk's critical attention. As far as the "Positive Theory" is concerned, its "misdirected character" should, according to Professor Cassel, already be made evident by Böhm-Bawerk's "statement of the problem". "Does the

[1] It is easy to explain, as Professor Cassel does *ex poste facto*, that the older attempts must be allowed "to lapse into oblivion" (p. 185), but it would certainly not have been so before Böhm-Bawerk had written.

[2] But we can still find a faint echo of them in his statement (p. 51) that a piece of land in part yields a certain return, and in part "obviously" has a certain capital value. *Actually* that is so, but it only becomes "obvious" with a rational theory of interest—i.e. one opposed to Turgot's.

[3] "Das Recht auf den vollen Arbeitsertrag," p. 124 ff. There Cassel, in his eagerness to obtain fairly plausible figures, makes an arithmetical blunder, which as a teacher of mathematics he would scarcely have excused in his pupils. When the book appeared I privately drew his attention to this serious error; but this circumstance did not prevent him from later quoting his work without any qualification, as if no exception could be taken to it.

appears most clearly if we regard the services (of durable capital-goods) as the ultimate products and thus include waiting for their services in the production process in its wider sense", etc. Here, therefore, we must necessarily deal with a significantly long "period". Yet he makes no attempt to complete his previous "equilibrium equations" by taking account of this omission, and the cardinal question of whether the "price for waiting" (interest) is determined by its own scarcity or the "scarcity of capital" remains shrouded in darkness. The problem is indeed difficult; it is only Professor Cassel's claim that he has made it so much easier than his predecessors that gave occasion to these reflections.

Professor Cassel's favourite expression "capital-disposal" (it used to be called *Kapitalnutzung* or the use of capital) is not particularly suited to the clearing up of the matter. This "capital-disposal" soon becomes synonymous with waiting (in which case it is superfluous as a term), and then a *condition* for waiting (and therefore not synonymous with it) in the waiter himself[1]; and later we take it to be the waiter who puts his capital at the disposal of another. "Waiting," we read (p. 199), "implies that a person foregoes for a time the disposal of *capital*. Capital-disposal is the right of disposal over *capital* thus rendered possible for this period."[2] But what is the word "capital" doing here? The man who saves and waits certainly foregoes the consumption of some of his *income*, and eventually places this *income* at another's disposal in exchange for a future (greater) income. A house costs £5,000. I have an income of £1,000 per annum plus 95 shares of £50 each, and either get the house built or want to buy it. I forego the consumption of a quarter of my income, or £250, and sell my shares in lots of £250 to nineteen other similarly situated persons, each of whom saves a quarter of his income in order to obtain possession of the shares, which thus only change hands. (Alternatively, they might have taken out mortgages on the house.) With these twenty parts of twenty different persons' incomes the house is paid for, and no house

[1] "In order to be in a position to take over this function (waiting), we must dispose of a certain amount of capital in the abstract sense" (p. 199). The words "in the abstract sense" admit of no explanation, for nowhere previously has there been any definition of capital other than real capital.

[2] The italics are mine.

has ever been built or purchased in any other way when payment was made in cash. The builders of the house obtain a new income, which they can dispose of as they think fit. The matter is just as simple in practice. Why make it more complex purely for the sake of jargon? Professor Cassel also has a predilection for the phrase "capital-market", but fundamentally it is only a metaphor, for no capital in the physical sense is either demanded or supplied on this market, but simply and solely portions of income, which are supplied by savers and demanded by entrepreneurs.

Characteristic of Professor Cassel is his sharp distinction between durable goods and consumption goods. Here also he must have been primarily inspired by Walras, who as we know defined the former exclusively as capital and the latter as "*revenus*"; for the simple reason that the total value of the future services of a durable good is as a rule greater than its present value, the difference constituting interest. This distinction, however, cannot be justified. Even the goods which are consumed in a single act must be counted as capital when the act of consumption occurs in the future and the goods obtain a greater value through the very act of waiting. Broadly speaking, the manufacturers' and merchants' stocks of raw materials and finished goods belong to this category, as Professor Cassel himself admits, although he is apparently inclined to belittle their importance.[1]

None the less he wishes to maintain without qualification that this distinction is essential. Even in the Introduction he

[1] Whether stocks of such goods arising from the intermittent nature of production should be included in the capital concept is a detail of mainly theoretical interest. The answer to the question is in the negative. Professor Cassel attempts to show that the need of an economic distribution of the consumption of such a stock, e.g. the stock of wheat until the next harvest, would of itself produce interest (p. 216 n.). But he succeeds in showing something entirely different, i.e. that if money interest originated in another way, the price of wheat during the consumption-year must successively rise. Here it is precisely the rate of interest which brings about an increasing inequality in the consumption of the stock of corn, whereas according to Professor Cassel the function of interest should be the reverse. The explanation should be simply that the scarcity of the stock has no effect on the height of the rate of interest. If everything is completed in terms of corn there is no rate of interest even in Cassel's example, and if wheat were the only commodity produced, it it is difficult to imagine how transactions *within* the harvest-year could produce interest. But not so when they are extended from one harvest-year to another, when the rate of interest would be a symbol of the discontinuity of production itself.

devotes to the subject space and attention which seem to me to be wasted. Again, when he is explaining the origin of interest, he clearly distinguishes between "the gradual wearing-out of durable goods" and "time-consuming production in the real sense", and he accuses (p. 194) Jevons (and Böhm-Bawerk) of "artificial constructions", when they try to "force" both processes "into a single form". We may well admit that the *technical* aim of capitalistic production is, or at least can be, different in both these cases. One or more time-intervals can deliberately be inserted in production in the latter, mainly in order to utilize the free forces of nature (the storing of wine in cellars, the effects of sunlight on vegetation, etc.). With durable goods, however, it is largely a question of joint supply. A capital-good is given durability in order that it should yield more services, but these must, on the average, necessarily be postponed to a more or less remote future. From an economic point of view the difference is therefore unessential—the less so because increased durability often goes hand in hand with an all-round increase in efficiency; and it entirely disappears if, as in other cases of joint supply, we employ the method of variations (the marginal method) and thus obtain a picture of the whole process in flux. A farmer has to choose between two ploughs, one of which lasts ten years, and the other, equally useful, lasting eleven. If he chooses the more durable (and dearer) plough, he has the benefit of an extra year's service, which, however, only comes into being after the lapse of eleven years, and must therefore replace the difference in price between the two ploughs accumulated by the total interest for the eleven years. Similarly, the price of old wine must exceed the price of newly-pressed wine by the interest for the years of storage.

Professor Cassel holds that the real practical reason justifying this distinction is that "incomparably the largest quantity of capital-disposal is required for the services yielded by durable goods" (such as houses, railways, etc.). Translated into everyday speech this means that the greater part of annual savings, together with the annually disinvested portions of capital, are invested in this way. And this is what undoubtedly happens in present-day society, but only because of its outstandingly progressive character. In a stationary state, the situation would be entirely different. The whole of this analysis furnishes but

one example out of many of Professor Cassel's irrational inclination to regard as normal what is from a quantitative point of view a violently progressive society.

We come now to an undoubtedly valuable contribution to the practical problem of interest. We are, or course, referring to his celebrated calculations on the strong impulse to individual capital-consumption and to a reduced total of capital accumulation which a very *low* rate of interest would induce. (It is on account of this tendency that such a low rate cannot exist.) This element deserves all attention, but one cannot with certainty infer any other conclusion than that saving and capital accumulation will progress at a slower *tempo* the more the rate of interest falls. And this seems to be clear *a priori*. Assuming a sufficiently clear insight into the urgency of future wants as compared with present wants, and also a sufficiently vivid interest in the welfare of future generations, it will appear that capital accumulation cannot cease, as long as it is generally possible to gain *more* in the future by sacrificing *less* (computed in terms of subjective values) in the present, i.e. as long as there is a positive rate of interest, however small. In a socialist state, the conception of which presupposes the fulfilment of both these conditions, the rate of interest would therefore tend to fall to a minimum, until it finally became zero. Cassel's own views on "interest in the socialist state" are rather obscure, and appear to be a survival of his bizarre ideas in "*Das Recht*".

The important practical question of the structure of the rate of interest in the immediate future, that is to say, until the losses in capital incurred in the war are more or less made good, depends above all on what happens to the population. This book contains no chapter on the theory of population—only a couple of pages in the chapter on wages are devoted to it, out of sheer necessity—and the author's own views on the subject seem to be hopelessly vague. It appears as if his whole system of economics is so inextricably bound up with the idea of a continually and rapidly increasing population that he cannot depart from it, even when it is all too patently opposed to the facts. Before the war, Bortkiewicz had already predicted that the population of Germany would have become stationary within perhaps twenty-five years. Since the war the probability

has become much greater, and the prediction need not be confined to Germany alone!

Professor Cassel maintains that, even in a stationary state, every fall in the rate of interest would produce an enormous rise in the demand for fixed capital, e.g. for houses for labourers. But this is by no means certain. The price of a house is not made up of interest only, and, besides, the habitation of a large house involves other outlays. Of these fuel was quite as expensive in Sweden as the rent of the house itself—at least during the war. The situation is entirely different when there is a great rise in the standard of living of the labouring population, for then it becomes certain, as the example of America shows, that the workers' demands for dwelling space will increase even without any fall in the rate of interest.

All in all I fear that Professor Cassel has not succeeded in throwing light on the problem of the probable future rise or fall in the rate of interest, whether in its theoretical or practical aspects.

We must add that this chapter undoubtedly contains many sound observations, e.g. on the question of the tendency towards the concentration of firms (increasing returns proper)—a subject which has hitherto been very much neglected by theorists. But queer and arbitrary statements, whose only motive apparently is a desire to controvert accepted principles, are to be found in plenty, e.g. on pp. 227 and 228 among others. For reasons of space, I must forego any closer examination of them.

No less than thirty-eight pages are devoted to the theory of rent. We may well doubt the need for so exhaustive a treatment, for nothing new is added to a subject which has been discussed almost *ad nauseam* and which is yet so simple in essence. The pertinent criticisms of the Ricardian theory had already been made by Walras and should by now be considered common property in economics, even though no less an economist than Marshall attempts to maintain Ricardo's teachings in their old formulation. It is in any case an abuse of words to dismiss, as Professor Cassel does, Ricardo's famous thesis that "the price of corn is not high because rent must be paid but that rent must be paid because the price of corn is high" as merely "false". Rightly interpreted, it contains an extremely important and often misunderstood truth, and it should not give rise to any real misconception.

Naturally, in actual fact, as Professor Cassel (following Walras) rightly maintains, the price of land and its services is determined in more or less the same way as the prices of other factors of production, and is only a link in the whole chain of price-relationships. But if one tries to deal with the whole problem in all its ramifications at once, it becomes so much more complex and so much less susceptible to a general survey that the whole exposition peters out in vague generalization. If we are to obtain some real insight into the interrelations of the phenomena, it is therefore necessary [1] to start with a first approximation or abstraction, in which the *quantities* of goods on the market are taken as given, and then go on to a second, in which the *prices* of the goods are taken as given. This procedure is equivalent to treating the problem of production (and distribution) on the assumption that only one commodity is produced—and yet even in this case it is complicated enough!

As an example of the looseness of analysis in this book, we may cite the statement (p. 286) that in comparing two pieces of land of different quality, we must not, as Ricardo does, assume them to be worked by the same amount of "labour and capital", but by the amount of labour and capital adapted to each. What is he driving at? Ricardo himself says that the better land is cultivated more intensively, whether alternatively to or simultaneously with the cultivation of the worse, but that does not imply that there is a lacuna in his deduction of differential rent.

Professor Cassel's peculiar and mutually inconsistent definitions of "increasing and decreasing returns" have already been discussed.[2] On p. 279 he adds yet a third, when he says that if with a given *price* for labour, land, and capital an entrepreneur can increase the value of his product relatively to total costs by applying more labour and capital to a given piece of land, a firm is still "in conditions of increasing returns". But on the same assumption the entrepreneur could have obtained the same addition to his relative profits by diminishing the

[1] It was this method that I for my part adopted.

[2] As in agriculture, we can usually manage without the terms increasing and (after a certain point) diminishing returns. For as soon as the population has increased to such an extent that the free products of nature (wild grass, timber, etc.) have an exchange value, diminishing returns have already set in and cannot be counteracted (but rather can only be shifted to a higher plane by technical progress)—which Cassel himself seems to admit.

amount of land employed and thus reducing total costs. On this excellent definition "increasing" and "decreasing" returns are therefore identical!

The whole analysis is here very nebulous and diffuse. Naturally, the entrepreneur strives to attain the maximum absolute and not relative profit; we must therefore necessarily start from something fixed and given, or else the whole edifice will vanish into thin air. We must assume that the entrepreneur disposes over *either* a given amount of capital (his own or borrowed), or else a given area of land, or finally a given amount of labour (as in co-operative agriculture). But in this case the Principle of Substitution only comes into operation for the factors of production demanded by him and not for those he already possesses.[1] Only in a general equilibrium resulting from competition between entrepreneurs, where their profits are theoretically forced down to zero, does the Principle of Substitution or marginal principle hold universally. And yet we must always introduce a reservation for "the marginal productivity of capital" regarded as a sum of value. This I have explicitly proved in my writings, but Professor Cassel completely neglects it. His own rather vague and diffuse theory of capital is wholly unadapted to more clear-cut conceptual distinctions.

Greater store must be set on his really exhaustive treatment of the rent of mines—"the price of natural materials". And yet in my opinion his discussion would have gained in significance if he had first dealt with what is theoretically the simplest case, that in which the mines are regarded as inexhaustible, and at the same time the annual output can be increased within certain limits without increased general costs. This is clearly the assumption from which Ricardo starts in his only too sparse reflections on the subject. If in these conditions all mines should be regarded as equally productive, there would, says Ricardo, be no rent for the mine, and the price of the minerals would include only labour and capital costs. When, on the other hand, some mines are more productive than others, the owners of the

[1] This limitation is also to some extent applicable to my analysis in *Lectures on Political Economy* (p. 131 above), where I was dealing with increasing returns proper. It was therefore possible to conceive of (e.g.) a trust with a large capital consisting of many individual firms and striving after an optimum size for each; in this way it obtains a maximum profit on all its individual investments of capital and on its capital as a whole.

better mines enjoy a rent, which is determined in the same way as the ordinary rent of land.

But here Ricardo must be wrong. If on this assumption the better mines were released for free exploitation, labour and capital would flow from the worse to the better mines, the annual output would rise and the price of ore fall. We maintain on the contrary that there would be no such change in the price of agricultural products when the rent of land is confiscated or remitted by the State. The owners of the better mines can therefore only procure incomes by an artificial lowering of the gross product, and even in this case there would be an essential difference between "royalty" and "rent". The former is a monopoly rent, the latter a pure scarcity rent. When we take an imminent exhaustion of the mines into account, the difference is naturally accentuated, but it tends to disappear to the extent that relatively increased costs are involved by increasing the annual product of either mines or agriculture in general. In any case, it is to Professor Cassel's credit that he has gone into the details of a subject which has been only too cursorily dealt with in economic theory.

We now come to the special chapter on wages. Here also Professor Cassel claims to have constructed an independent theory, but I cannot discover wherein its originality lies. The division of wage-theories into "pessimistic" and "optimistic" is certainly not new. All wage-theories without exception—or with the exception of those which are merely confused—are necessarily pessimistic, if we start from an unrestricted tendency for the population to increase, otherwise *no* wage-theory would be pessimistic if pursued to a logical conclusion. Even the Iron Law of Wages is converted into "a standard of life" theory or a "Golden Law of Wages" [Gide]—a change which was by no means alien to Ricardo's train of thought.

Why the Wage-Fund theory should be singled out from all others for description as pessimistic is difficult to understand. If we assume the "dividend" or fund to be sufficiently large and the divisor (the number of workers) sufficiently small, the quotient—the *per capita* wages—can, at least at first glance, attain any magnitude whatsoever. I willingly concede that the Wage-Fund theory, in its classical form, where the fund was mainly regarded as of a *single* year's duration, was completely erroneous.

As we have already remarked, it led even Ricardo to draw patently false conclusions, and in this form it unfortunately became a weapon in the struggle against the shorter working day. In the extended form it assumed at the hands of Bohm-Bawerk, it can easily be defended from a purely theoretical point of view, but it has, as we have said, a severe disadvantage from a practical point of view. Eminently durable capital-goods cannot be fitted into such a fund without involving the consideration of altogether unmanageable periods of time. For shorter periods, however, these durable capital-goods take on the same economic status as land; they are "Rentengüter", and their share (or their owners' share) in the product is determined, at least in the stationary state, quite simply according to the principle of marginal utility or of marginal productivity. A fusion of the Wage-Fund and the marginal productivity theories, however, would then be impossible. Or else one can [1] throw overboard the whole concept of the Wage-Fund, or the subsistence-fund, and adopt instead Böhm-Bawerk's brilliant suggestion. The idea of considering *capitalistic* production as primary and capital itself as secondary was put forward in the second book of the *Positive Theory*, but of course Böhm-Bawerk himself did not carry it to completion. By this means everything is dominated by the marginal principle applied to land, labour, and *time* (the period of waiting or capital-investment) as the factors of production.

Remarkably enough, in this chapter, Professor Cassel also rejects marginal productivity as a ground for the determination of wages; he asserts *inter alia* that it provides no "elucidation of the dependence of wages on the workers' efforts and ability". This we fail to understand. In the individual case wages are of course proportional to the worker's efficiency—in all cases in the bargaining system. If the efficiency of labour increases all along the line this theory drives us to the conclusion that wages fall relatively (or possibly absolutely), but this sad result cannot be ascribed to the fault of the theory! Cassel adds that he is afraid that efficiency and marginal productivity will be confused, and in support of this view he quotes a passage from Professor Seligman which is not very remarkable in its penetration, and however prominent a thinker Professor Seligman may in many

[1] For my own part, I have already made the attempt.

respects be, we cannot hold him to be a typical representative of modern economics in any way.

What then are Professor Cassel's own views on the theory of wages? It is not so easy to say. He begins by going back to the principle of supply and demand, which always provides a starting-point at any rate, if nothing else. But in its elaboration he expresses himself, contrary to his wont, in quite loose phraseology, as though he were afraid of certain unavoidable conclusions. The policy of "the open shop" described by the Webbs he praises discreetly, without, however, completely binding himself to it. "The Webbs' doctrine has the great merit that it has changed the study of the supply of labour from a pure computation in terms of arithmetical magnitudes to an examination of the underlying economic and social processes which determine the supply of labour"—which sounds very much like a verbal flourish. And as verbal flourish number two I shall cite the following (p. 333): "The most advantageous position for labour on the whole is attained if the supply of labour is as nearly as possible adapted to the demand, i.e. if the price of different kinds of labour is merely the expression of their inevitable natural scarcity." Can this theory be applied without closer examination to those earning the lowest wages? More than once Cassel talks of the necessity of an "amelioration" of these unfortunate wage conditions or of the "market", and still more often he warns us against any "misdirected" attempt at such an amelioration, but he never tells us how the desired amelioration should be introduced.[1]

[1] Undoubtedly, the cardinal mistake in his approach is that he here, as in his treatment of parasitic occupations and the like, always proceeds from the hidden assumption that wages as such must necessarily be sufficient to cover the labourers' subsistence. Neither theoretically nor often practically is this hypothesis justifiable. Since the theories of a science must be generally valid, it is perhaps permissible to conceive of a "strong" case. We shall assume that in equilibrium, wages for most workers are considerably below the subsistence-level, but that, at the same time, the total product is so great that, with a different distribution, it would abundantly cover the needs of all. From Professor Cassel's point of view the question of an amelioration of the conditions of labour in this case constitutes, as far as I can see, *an absolutely insoluble problem*. For wages as such cannot be raised, at least not safely, unless the population diminishes to such an extent that the marginal productivity of labour is considerably raised. But such a diminution is a slow and, in most cases, painful process, and, moreover, in this particular case it would, on our assumption, be completely unnecessary and therefore to be repudiated. The only way out is to grant subsidies, of the consequences of which Cassel is so fearful. If necessary, they must of course take on such a form that they do not imply any humiliation for anybody.

When he discusses (p. 334) the question "of a limitation of the total supply of labour", he expresses himself so vaguely that we cannot tell whether he is considering a shortening of hours or even—and this more or less follows from the context—a reduction in the number of workers, which naturally makes an immense difference. On p. 349 he says that too small a relative birth-rate in the higher classes and the upper sections of the working-classes may "perhaps lead to a relatively too great scarcity of qualified workers, especially in the key positions". This in its turn would involve a particularly disadvantageous development of the market position for the lower classes, and would press down their wages considerably. No doubt such a chain of events is conceivable, but in any case it would be hard to point out a historical example of this kind. A general fall in the birth-rate is a phenomenon confined to comparatively modern times.

As an explanation of the high wages of North American workers we are offered (p. 339)—as far as one can gather from a phraseology which is repeatedly loose—the theory that the European demand for agricultural products prevented their internal price in America from falling as much as they would otherwise have done. If that is his real opinion, it is wrong. This demand—as he himself admits immediately afterwards—was responsible for the emergence of rent and to this extent for a fall in wages (in terms of corn) in America. Whether this disadvantage has been counterbalanced by the cheapness of European industrial goods is more than doubtful.

The chapter closes with several reflections on "wages in the socialist state", which, like his previous remarks on the same subject, suffer from being *excessively* critical to the point of ineffectualness. He asserts *inter alia* that much, perhaps most, of the incomes of the "leisured classes" to-day would *not*, after redistribution, accrue to the benefit of consumption in the socialist state, because "probably" it "will have to be claimed for the requisite accumulation of capital". Which presupposes a large continuous increase in population inconceivable in the long run, whether in the socialist state or in present-day society.

On the whole, in spite of much that is interesting in detail, Professor Cassel's inquiries into the theory of wages are too much devoid of rigour and—so to speak—backbone, to provide

the basis for fruitful social investigations, although, appealing to a well-known monograph, he very emphatically states that such has been the case.

III

The third book is devoted to the *nature of money* and to some extent to actual monetary systems. Even here the author's theories are not too rigorous or consecutive. As far as one can see he is still completely dependent on the Quantity Theory, as in Section 43 on " Free Standards ". The only concession he makes to the " bullion " theory is to be found in the statement that the hope of a future conversion of paper money into bullion can to some extent affect its value. Indeed, this is not incompatible with the quantity theory ; some-bank notes are hoarded for future conversion and, for the time being, take no part in circulation. Besides there are cases on record where paper money has attained a value even higher than that of the bullion it originally represented.

But in the chapter on " Bank Money " we suddenly stumble on the following passage, which might almost have been culled from one of Jacob Riesser's pre-War works. No one should doubt, at this time of the day, that these works exercised a baleful influence on Germany's monetary system during the War. I quote the passage in full :—

> " There is moreover a possibility of a continuous multiplication of the means of payment only as long as confidence in the bank's capacity to cash its notes and deposit is undisturbed. But as we know from experience, this confidence cannot be maintained, unless the bank keeps a reserve which is in a sufficient proportion to the obligations daily falling due and particularly to its notes. In this respect an international desire for an appropriate reserve has arisen, a desire which has not fixed upon a constant numerical proportion without seriously upsetting confidence at home and abroad in the maintenance of the foreign exchange rate. We therefore find that a minimum reserve which is never actually used is regularly kept against bank money (!). This minimum reserve will be left untouched even in cases of the direst necessity, as in wartime. What is more, it is just in such cases, as the most recent experience has shown, that an earnest attempt is made to

> protect the reserve and even to strengthen it by diverse means by abolishing the obligation to redeem the notes in cash." [1]

Well might we ask—*what* is Professor Cassel's *true* opinion? Is it the "scarcity of bank money", ultimately ensuing from the interest policy of the banks, which determines the value of money? Or is it confidence in the conversion of bank-notes and deposits in gold—a confidence so touchy that it must always, so to speak, have its object, gold, visible before it, but at the same time so impregnable that it cannot be perturbed when it is patently deceived by the banks' indefinite postponement of conversion? Of course one can only accept *one* of these views to the exclusion of the other. There is no doubt, at any rate for me, as to which has most to be said for it. In the nature of the case, Professor Cassel should tend to hold the former—the experience of the War, as he himself admits, must influence him in this direction.[2]

When he is unravelling the influence of the rate of interest on commodity prices, we meet the same regrettable half-heartedness and uncertainty. Judging by many of his statements, he is clearly aware that the essential factor must be the *relative* height of the rate of interest in relation to the return the borrower expects to get from the loan, i.e. to the real rate of interest. None the less he says [3] that "a real rate of interest in any other sense than the market rate does not exist". Very strange! The rate of interest on the so-called open market, i.e. the discount for *first-class paper*, which in fact constitutes a kind of intermediary between prime bills of exchange and mere cash, stands indeed in a looser relation to the average yield on capital than does the bank-rate. Again, as far as this yield, i.e. the real rate of interest, is concerned, it is actually

[1] [This passage, which occurs on p. 366 of the first German edition, has been substantially modified in the second English edition.]

[2] It is a matter for separate consideration that, at the critical moment he goes back on his convictions, at least apparently; for instance, at the beginning of last spring (1919) he unexpectedly supported the lowering of the Swedish bank-rate. Since then, judging by newspaper articles, he has again held that the bank rate (though only the *loan* rate) should be kept up.

[3] [See first German edition, p. 382; first English edition, p. 418. In the second English edition (p. 439) the passage has been modified and reads as follows: ".... 'a real rate', in a sense other than that of the market rate, is a very unreliable indicator for the banks' interest policy, since the market is, as already shown, directly and powerfully influenced by the banks' interest rate".]

not observed on the Stock Exchange apart, perhaps, from its indirect effect on the price of shares. Of course it cannot be strictly determined numerically, but it does not on that account cease to exist and exert its full influence on economic phenomena. *Heat* would still exist even if there were no thermometers, and so would electric currents even if we did not know how to measure them by means of a galvanometer.

My own statement that a persistent, abnormally high or low money rate must be *cumulative* in its effects on the level of commodity prices Cassel calls "a paradox which is obviously only possible if we overlook the reactions on the capital market of an unjustifiable lowering of the rate of interest".[1]

But how can the fact that a cause operates in the same direction as long as it persists be called a paradox? Clearly my theory coincides with Ricardo's theory of the effects of a continued flow of gold into the banks. On the other hand, it must be admitted that some forces come into play as a reaction. Professor Cassel's own discussion of these forces (on the preceding page) does not appear to be particularly lucid. There is no doubt that when a sudden violent rise in prices has set in, people with fixed incomes or with incomes which have not increased sufficiently are compelled to curtail consumption. This process is equivalent to a real accumulation of capital, and to that extent should lower the real rate. In normal conditions, however, such a reaction should only be of secondary importance. Otherwise, as Ricardo says, the banks are "potent engines indeed", they will be able to determine arbitrarily the height of the rate of interest without any risk other than that attaching to a *single* rise or fall in the level of commodity prices. Professor Cassel was formerly wont to be the first to maintain that the banks do *not* have this power.

What appears to me to be a still more serious defect is Professor Cassel's tendency to expound the theory of money in such a way as to make it serviceable for some of the practical ends in which he is interested. He holds *inter alia* that the present high margin of profits of private banks is especially beneficial and must be left undisturbed. He therefore attempts to render credible the theory that the rate of interest does not normally have any effects worthy of mention on the volume of

[1] [See first English edition, p. 479, n.]

saving. Naturally, he cannot substantiate this view. Accordingly, he explains in the Introduction to his chapter on Bank Money (p. 412) that "in an inquiry into the nature of money, it is *clear* that we must abstract from all deposits representing investments of capital, and confine ourselves only to cash entrusted to the bank on current account". It is no accident that this is a preliminary to his thorough-going refusal to attach any importance to the deposit-rate in the determination of the value of money. In the important Section 47 on the "Cover on Bank Money and its Reflux", Cassel assumes for the sake of simplicity that "the capital left in the bank for longer periods or permanently *remains constant*",[1] and this provisional assumption is never later discussed. And yet he himself must recognize (p. 438) that a rise in the discount-rate will only have a sufficiently powerful effect on the "provision of money", if "the sum of money lent is *large* in relation to the bank money" (bank-notes or current accounts), in other words that the money consisting of interest-bearing deposits (just as much as the banks' own capital) constitutes a significantly large part of total liabilities. The importance of deposit rates for a rapid regulation of the issue of bank-notes (or bank money in general) clearly follows; at the same time it is the basis of the modern demand that the central banks should also be allowed to receive deposits in return for the payment of interest—as the Bank of England actually did during the War, at least for the private banks.

Although he elsewhere keeps only to the closed economy "on principle" (!) Professor Cassel also deals here with international payments and the foreign exchanges. Characteristically enough, he begins with "free independent standards". This is indeed a very difficult and complicated question; in any case his theories do not seem to me to be well developed. He asserts that a high exchange rate in one country—e.g. Germany—acts as a stimulus to borrowing from abroad on short term and to the export of securities, because in both cases "there is a profit to be earned on the high exchange" (p. 512). That may well be, but is this result certain? The man who procures a deposit abroad will one day have to pay for the loan. If the exchanges continue at the same rate, he has gained nothing, and has only had to pay

[1] The italics are mine.

what was probably an exorbitant rate of interest in the interim. Similarly, the price of foreign—e.g. Swedish—securities must rise in Germany, while German securities fall in Sweden if the mark depreciates relatively to the krone; how then can it pay to export them from Germany to Sweden?

The explanation must be as follows. The man who buys securities in Germany in order to sell them in Sweden is not—as Cassel says—a speculator in the proper sense of the word, but is merely conducting an arbitrage operation, the gain from which, if any, he can calculate directly. Actual speculators are the final buyers or sellers of these securities. The German owner of *Swedish* securities sells in the hope that he will be able to repurchase at a profit when the *mark* exchange rises again. He can therefore sell them at a somewhat lower price than that corresponding to the rise in the exchange-rate, or otherwise it would not pay him to do so. On the same grounds a Swedish buyer expecting a future rise in the exchange on Germany offers a little more for *German* securities than would correspond to the present rate of exchange, and so on. The same holds for Swedish imports from Germany. If the payment is stipulated in *Swedish* money and the exchange-rate on Sweden rises, the German buyer obtains a postponement of his payment, if necessary, against the payment of a higher rate of interest, because he hopes for a future fall in the exchange-rate. If the payment is made in German currency, the Swedish creditor, for this reason and no other, allows his claim on Germany to remain outstanding instead of pocketing it at the current low rate of exchange on Germany.

One of two conditions is necessary, if a country having no interest-claims abroad is to be able to import more than it exports. *Either* a country offers its creditors an attractively high rate of interest by raising its discount rate, *or* its foreign exchange-rate has fallen sufficiently to attract speculation on its prospective rise.[1]

Of course the level of commodity prices and the exchange-rate always tend to move in the same direction in two countries trading with each other, at least as long as the exchange of goods can proceed freely, but this movement may just as well

[1] Cf. my article in *Ekonomisk Tidskrift* on "The Riddle of the Exchanges". To judge by an article in *Sv. Export*, No. 17, 1913, Cassel seems to have now accepted this view.

start from the side of the exchange-rate as from that of the price level. With a higher exchange-rate there is a rise in the price of exports as well as of imports, and if the banks do not appropriately react with a higher discount-rate, but let their bank-notes and credit flow out, the rise in prices is rapidly diffused to all commodities. Thus the *credit policy* of the banks—and above all of the central banks—is the dominating factor.

With some astonishment we find Professor Cassel repeating in this book without any further critical examination his celebrated speculations—more fantastic than trustworthy—on the relation between the *quantity of gold* and the level of commodity prices throughout the nineteenth century. No one denies that some such connection must exist, but in order that it should be demonstrable in detail, all the factors at work must naturally be considered, and this he has completely neglected to do. We have heard tell of an American humorist who once, probably in the great days of the Temperance movement, gave an evening lecture with the queer title of "Milk", which began with a promise not to mention the word milk again. He succeeded without any difficulty. Professor Cassel has solved the much more difficult problem of giving us a numerical analysis of the connection between gold-production and the price-level from 1800 onwards without as much as mentioning *silver* on a single occasion. I am by no means the first to draw attention to this omission, it has been done several times before now—in Sweden more than ten years ago by *Brock*. But it is still entirely ignored; he continues to "conjure" with his gold-curves. Of what use are such ingenious constructions? The more they succeed, the more suspicious become the very methods which, when rightly used, should inevitably lead to a demonstration of the gap in the argument, at least for what covers the nineteenth century, when the world's main metallic currency was silver. If he had extended his curve to cover the eighteenth century, then, as far as I can see, their disagreement with the facts—not to say their absurdity—would have immediately become apparent.

IV

My review has become exceptionally long, or otherwise I should willingly write at rather greater length on the fourth

book on *Trade Cycles* in order to compensate for my previously largely negative criticism. As I have already said, it is in my opinion incomparably the best part of his work. Professor Cassel's great gifts for concrete description based on facts and figures here show to advantage. Besides, the somewhat irritating Olympian omniscience of the rest of the book has entirely disappeared; he never claims to have propounded some new theory of crises, but is content to suffer the older explanations of crises calmly and objectively and to accept the most plausible of them. At the same time he illuminates all the phenomena associated with the trade cycle with interesting statistical tables and diagrams.

Considering the extraordinary difficulty of the subject (and my own far from adequate comprehension of it), I certainly cannot vouch for the correctness of all his conclusions, but on the whole they appear preponderatingly sound and just.

Some objections can certainly be advanced; the description of the period of depression, which is the weak point of most theories, hardly emerges in a clearer light in Cassel. From his older essay (*Ekonomisk Tidskrift*, 1904), on which this is otherwise a great advance, he has taken the idea that capital accumulation even in a depression mainly takes the form of *fixed* capital. He tries to show by means of the statistics of railroad construction (*inter alia*) that the increase in fixed capital-goods does not stagnate even in the downward phase of the trade cycle; so that society is *better* provided with fixed capital at the end of the depression than at the beginning. He forgets that all this must be judged relatively. The provision of fixed capital must always keep pace with the growing needs of the population. If its growth is actually accelerated in the boom and retarded in a depression, the latter from this point of view cannot serve as " a preliminary to the subsequent upward phase "—other than negatively by creating a relative vacuum which must be filled. Logically speaking, what Professor Cassel says must hold for *circulating* capital—stocks of goods. What in fact happens cannot, unfortunately, be ascertained owing to the lack of statistical material. Professor Cassel does not wholly deny this possibility, but he is generally tempted to keep it in the background.

The agricultural situation is particularly relevant at this point. If, as he also maintains, agriculture relinquishes some

labour to industry during a boom, it must on the other hand be possible to do some preparatory work in the subsequent depression, which will serve to provide food for the population in the next industrial boom. For during the depression a number of industrial labourers return to agriculture, which can also absorb part of the increase in the labouring population. Professor Cassel thinks—in my opinion wrongly—that agriculture is independent of trade cycles proper, thus differing from Dietzel and Petander, who perhaps go to the other extreme.

Here and there we still find inconsistent and loosely reasoned judgments. On p. 609 it is left an open question how far real wages (as distinct from money wages) rise or fall in a boom. But only a few pages later, without giving any really decisive reason, he is sure that they rise, at least if the services of those recently taken into employment are considered. Brock has maintained the opposite thesis, and the statistics he adduces would have deserved some scrutiny. The scepticism with which Professor Cassel here speaks of "statistics" does not well accord with his own diligent application of statistics as a method of proof.

All these are mere details. One reads this painstaking discussion with interest and advantage. And what is more with enjoyment. The very *tone* is different. Curiously and characteristically enough it is just at this point, where he has really so much that is new and valuable to offer, that an unassertive, quiet and scientific approach redeems the unpleasant aggressiveness of the preceding part of his work.

With a certain feeling of constraint we ask: why could it not all have been written in this spirit? Why has not Professor Cassel throughout contented himself with the rôle of continuer instead of that of a pretended innovator, for which neither his nor other men's powers suffice when it is a matter of so large a field as the whole of economics? Why has he not resolutely freed himself from the immature vagaries of his earlier writings —which he cannot seriously maintain—and, with the acuter view which he must have acquired, given us a simple objective survey of the present position of economic science? That the work even in its present form has many merits, I do not deny; but—and this is the highest compliment I can pay to his talent —he could have enormously improved his book if he had cared more for the subject than for his own self-esteem.

Macaulay mentions as a characteristic of James II that when a member of his court dared to contradict him and humbly warn him against the consequences of his explicit avowals, he used to repeat what he had said in identically the same way and then believed that *he had sufficiently refuted all objections*. Such a method may be all very well for kings in difficulties, although, as the example shows, it has its dangers even for them. For laymen who have not yet become the acknowledged monarchs of their subject it is decidedly not to be recommended. Professor Cassel must learn—unless it is indeed too late—to use his critical faculties on himself as well as on others, to give as well as to take—otherwise his life-work will not survive criticism.

2. Real Capital and Interest[1]

(a) Dr. Gustaf Åkerman's *Realkapital und Kapitalzins*

It has been a great pleasure for me to re-read in print a book in which I had already taken a keen interest in its manuscript form, especially as what remained rather obscure in the perusal of the manuscript now stands in a clearer light. This holds for the defects of the book as well as for its merits, but on the whole I believe that it is with a good conscience that I can give the author credit for having fulfilled his anything but easy task with rare energy, consistency, and deep penetration. The object of the book is to investigate the *co-operation* of social *durable* capital with free uninvested labour in production. This problem is clearly of great practical significance—no doubt much more so than the problems dealt with by Jevons and Böhm-Bawerk. They concentrated on the capitalistic process of production, in which labour resources (and probably land resources) ripened into immediate consumption goods, or what the author calls "variable capital". But his problem is so complex that the vast majority of economists, including the reviewer, have almost entirely passed it by as being much too difficult to be susceptible to analysis. In spite of the fact that Walras did touch on certain aspects of the question, our author has not much to draw from him, for Walras essentially regards capital-goods as *indestructible* or as constituted in such a way that they can be kept intact with a given amount of maintenance (or insurance) costs. This procedure naturally simplifies the problem, but on the other hand it neglects many of its most important aspects. For Walras does not take into account the fact that a longer or shorter duration for the projected capital-good may be more profitable, which is the crux of the matter for Åkerman. But as the author himself admits, the real starting-point, if nothing else, of his own

[1] This article first appeared in the *Ekonomisk Tidskrift*, 1923, Nos. 5–6, pp. 145–180.

treatment was discovered in the long-forgotten work of the Scottish-American, John Rae.[1]

From the very beginning the author has therefore to go almost entirely his own way. Our esteem for his work rises still more, when we remember that his problem is not elementary from a mathematical point of view, and that in order to master it he only had access to the ordinary high-school knowledge of mathematics. Nevertheless, it is for this very reason that he has been compelled to give his analysis such a form that the book can be read by anybody without any but the most elementary knowledge. But with one intractable condition—the unremitting attention of the reader is demanded. If we miss our way only once in the finely spun web of reasoning, everything we read later is bound to be in vain, and it only remains for us to begin again *de novo.* Which is naturally a shortcoming. The author ought to have relieved the reader's tension with a fuller and more pointed method of exposition, and would have been in a position to do so if he had more time at his disposal. We may mention as an example of the difficulties confronting the reader the magnitude representing the value of a unit-use of some capital-good, e.g. a machine. This magnitude b, together with l (wages) keeps on appearing in the whole of the latter part of the book, and is obtained in the following manner. We conceive of the productive services of this machine in a unit of time, e.g. a year, as being divided into as many equal parts as units of labour required to produce, not this machine, but an equally good and useful one of a single year's duration. This concept is indeed extremely abstract in character. Certainly it is developed with unfailing consistency

[1] *Statement of some New Principles of Political Economy.* Unfortunately, I only know his work through Böhm-Bawerk's quite detailed and largely eulogistic description of it (in the *Geschichte und Kritik der Kapitalzinstheorie*). Böhm-Bawerk's criticism is in effect identical with his celebrated objection against all "productivity theorists", who in his opinion constantly confuse physical and value productivity. As I have already attempted to show in *Über Wert, Kapital und Rente,* at the very most this confusion is nothing more than a methodological error. In the first approach to the solution of the problem of production and distribution, it is permissible, if not advisable, to consider the prices of commodities as constant (which in the last analysis is essentially what Böhm-Bawerk himself does); in the same way, we regard production as constant in the first stage of the solution of the problem of pricing. It is only at a later stage that we should combine both these approximations in order to obtain the final solution of the problem. Once this is grasped, then, as far as I can see, Böhm-Bawerk's objection loses its force.

and does lead to correct results, but only by inflicting on the poor reader the torment of keeping this "*b*", which is neither fish nor fowl, in mind. With a slight revision of the formula the book could have been made more intelligible in this respect.

But there is another more serious difficulty, which I fear is for the most part insuperable in the discussion of the economic phenomenon of durable capital. For we cannot, at least without further analysis, apply the celebrated principle that capital is or corresponds to a certain amount of "'previously-done' labour", i.e. the accumulated saved-up, or invested, resources of labour (or land). A machine fresh from the factory undoubtedly represents a certain amount of labour; if this were the machine's only cost of production, and if the usefulness of the machine is taken as known, we can theoretically calculate at what rate of interest these costs will yield interest for the lifetime of the machine at the same time as they are being repaid. But if the machine has been in use for over a year or for several years, there remains only one part of the "annual use", which, for the sake of simplicity, is assumed by the author to be constant in size or technical value. Clearly it is then quite impossible to decide how much of the previously invested labour resources still remain "stored-up" in the capital-good. In fact the question has no meaning to which any proper sense can be attached. For the annual uses successively following one another constitute a kind of joint-supply (to adopt Marshall's terminology) and fundamentally it is just as absurd to ask how much labour is invested in either one or the other annual use as to try to find out what part of a pasture goes into wool and what part into mutton. It is only at the margin of production that these quantities can be differentiated and have a concrete significance assigned to them.

It so happens that from the very beginning the author is convinced that the problem is capable of solution in one way or another. The whole of his intricate terminology bears witness to this conviction. In addition to the concepts of investment-capital and "real value capital", both of which have a perfectly real meaning, Åkerman employs those of amortization capital (in German, *Tilgungskapital*) so-called, transitory capital, maintenance capital, concrete real capital, etc. "Investment-capital," i.e. the labour costs of manufacturing a machine

is first divided into parts—into the so-called *i*-series. The first term of this series corresponds to the amount of labour required to make the machine last only a year, the next term is the additional cost of making it last yet another year, and so on. This idea borrowed from Rae, even if abstract, is quite scientific; but it only has practical significance at the margin of production where it pays to exchange a machine lasting ten years for one just as good in other respects but lasting an extra year. But, in addition, the author believes that the capital bound up in a machine is after a time disinvested or amortized (and in a stationary state reinvested) in the following order. In the first year we regard the machine as repaying part of the investment-capital and the interest accumulated on that part for a *single* year. Next year it repays another, rather smaller, part of the initial investment costs, but with a total accruing interest for *two* years, and so on, until the machine becomes finally worthless, but at the same time is finally amortized. These amortization-quotas, or rather the amounts of labour they are taken to represent, form the *u*-series, which of course is quite different from the *i*-series, although their sums are equal. (Similarly, if we use the rate of interest for a moment of time in our calculations, in equilibrium the last terms of both will be equal at the margin of production.) But in the first part of the book the *u*-series is often inextricably associated with the terms of the *i*-series in a most confusing manner. The author holds that this *u*-series, also called the "abstract amortization system" has a really scientific significance, or is at least of great interest for purposes of exposition. I shall not bother to deny the latter, but essentially it is only one of an infinite number of other conceivable amortization systems. Nor has it the advantage of leaving the capital situation of the owner of the machine intact, for if the amount amortized is reinvested on the basis of another amortization system, his supply of capital will clearly increase at the beginning only to diminish later. Consequently, it is only at the end of the machine's existence that taken together they become equal to the amount of investment-capital. (It is assumed that the interest received is consumed.)

If the owner of the machine wishes to maintain his capital intact, he has instead to choose either the "natural" or the

"theoretical" amortization system. As far as I can see these two systems really coincide. They can best be described in the following way. Each year we write off or reinvest the *difference* between the outstanding value of the capital-good at one point of time, and its value at the succeeding point, e.g. at the beginning and end of each year; this procedure may indeed be called perfectly "natural", but the concept does not therefore obtain any "concrete" content—neither more nor less than that of the "theoretical" system. (A fourth system, the so-called "practical" system, in which each year we write off an equal fraction of the original value of the capital is also applied now and then, but only because of its simplicity. It has no other *raison d'être.*)

Now if production is "staggered" (*durchgestaffelt*—to use Böhm-Bawerk's term), machines of all sorts of durations manufactured in different years are employed side by side in the same firm or group of firms, and the oldest machine (or machines) is annually exchanged for a new one. In this case it is a matter of indifference which amortization system we choose, provided that we apply it consistently.[1] For in all so much is always written off from the estimated value of existing machines as is required (under stationary conditions) to repurchase the new machines and consequently to maintain all the machinery at a constant magnitude and composition. On the other hand it is not a matter of indifference for the *book* value of the existing capital, for if we write off more at the beginning of each machine's "life-time" and less later, the total book value of all machines clearly becomes *less* than would be the case if we chose the reverse method. Here also the "natural" system is to be preferred.

The book value of all the existing machinery becomes exactly such that the yearly interest in them, computed at the same rate as that actually yielded by the amortized or newly invested capital, corresponds to their total yield per annum. In perfect equilibrium this rate ought to be identical with the prevailing rate of interest. This principle is demonstrated by the author (on p. 151), but at bottom it is a mere truism, for the outstanding capital value of the machines which have been

[1] e.g. an amortization system for the i-series could here have been chosen, which would naturally have been impossible in the case of a solitary machine.

partly used up has in fact just been computed by applying this very rate of interest.

It might appear strange that the same physical capital can just as well be taken to have a greater as a smaller amount of labour resources invested in it. But if we remember that "static" capital always has a dynamic pre-history, the paradox is resolved. The more the owner reinvests, the less the capital that has to be supplied from outside before the collection of machines of different durations becomes complete, so that a perfectly stationary state has been reached. The smaller the portion of the present value of fixed capital he can, if he wants, regard as invested wages—and in this sense as "capital"—the greater the part he may regard as interest which has been accumulated but not yet consumed. If the firm is sold, he will receive this interest probably in the form of profits over and above the book value of the stocks. (But naturally we ought not to think that this form does in fact yield a rate of interest corresponding to the relation of the net gain per annum to the book-value of the capital. When, after a time, the owner buys new machines to replace those which have been worn out, and thus reinvests some of the successively uninvested capital, in equilibrium the reinvestments will only yield the current interest.)

The author's adherence to the idea of "concrete" capital, consisting of invested labour, leads him to hasty conclusions which I shall discuss later. In my opinion, he would have saved himself much unnecessary trouble if the u-series and the whole discussion, however interesting in itself, of the different amortization systems had been completely omitted. For they have no special function to perform in the actual solution of the main problem. Their irrelevance is due to the fact that the annual costs of maintenance of real capital are always amortized and reinvested in their totality, whichever the amortization system adopted for particular capital-goods. This quantity is obviously proportional to the amount of labour invested per annum, and also determines the amount of free uninvested labour.

We have now reached the stage where, with only a few simplifying assumptions, we can ascertain and describe numerically the connection between all the essential constituents

of the economic phenomenon of durable capital, viz. the yearly product, wages, and interest for each given amount of capital per head of the labouring population. The author considers in turn different economic situations where capital receiving its maximum remuneration only suffices for an investment lasting one year, two years, or three years, or for an investment lasting for an intermediate period. (Clearly the different amortization systems, and consequently the book-value of fixed capital, will not play any decisive rôle if this method of approach is employed.)

The author makes two basic assumptions about the forms of the productivity functions. The first is concerned with the i-series, i.e. the amount of labour which has to be invested in order to produce a capital-good of a given size and utility and make it last for one, two, or three years, and the second with the form of the productivity function, given the (most advantageous) co-operation of a certain amount of "free" labour with a certain amount of capital. Both these functions must be regarded as *technically given.* To the latter Åkerman gives a definite mathematical form, but the former, later called $f(n)$ is only empirically determined by the successive differences in the i-series.

If the relation between l wages and b the value of the unit-use of a machine is taken as given, it can be shown that a particular "life-time" for each newly-manufactured machine produces the maximum interest on the capital so invested. The author solves this by no means elementary problem of maximization with elementary tools, and in a particularly ingenious and lucid manner (pp. 110–14). From a purely expository point of view this is one of the best passages in the book. He then introduces a situation in which a number of different machines co-exist, although they were all manufactured in different years. We thus obtain a static state in which there is a "staggered" and constant production of machines and consumption-goods. For its actual renewal or "maintenance" this complete equipment of machinery demands the exact cost incurred in making a single new machine. Thus to each labourer who is continuously occupied in manufacturing machines, there corresponds a definite amount of machines now being used (and of course an equal amount of "machine-uses" available per

diem or per annum). Similarly we can calculate the present discounted value of the outstanding uses of all the remaining machines, and consequently their present capital-value "Realwertkapital". This we do simply by applying the most advantageous life-time, which has already been provisionally determined, and the yield of every machine which has recently been manufactured. (Adopting a different amortization system the author also works out the results for two other concepts of capital. But I pass this section by.) Given the most profitable life-time for machines, the number of labourers employed in the production of machines and the value of the machine-capital are mutually determined. As soon as we know the former we also know the amount of free labour resources, for these two are together equal to the whole of the available supply of labour, or the annual labour resources of the society.

Now the free labour resources are combined with the unit-uses of the machines available in each year. At this point the productivity function is assumed to be technically given. In perfect competition it must be homogeneous and linear, i.e. such that a uniform increase in all factors of production produces the same percentage increase in the product, in other words, such that, after a certain optimum size has been reached for the individual firms, production on either a large or a small scale is relatively just as profitable. This function gives the hypothetical size of the national dividend per annum, and by its partial derivation we obtain—also hypothetically—l the level of wages and b the value of the unit-use of the machine.[1]

Now in equilibrium these quantities, l and b, must clearly coincide with their initial hypothetical values. In other words, we have to determine six or seven unknowns, i.e. the duration of the capital-good, the rate of interest, and the distribution of the existing labour force between machine-labourers and free labourers, in addition to the three quantities already mentioned. In mathematical parlance, these six or seven unknowns are

[1] This is the only point at which Åkerman makes use of higher mathematics—following my "Lectures" more or less closely. It should not, however, have given rise to some of the insuperable difficulties which crop up in the treatment of this part of the problem; even in its elementary form it would have been better had he proceeded much as I did in my perfunctory attempt to solve the problem in the passage dealt with.

determined by the same number of simultaneous equations, which are transcendental to boot. The author solves this formidable problem empirically and approximately by the construction of arithmetical tables of the same kind as those used by Böhm-Bawerk, though they are naturally more complicated and more awkward to handle.

The book's most brilliant and most significant contribution to economics is not only to have put this problem (which I have merely outlined with the greatest brevity) in all its detail, but also to have solved it empirically. It can be argued against the author's use of figures that it is often hazardous to decide to what extent the results gained are of general validity or are dependent on the actual selection of the arithmetical data.

An increase in capital must bring about an extension of the life-time of a capital-good, so that capital grows not only in "breadth" but also in "height". Otherwise the marginal productivity of labour would necessarily rise in comparison with that of the use of a machine. This consideration, as I shall show later, always makes it advantageous to increase the durability of the machine, and this is further corroborated by the author's tables, though the result is somewhat obscured by his assumption that the extension of the life-time of a machine occurs not continuously but in one-year stages.

On the other hand, how far capital, when it grows, must also grow in breadth remains less clear. The author's Table III (p. 144) shows that the amount of labour $u = i$, which is needed for the maintenance (renewal) of durable capital, increases continually, though not at a particularly violent rate, when capital itself, and with it the life-time of capital-goods, is increasing. We ask ourselves whether the solitary exception here is perhaps merely apparent and whether therefore we are even here dealing with a general rule. This appears to be the author's view on p. 28, where he says that when there is an increase in capital "a greater amount of labour than before must each year be employed in investments which partake of the nature of the replacement of durable capital-goods, and thus a smaller amount than before co-operates with the existing capital-goods". But this passage might only be a lapse, for an increase in capital need not have the results here indicated by Åkerman. We can, as I shall show later, construct

a productivity function proper and a function for extending the life-time of durable capital-goods (the author's $f(n)$ or i-series) such that, given no changes in population, both the labour invested in machines and free labour remain *constant* when capital increases. In this case capital grows exclusively in height and not at all in breadth. With the appropriate assumptions it is possible to make the former *diminish*—though not of course—indefinitely—with a growth of capital.

But Table IV (p. 149) shows a continual rise in the *value* of the annual *product* when capital increases and the rate of interest is still positive. Is this rule general ? Clearly it is not. For as long as the process of prolonging the life-time of the machine always results in relatively smaller costs of maintenance, it might appear to be in the interest of the capitalists to undertake such a prolongation, even if the value of the gross product is thereby diminished. If the capitalists combined, it would certainly be possible for them to prolong the life-time of the capital-good to their own advantage, even if it involved a fall in the annual product, and would therefore be anti-social in its nature.[1] Can this also occur even in free competition ? No.

Actually it was this point which more than anything else attracted my attention when reading the manuscript, and it is of such intrinsic interest that Åkerman might well have discussed it in greater detail. In the manuscript version the author had in place of Table IV a table from which it apparently followed that the product per annum does *not* continually grow with a rise in the amount of capital, but ultimately begins to fall, *even before the rate of interest has fallen to zero.* Åkerman and I had a prolonged discussion on this point, and we finally arrived at the conclusion that this result depended on the fact that the productivity function, which after all was quite empirically chosen by him, did not satisfy the preconditions for free competition—in other words it was not homogeneous and linear. The author later reconstructed this table and thus opened the way to a consideration of the function $P = k\sqrt{c\,r}$ mentioned on p. 137, which is applicable to free competition.[2] But it has the

[1] We have a parallel case in investment in " variable capital ". Cf. my *Uber Wert*, etc., p. 104.

[2] Clearly, if the factors of production c and r are both increased in the same proportion, then, since k is a constant, the product P is also increased in the *same* proportion. [P is the product, c is free labour, and r the machine-capital with which it co-operates. Cf. Åkerman, *Realkapital*, p. 41.]

disadvantage of holding (in my opinion needlessly) only for a special case, so that the figures for the product increase without intermittence. As I shall show later, this result should also be perfectly general.

Similarly, if we postulate the existence of free competition and disregard the effects of inventions, wages should rise in all cases with an increase in the amount of durable capital. But as Table IV clearly shows, they will rise less than proportionately to the increase in capital. In other words, although the extension of the life-time of capital-goods cannot entirely frustrate a rise in wages, it is adopted in reaction to such a rise, which has already taken place.

I must adopt a more sceptical attitude to the statement on p. 152 ff., even though it is made with certain qualifications. On "variable capital" I have observed in my own writings that von Thünen's thesis that the rate of interest is determined by the addition to the product due to the "last" portion of capital does not hold for an increase in the whole of the social capital. It is only valid for a low rate of interest, since part of the increase in capital is absorbed by increased wages (and rent) so that only the residue of the increase in capital is really effective as far as a rise in production is concerned. The author now says that von Thünen's thesis may hold even for social capital if only we take into account the increase in "concrete" capital, i.e. the amount of labour recently invested to the value of the previous increase in capital. This should probably prove to be right, if only we could always, so to speak, catch hold of this concrete capital. For example, the principle holds perfectly for Böhm-Bawerk's schema (*vide* Appendix). But in the arithmetical demonstration here given, it only depends on the fact that capital-goods invariably last for a single year and no more, so that capital grows exclusively in breadth, and thus proportionately to the amount of labour annually invested. Åkerman further assumes that it takes a year to manufacture any capital-good. To obtain a picture of the process as a whole, we can imagine a supply of free labour always co-operating with another supply of labour, which has already been invested for exactly a year and is now "maturing". The problem now becomes extremely simple, and the result is really only an application of the principle that "interest is the difference

between the marginal productivity of saved-up (accumulated) labour and that of current (free) labour", but it is actually much too simple to permit of drawing any conclusions for fixed capital lasting for several years. For the inter-relations are much more entangled here, and as we have said concrete capital (so-called) has no proper significance in this case. Åkerman himself admits that his tables cannot provide any complete corroboration of this definition of interest. Characteristically enough, he does not seem to be certain which of the numerous capital concepts he has defined should be used as the basis of his calculations, but he believes that better results will be obtained by adopting the rate of interest at a moment of time and by applying "higher mathematics" to the problem. As I was rather interested in the subject, I undertook a minor piece of research of this kind, which I append at the end of my review. It leads to a particularly interesting result, but the above definition is not corroborated.

Böhm-Bawerk (and in fact Jevons also) describes interest as being determined by the relation of the last addition to the product to the extension of the period of investment, or to put it in another way—by "the marginal productivity of waiting" Much to his disappointment the author has not succeeded in showing that this definition, closely related as it is to the one just discussed, is compatible with the results of his tables; this is because he is dealing with a constant investment period of a single year. This discrepancy depends on an omission on the author's part—an omission to which, I believe, attention was already drawn at Åkerman's *viva voce* examination. With his formulation of the problem, he should have taken not the value of the annual product, but the (total) sum of wages paid out in the course of the year as the divisor. (If simple interest is applied, as in Åkerman's analysis, we ought generally to calculate the interest accruing on the original capital and not on the increasing products.) Once this factor is taken into account, his tables are brought into agreement with Böhm-Bawerk's definition, though it does not follow that anything is demonstrated for the general case. We are here confronted with the thorny question of the *average* investment-period. In this case it was due to the simple character of the problem that the author could—apart from the above omission—deal with

this concept, the average investment period is here only another way of expressing the proportion between labour which is and labour which is not invested. But not so for "staggered" production. For instance, in the Böhm-Bawerkian scheme the average period of investment for capital in the process of maturing at each moment of time is half the period of production, and this magnitude constantly appears and reappears in the formulæ. But it can easily be shown (*vide* Appendix) that the average period of investment for all capital is a *third* of the period of production; and I do not see how this magnitude and its successive modifications could possibly be put in a simple relationship with the variations in the net product. Perhaps I have misunderstood the author or else am merely mistaken—if so I earnestly hope that I shall be corrected. But it really does appear to me that Åkerman has here been involved in an attempt to solve an insoluble problem. Clutching at any straw, he says that if the two quantities are compared in a certain position, they both become *zero at the same time*, which of course does not prove that they are generally identical.

Actually the disagreement lies in the nature of the subject-matter, and we cannot blame Åkerman save for pronouncing a judgment he could not satisfactorily substantiate. At the end of the book he also promises to analyse the dynamic aspects of the problem,[1] and he will probably succeed in illuminating these obscure and intricate points, of which I for my part am far from believing myself the master.

Our analysis is naturally valid for the construction of machines. For firstly machines, the uses of which have not changed, will be constructed to last long enough to be economically remunerative, and secondly, if we are considering a change in the life-time of machines, those machines which only last as long as before will be given as many useful qualities as possible from all points of view. This property, which Åkerman deals with in his Introduction, he sums up in the name "automatism". It is well known that machine technologists talk of an automatic power of 100 per cent and an automatic power of 50 per cent according as machines "save" more or less labour. The author deserves all praise for seeking to give greater scientific precision

[1] [The second volume of *Realkapital und Kapitalzins* (Stockholm, 1924) deals with durable real capital in dynamic conditions.]

to an idea which is so vague in ordinary everyday speech. Yet his treatment of the question does not seem to be as clear and definite as would have been desirable ; if it is at all possible to obtain perfect clarity in this sphere. He says (pp. 27–8) that "any durable capital-good, in the production of which some labour has been invested, has thus attained a degree of automatism such that it later requires a given amount of co-operating labour, neither more nor less, if the maximum amount of efficiency *per co-operating labourer* is to be obtained". Automatism, he continues, is to be regarded as high or low according as the machine in question requires "a smaller or greater amount of co-operating labour in proportion to the labour originally invested, in order to reach this maximum return per unit of co-operating labour".

To say the least, this description is not very lucid. If the words italicized (by the author himself) mean the free labour resources co-operating with machines, as the context appears to require, then the statement is incorrect. For whom would it benefit that the product per unit of this labour and no other should be as large as possible ? But even if by "co-operating labour" we understand the whole supply of co-operating labour, both free and invested, Åkerman's thesis still remains incorrect, unless the rate of interest has fallen to practically nothing. In equilibrium, the distribution of the available supply of labour between free and invested labour must rather be such that the capitalists obtain the maximum interest compatible with the current rate of wages, and labourers, taken as a whole, the highest wages compatible with the current rate of interest. But in these circumstances "Automatism" becomes an integral part of the whole problem of production, from which it cannot be separated. Nor can it acquire an independent significance. On the other hand, there ought to be no serious difficulty in attempting a theoretical treatment of the question, in which we start with a state of economic inertia, all machines being of identically the same kind with reference to their potential uses.

The book is not without its shortcomings and weaknesses, but as far as I can see they are fewer and less important than one might have expected in the treatment of so extraordinarily difficult and exhausting a problem. The normal reader cannot imagine the practical difficulties encountered in carrying out the

calculations. The unreality of the arithmetical tables is striking enough; for example, one cannot help noticing that they record a precipitous decline in the rate of interest after a comparatively modest increase in capital. Again, according to Table IV, when a society's capital increases there is an almost uninterrupted fall in the *total* capital gains—a circumstance which, in this respect, is very discouraging for capitalists. This result is largely due to Åkerman's actual choice of the terms of the *i*-series—the additional labour necessary for making a machine last longer. If they are to correspond to the facts of the real world, they should from the very beginning decline more rapidly than he makes them do. It was impossible for convenience of exposition to adopt this procedure, for in the author's view the terms of the *i*-series should be chosen so as not to infringe the principle that in general the duration of some capital-goods cannot advantageously be extended beyond certain limits. It is, therefore, not sufficient to make the terms of this series stop falling at some point or other, but, as the author rightly maintains against Rae (pp. 22 and 118) it is also necessary that their *average* size (per year of life-time) should cease to decline. If he had wanted to obtain figures more closely approximating to the real world he would, in the first place, have been compelled considerably to extend the *i*-series. In the second place, the tables would then have become too full, and it would have necessitated the use of higher powers for the rate of interest for a moment of time, and the calculations would have become extremely tedious and difficult.[1] Most of these obstacles might be overcome by the use of more powerful mathematical tools, but this must be left to the future. As they stand, most of the columns of figures in all cases fulfil their function of illuminating the most significant aspects of the phenomenon.

In my opinion, the more purely critical sections of the book testify to Åkerman's erudition and soundness of judgment.[2]

[1] The series employed are all recurrent and can therefore be reduced to a few terms—a fact with which the author does not seem to be acquainted, except in the case of geometrical series.

[2] I may mention *en passant* that the passages from my *Uber Wert* and *Lectures* quoted on p. 135 are hardly inconsistent. In the earlier passage I am dealing with the antithesis between short- and long-term investment. Åkerman does not make this point clear. I maintained that arithmetical averages are still of some use in handling short-term investments. I did not say that this method was exact, for if that were so they would also be applicable to long-term investments. I must express my gratitude for an acknowledgment

I am convinced that on the whole the author has made a really significant contribution to the theory of capital, and it is with great interest that I look forward to the continuation of his work. Only I should advise him to remember in his new exposition that the contemporary reader, even of scientific works, seldom has unlimited time and patience at his disposal.

of my own work which if anything is only too generous. He wishes to associate my treatment of the Wage-Fund with Böhm-Bawerk's well-grounded Wage-Fund theory. Actually my more rigorously mathematical analysis of Böhm-Bawerk's arithmetical exposition was much too derivative to have any particular merit of its own.

2. Real Capital and Interest (*Continued*)

(*b*) A Mathematical Analysis of Dr. Åkerman's problem

In the following pages, we shall attempt a mathematical solution of the problem we have just been discussing. We start with the assumption that production is continuous and that capitalization takes place on the basis of the rate of interest for a moment of time. Since machines are in fact discrete and are not therefore capable of being divided into infinitesimal parts, our result will of course only have an approximate validity. But no more can be obtained by any other method of approach.

Using an amount of labour a, a labourer (or group of labourers) produces a capital-good, e.g. an axe, which is instantly taken into employment. If used normally the axe can remain in use for n years after which it is devoid of any value. We assume that the axe is so small (or that the group of labourers required so great) that the *length of time* required for its production compared with its actual life-time need not be taken into account. Our calculations are thus simplified to a considerable extent without, however, losing in force. Naturally it does not follow that a is a negligible quantity.[1] If, however, a labour-year (or else the work of a whole group of labourers for a year) is taken as the unit for the services of labour, a becomes quite small and its reciprocal $\frac{1}{a}$ quite large.

The exchange-value of an axe to the man who buys or employs it naturally depends on its utility for his purposes. We make the additional assumption that this value is known, and that it is estimated to be b (shillings) per annum ; b is therefore the *sum* of the undiscounted value of all its uses for one year. Let us assume that the axe is applied uniformly throughout the year (or years). If Δt is a fraction of time, then the value of the axe's uses for this time is clearly $b.\Delta t$. If we relate the

[1] For example, in modern house-building all the different parts and accessories of the house are manufactured at the same time as the foundations are laid, so that the whole house, even though actually requiring an *amount of labour* corresponding to ten labour-years, is in fact completed in the course of a few months, perhaps only a few weeks, i.e. in a negligibly small period of time as compared with the house's own probable duration.

axe's employment through t years to the present moment and let r be the rate of interest, we obtain its present value by dividing $b\Delta t$ by the binomial expression $(1+r)$ raised to the power t. Thus—

$$\frac{b.\Delta t}{(1+r)^t} \tag{1}$$

Let $1+r=e^\rho$ where $e=2\cdot718\ldots$ is the base of the natural system of logarithms and ρ is thus the "natural" logarithm of $1+r$, i.e. the ordinary logarithm divided by $\cdot434\ldots$ It can also be expressed in terms of r by means of the logarithmic series, $\rho=\log_e(1+r)=r-\frac{r^2}{2}+\frac{r^3}{3}-\ldots$ which is convergent for $r\leqq 1$. ρ is the instantaneous rate of interest for a moment of time, or what is called in German "Verzinsungsenergie". ρ and r more or less coincide with sufficiently small values for r; otherwise ρ is always less, if only insignificantly, than r (if r is 5 per cent, $\rho=4\cdot88$ per cent, and if ρ is exactly 5 per cent r is $5\cdot13$ per cent, and so on). In each case they stand in a definite arithmetical relationship to each other, and it is not very incorrect to assume them to be wholly substitutable for each other.

Substituting in this manner, we obtain for the value of each of the axe's uses discounted to the present—

$$b.e^{-\rho t}\Delta t \tag{2}$$

Since t is to be taken here as continuously variable, we obtain the present value of all the axe's uses and therefore its own present value by the summation (integration) of the above expression between 0 and n, two points in time

$$b\int_0^n e^{-\rho t}dt=b\frac{(1-e^{-\rho n})}{\rho} \tag{3}$$

(corresponding to the normal calculations for annuity-loans). If r, and consequently ρ also, were so small that in expanding the series for the exponential function—

$$e^{-\rho n}=1-\rho n+\frac{(\rho n)^2}{1.2}-\frac{(\rho n)^3}{1.2.3}+\text{etc.},$$

we need only include the first two terms, the above expression is reduced to $b.n$; in other words, the present value of the axe is equal to the (undiscounted) value of all its uses. If we include the first three terms, we get $bn\left[1-\frac{\rho n}{2}\right]$, i.e. the total use-value discounted by simple interest on it for half its period of use.

In equilibrium, the value of the axe coincides with its costs of production. Let l be wages per head per annum. Then—

$$b.\frac{1-e^{-\rho n}}{\rho}=al.^{1} \qquad (4)$$

This equation holds for a, b, l, ρ (or r), and n, as they are determined in an equilibrium situation. If equilibrium is not yet reached, equation (4) describes the following conditions instead. Let us assume that not only is b (the value of the axe's use for a year) given, but also ρ and r, r being taken as the usual rate of interest current at the time. Now if n and a, the life-time of the axe and the amount of labour needed for its production respectively, were also to be technically given (as we often take them to be), the R.H.S. of the equation would represent the sales-value of an axe (l the wages per annum multiplied by a the unit of labour) which is received by the axe-manufacturers. Now although the magnitude of neither n nor a is given, they are *technically related* to each other. By investing more labour on an axe we can increase its durability, all other properties remaining constant; n is thus a function of a and a of n, i.e. of the period for which it is sought to make the axe last while it is being manufactured. Clearly, both increase together, but n must increase *more than* proportionately to a, otherwise, however low the rate of interest, labour could not be employed in producing axes of longer duration, but it would be employed in producing *many* less durable axes instead. We assume therefore that a varies as a fractional power of n, i.e.

$$a=kn^{\nu} \qquad (5)$$

where k is a constant and ν a proper fraction. If, for example, $\nu=\frac{1}{2}$, a would grow proportionately to the numbers 1, 2, 3,

[1] If the yearly services of a whole group of labourers—say of ten men—is taken as the unit, the amount a in terms of this unit falls in proportion as l (in terms of shillings) increases.

4, etc., whilst n grows as the numbers 1, 4, 9, 16, etc. In other words, n increases geometrically in relation to a. Of course the form of this function is too special to reflect the actual relation between a and n when both are undergoing large changes, but with smaller variations which, as a rule, are the only ones likely to occur in practice, it may be as good an approximation formula as any other.[1] If we assume, for example, that it held for axes lasting for 16 to 36 years, and that $\nu = \frac{1}{2}$, then the constant k represents a *quarter* of the amount of labour required to give the axe in question a life-time of 16 years; or else, and it here comes to much the same thing, a *fifth* of the labour needed to produce an axe which is intended to last 25 years, etc.

At this stage, we could, of course, eliminate a from equations (4) and (5), and then l and b would be the only unknowns outstanding. But we prefer to retain both equations in their present form.

For the labourer, or group of labourers, if they themselves are the entrepreneurs, the most advantageous value of n is that which makes the selling price of the axe a maximum in relation to the amount of labour invested, i.e. makes l attain its maximum.[2] Since a variable magnitude at its maxima (or minima) behaves like a constant, we have to differentiate equation (4) as though l were a constant, which gives

$$be^{-\rho n}\Delta n = l\Delta a.\text{[3]} \qquad (6)$$

We have again obviously obtained on the L.H.S. an expression of the form of equation (2), n and Δn taking the place of t and Δt. The obvious implication is that at its maximum $b\Delta n$, the last addition to the value of the axe, when discounted to its present value exactly corresponds to $l\Delta a$, the last increment to the cost of its manufacture.

We get by logarithmic differentiation of (5)

$$\frac{\Delta a}{a} = \nu\frac{\Delta n}{n}. \qquad (7)$$

[1] On the other hand, there is no expression to correspond with Åkerman's i-series, which would describe the condition that the durability of some capital-goods cannot successfully be increased beyond a certain point.

[2] We might also assume that they do not sell their axes but hire them out. Here they must themselves borrow at the rate of interest r or (ρ) for maintaining them—the theoretical result is the same in both cases.

[3] That the remaining condition for the maximization of l, as of ρ in the next case, is here always fulfilled will be shown later.

Substituting in (6)

$$be^{-\rho n}\frac{n}{\nu} = la \tag{8}$$

and combining with (4), we obtain finally

$$e^{\rho n} = 1 + \frac{\rho n}{\nu} \tag{9}$$

This result is rather peculiar. The product ρn is here the root of an equation, in which ν is the only variable. In other words once the particular function we have used for extension of life-time is taken as given, it follows that the product of the rate of interest (with continuously compound interest) and the optimal lifetime of the axe is a *constant*, independently of the size of b, as soon as we regard ν as a technical datum. Even with the choice of a less simple function, the connection between n and ρ remains independent of b, provided a is a function of n. (9) is of course a transcendental equation, but we can easily obtain an approximate result for the larger of the real roots.[1] (The other $= 0$ for every value of ν.) If, for example, $\nu = \frac{1}{2}$, ρn is roughly $1 \cdot 27$, so that if ρ is $\cdot 05$ (and the ordinary rate of interest therefore a little over 5 per cent) the axe's optimum life-time is always *circa* 25 years, however much the value of its uses, calculated per annum, may vary. We shall indicate this root by $\phi(\nu)$ with the proviso that it is a *constant* as soon as ν is taken as a technical datum. The following analysis depends to a great extent on this result.

We have hitherto regarded the rate of interest (r or ρ) as given. Now if we consider capitalists as entrepreneurs, l must be taken as given instead. Those capitalists, who at a given wage manufacture axes to be later applied, are confronted with the problem of making the axes last so long that the capital invested in their manufacture receives the maximum rate of interest. From a mathematical point of view, this problem leads us to exactly the same formula as the first, for when ρ reaches its maximum, it behaves as a constant, and we have therefore to differentiate equation (4) as though l and ρ were constants. We obtain precisely the same equation as before, and also equation (9) in a similar manner.

[1] This can be solved by expanding according to Lagrange's theorem, taking out the root $\rho n = 0$.

$$e^{\rho n} = 1 + \frac{\rho n}{\nu} \tag{9}$$

But it is no longer ρ but l which is the datum. To find n we substitute in (8) the value discovered from (9) for $\rho n = \phi(\nu)$ (e.g. $1{\cdot}27$ if $\nu = \frac{1}{2}$), and eliminate a by means of (5). Thus

$$n^{1-\nu} = \frac{l}{b}\, k\nu e^{\phi(\nu)}, \tag{10}$$

or what comes to the same thing, as $\phi(\nu)$ is the root of (9).

$$n^{1-\nu} = \frac{l}{b}\, k[\nu + \phi(\nu)] \tag{10 bis}$$

If $\nu = \frac{1}{2}$ and $\therefore\ \phi(\nu) = 1{\cdot}27$, we get

$$\sqrt{n} = \frac{l}{b}\, 1{\cdot}77k$$

We are here restating the principle with which we were acquainted before, that an increase in wages produces a tendency to increase the durability of a capital-good, in this case in geometric proportion to the rise in wages.[1] This tendency corresponds to the extension of the period of production in the case of "variable real capital" (circulating capital).

Before going any further, we should like to mention an interesting fact with reference to the *average* investment-period of capital tied up in a particular capital-good. Under normal circumstances, the annual yield of a fixed capital-good will afterwards repay as well as yield interest on the costs incurred in making it. As we have maintained in our review of Åkerman, the question of the *order* in which either the former or latter occurs is of merely formal interest. But we should be able to represent the average investment-period of this capital as a period such that if all the uses of the capital-good were finally turned out *at the same time*, they would yield the same interest on the capital as the owner actually obtains. Let this period be m. Since in our example the total value of all the uses is clearly $b.n$, with equation (4) we get

$$bne^{-\rho m} = b\frac{1 - e^{-\rho n}}{\rho} = al \tag{11}$$

[1] We shall later try to show that this result is perfectly general, quite apart from the function for extension of lifetime.

if a is here increased, and therefore according to (5) n too, m must also be increased.[1] Now since n is at its optimal value and we can regard l and ρ as constants (for one is assumed to be an actual constant and the other has attained its maximum), we obtain by logarithmic differentiation of (11) the equation—

$$\frac{\Delta n}{n} - \frac{\Delta a}{a} = \rho \Delta m \qquad (12)$$

describing the relations between the simultaneous increases in n, m, and a. This result is not difficult to interpret. Since a is the amount of labour required to produce one axe, $\frac{1}{a}$ is the number of axes produced by one unit of labour[2] and $\frac{n}{a}$ the number of (potential) yearly uses of $\frac{1}{a}$ axes. Therefore $\frac{bn}{a}$ is the value of all their uses. If for the moment we call this expression P, and retain our assumption that b is a constant we obtain by logarithmic differentiation—

$$\frac{\Delta P}{P} = \frac{\Delta n}{n} - \frac{\Delta a}{a} = \rho \Delta m$$

or

[1] It can easily be shown that if m, the average investment-period, is reckoned on this principle (i.e. of the annuity-loan), it is rather less than half the "amortization period" $\frac{n}{2}$. But the lower the rate of interest, the more closely does it approximate to $\frac{n}{2}$. Since ρ the rate of interest varies inversely with n in our example, m must necessarily increase at the same time as n, perhaps even in a somewhat greater proportion. (We have here another example of the fact that compound interest is superior to simple for purposes of computation; for with the ordinary annuity-loan calculated in the same way, the average amortization period sometimes falls short of half the loan-period and sometimes exceeds it, according to its length and the height of the rate of interest. If, for example, a man has to effect an outlay of £50 at the end of every year for the next twenty years, the best thing for him to do would be to pay the whole lot at once after ten years, if the rate of interest is above 5 per cent, but not otherwise.)

[2] Since a is small, $\frac{1}{a}$ is large. But to make matters more intelligible we can imagine the number of axe-makers to be so large that even this number of axes can be produced almost simultaneously, so that taken together they can be regarded as a single capital-good.

$$\rho = \frac{\dfrac{\Delta P}{\Delta m.}}{P} \qquad (13)$$

We might have derived this result directly from (11); it holds, therefore, even if b is not taken as constant, but is allowed to vary in some proportion or other to the lifetime of the axe, as soon as ρ or l attains its maximum. Thus in dealing with fixed capital we obtain a counterpart to the Jevonian principle that interest is "the rate of increase of the produce divided by the whole produce", or is the "marginal productivity of waiting", i.e. with reference to *average waiting* reckoned according to the above principle. At this point we must note that the amount of labour invested is taken as fixed (= 1 unit of labour) so that the average period of waiting becomes capital's only variable dimension. It is also worthy of notice that the principle holds for the *whole* duration of the capital-good, and not merely for the period for which the stock of machines of different ages (= the existing fixed capital) still has to last. On the other hand, it is fairly clear that our principle is completely independent of the assumption we made about the form of the function for extension of lifetime.

We turn now to consider the stock of fixed capital. If the labourer (or group of labourers) continues to produce axes, he (or it) will produce $\frac{1}{a}$ axes in one year and $\frac{n}{a}$ axes in n years.[1] Within this period the number of axes in use will obviously continually increase, but once we get beyond n, it ceases to do so, since the oldest axes are discarded *pari passu* with the manufacture of new ones. Thus we have got here a *fixed* capital consisting of axes, which is "staggered" in structure and which includes $\frac{n}{a}$ axes of various ages, *and as a matter of course the number of uses available is the same at any moment.* The total (undiscounted) value of all the uses available in one year is

[1] The expression $\frac{n}{a}$ has thus a double significance; it is the amount of potential uses of the number of axes produced by one unit of labour in the first place, and the total number produced by a labourer in n years in the second. Because of its second implication it is described in the text as the total number of uses available at one and the same time.

therefore $\frac{b.n.}{a}$. Again, the total value of all the potential uses which the fixed capital, consisting of axes and existing at each moment, represents, is clearly

$$b\frac{n}{a}\frac{n}{2} = b\frac{n^2}{2a}$$

For the *time* elapsing during the manufacture of an axe is assumed to be so short that the age of the axe grows continuously from 0 to n years. This proceeds on the assumption that only a single labourer or group of labourers is employed in producing axes. If, however, M labourers or $\frac{M}{10}$ groups of labourers with ten men in each group are occupied in manufacturing axes, all our quantities will naturally have to be multiplied by M; *from now on we take the annual services of one labourer as the unit of labour.*

Now in order to find the value of the capital itself we employ in our calculations that rate of interest which is attained when the best possible line of action is adopted in the use of each individual axe for the *whole* of its life-time. Once equilibrium is finally reached this rate must coincide with the current rate. According to (3) the value of a new axe with n years to live is $b\frac{(1 - e^{-n\rho})}{\rho}$. Therefore the residual value of an axe already used for t years must be

$$\frac{b(1 - e^{-(n-t)\rho})}{\rho} \tag{14}$$

Since Δt is an infinitesimal period of time we regard the axes between the ages $t + \Delta t$ as having the same value. Now since one labourer produces $\frac{1}{a}$ axes per unit of time (one year) and M labourers therefore produce $\frac{M}{a}$ axes; the number of axes in the moment Δt produced t years ago is $\frac{\Delta tM}{a}$ and their total outstanding value is according to (14)

$$M\frac{b}{a}\frac{1 - e^{-(n-t)\rho}}{\rho}\Delta t.$$

Summing all these values, we obtain the value of all the fixed capital by integrating between $t = 0$ and $t = n$. Thus

$$K = M\frac{b}{a}\frac{1}{\rho}\int_0^n (1 - e^{-(n-t)\rho})dt = M\frac{b}{a}\frac{\rho n - 1 + e^{-\rho n}}{\rho^2} \qquad (15)$$

This equation corresponds to the sums of the recurrent series in Åkerman's analysis, which he does not however summate. It can be checked, for if ρn is so small that we need only consider the first three terms in the exponential series $e^{-\rho n} = 1 - \rho n + \frac{(\rho n)^2}{1.2} - \frac{(\rho n)^3}{1.2.3} + \text{etc.}$, our equation is then reduced to $M\frac{b}{a}\cdot\frac{n^2}{a}$, corresponding to the undiscounted value of all the potential uses of the axes, as we have already seen. Even if the fourth term is included, we obtain the same expression multiplied by the binomial $\left(1 - \frac{\rho n}{3}\right)$, i.e. the value of all the potential uses minus the simple interest on them for a *third* of the *whole* lifetime of each axe—a new but naturally incomplete approximation. The quantity $\frac{n}{3}$ is the distance of the centre of gravity from the base of a triangle, the height of which is n and the base the number of axes in existence. If the potential uses of the whole existing stock of axes are discounted back to the present, the average period of discounting should in fact be $\frac{n}{3}$ (cf. review, p. 270), if we use simple interest.[1]

[1] If a capital-good lasts altogether N *weeks*, and if the same number of capital-goods are all of various ages, the number of remaining weeks' uses of a good already in existence for T weeks is clearly $N-T$, and its average period of discounting, using simple interest, is $\frac{N-T}{2}$ weeks. We obtain the average period of discounting for the whole stock from the formula :—

$$\frac{\sum_{T=0}^{T=N-1} \frac{1}{2}(N-T)^2}{\sum_{T=0}^{T=N-1}(N-T)} = \frac{\frac{1}{2}(N^2 + (N-1)^2 + \ldots + 9 + 4 + 1)}{N + N - 1 + N - 2 + \ldots + 3 + 2 + 1} = \frac{2N+1}{6}$$

or since N is here a large number, $\frac{N}{3}$ weeks approximately. And in the same way, still using simple interest, we get the average *period of investment* for a "staggered variable real capital". (Cf. the relevant passages in my review.)

We can easily prove that at any moment the net value of the uses of the whole of the axe-capital, i.e. the gross value minus the cost of renewal of capital, is the interest on the total value of the capital at the same moment. For it follows from what we have just said that the former is $M\left(\frac{nb}{a} - l\right)\Delta t$, which, using (4), becomes

$$M\frac{b}{a}\frac{\rho n - 1 + e^{-\rho n}}{\rho}\Delta t = \rho K \Delta t. \tag{16}$$

(16) is of course bound up with the fact that the residual capital-value of the axes already in use is precisely estimated according to this rate of interest, and may therefore be called a truism.

We have not yet made any use of our assumption about the nature of the function of "extension of lifetime", i.e. equation (5). Once (5) is taken into account, K, the amount of capital, becomes a much simpler expression, for in this case ρn is a constant $= \phi(\nu)$, and so the numerator of our fraction also becomes a constant. Further, ρ and a can be simply expressed in terms of n, so that we can express K in terms of M, b, and n. Since according to (10) n is proportional to some power of the ratio $\frac{l}{b}$, we can express K in terms of l and b only, but always with the proviso that it is also a multiple of M and includes a constant factor, which is solely dependent on the value of ν, which is technically given. The significance of this consideration will become apparent later.

In actual fact neither l nor b is given, but the value of both is ultimately determined by the co-operation of free labour with real capital in the production of commodities. For we assume that under free competition wages l are the same for *all* labour, whether it is free labour or "replacement labour" (Åkerman), which is annually invested in machines. To obtain this economic nexus and the data necessary for solving the whole problem, we must now make the further assumption that all the capital of the community consists exclusively of only one kind of capital-good, in this case axes, and that only one kind of product is produced. Since we have previously only been occupied with capitalistic production in its simplest form

we are doubtless justified in making an assumption which is rather fantastic if taken by itself.

Let x free labourers co-operate with y units of capital (axes) in a given form. Now with the optimal employment of resources, the product, or the value of the product, will clearly be a *function* of both x and y. We can decide *a priori* that this function must be *homogeneous* and *linear*, i.e. such that a *uniform* increase in x and y produces exactly the same percentage increase in the product. For if two labourers, each having his own axe, could together produce *more* than twice as much as one labourer with one axe, or if the product of three labourers and three axes was proportionately even more, and so on, then we should obviously have to let the labourers co-operate in groups in such a way that the maximum efficiency was reached. But once this maximum has been attained, a further increase in labourers and axes, i.e. an increase in the *number* of such groups, would only produce a proportionate increase in the product. On the whole we can therefore assume that with a constant "stock" of axes per labourer, the product grows in proportion to the number of labourers, but with an increasing or diminishing stock of axes, labour remaining constant, the product certainly increases or diminishes in some degree, although less than proportionately to the change in the number of axes. In other words our productivity function, which we represent by $F(x, y)$, must take the form,

$$F(x, y) = x\Phi\left(\frac{y}{x}\right),$$

where Φ is a function of a *single* variable, i.e. of the ratio $\frac{y}{x}$. It increases or diminishes simultaneously with its variable, but to a lesser extent. For if it increased in the same proportion, the whole expression could be reduced to $cx\frac{y}{x} = cy$, where c is a constant; in other words, we should arrive at the ludicrous result that the product was solely dependent on the number of axes and not at all on the number of workers. We should get a still more ludicrous result if the function Φ increased more than proportionately to its variable.

Since we are chiefly concerned with expressing this relation

in as convenient a form as possible for our calculations, we may simply let the Φ-function vary as a root of its variable, i.e. we may put

$$F(x, y) = cx\left(\frac{y}{x}\right)^{\beta} = cx^{\alpha}y^{\beta},$$

where α and β are both positive fractions and their sum $= 1$. P, the value of the product computed for a moment of time,[1] thus becomes

$$P = F(x, y) = cx^{\alpha}y^{\beta}. \tag{17}$$

If this equation is partially differentiated with respect to x and y, we obtain

$$\frac{\partial P}{\partial x} = c\alpha x^{\alpha-1}y^{\beta} = \alpha\frac{P}{x}$$

and

$$\frac{\partial P}{\partial y} = c\beta x^{\alpha}y^{\beta-1} = \beta\frac{P}{y}$$

Let us postulate a stationary state in which there is perfect competition between employers and labourers. Once equilibrium has been reached, the first partial derivative must necessarily equal or l the wages per head per annum, and the second b, or the payment received for the yearly use of an axe. Thus

$$l = \alpha\frac{P}{x} \text{ and } b = \beta\frac{P}{y}, \tag{18}$$

from which, among other things, it follows

$$xl + yb = (\alpha + \beta)P = P, \text{ since } \alpha + \beta = 1.$$

In other words payments, so determined, made to the labourers and the owners of the axes, will together absorb the total value of the product; which is as it should be. Similarly, assuming a continuous productivity function, we obtain the simple ratio of b to l—

$$\frac{b}{l} = \frac{\beta x}{\alpha y} \tag{19}$$

Let A be the total number of labourers or the supply of labour annually available. If M is the number of labourers

[1] We might also have calculated it for an infinitesimal period of *time*, i.e. multiplied both sides of the equation by Δt. But once production is taken as stationary, this procedure would make no difference whatsoever.

always employed in the manufacture of axes in order to renew or maintain the fixed capital consisting of axes, then the amount of free labour is plainly $A - M$. It follows that the number of axes in use at the same time is $\frac{Mn}{a}$ and that in equilibrium just this proportion between free labourers and axes employed must obtain in each firm, as the result of reciprocal supply and demand; otherwise some of the labourers or axes would be unemployed. We can therefore substitute $A - M$ and $\frac{Mn}{a}$ for x and y in our previous formulæ, and replace P by π, the value of the whole social product. Thus we obtain

$$\pi = c(A - M)^{\alpha} M^{\beta} \left(\frac{n}{a}\right)^{\beta} \qquad (17\ bis)$$

and

$$l = \alpha \frac{\pi}{A - M} \text{ and } b = \beta \frac{\pi}{M} \frac{a}{n} \qquad (18\ bis)$$

and

$$\frac{b}{l} = \frac{\beta}{\alpha} \frac{A - M}{M} \frac{a}{n} \qquad (19\ bis)$$

By making a simple change in equation (8) and then combining it with (9), it follows that if the most profitable lifetime is attained for every axe, then

$$\begin{aligned} \frac{b}{l} &= \nu e^{\rho n} \frac{a}{n} \\ &= \nu e^{\phi(\nu)} \frac{a}{n} \\ &= (\nu + \phi(\nu)) \frac{a}{n} \qquad (8\ bis) \end{aligned}$$

where $\phi(\nu)$ is the root of (9).

We finally obtain—

$$\frac{A - M}{M} = \frac{\alpha}{\beta} (\nu + \phi(\nu)) \qquad (20)$$

This result is calculated to create some astonishment. All the magnitudes on the R.H.S. are *constants* irrespective of the *amount* of social capital. These constants reflect the assumptions

we made (1) for the technical conditions under which our capital-goods are manufactured, and (2) for their co-operation with free labour in the production of consumption-goods. Our assumptions have thus shown that, however much the amount of capital itself changes, the distribution of the existing supply of labour between free labour co-operating with capital-goods and labour employed in the maintenance or renewal of capital itself[1] remains *unchanged.* And yet only within limits, since the form of our function is too special to be valid beyond a certain field of variation, even if it contains one arbitrary constant.[2] Within these limits, however, capital, when it does grow, grows *exclusively* in height and not at all in breadth. *N.B.*—When capital first increases and there is a consequent disturbance of equilibrium, capital will also—or rather exclusively—grow in breadth, since in the beginning the additional number of new capital-goods will be of the same type as those already in use. If, on the other hand, the amount of labour invested per moment of time is temporarily increased and the amount of free labour diminished, there will be a rise in wages and a fall in the value of the use of capital (axes), more or less in this sequence. Further, according to (10), the new capital-goods now produced will be manufactured to last longer, as this method of investment has become most profitable. But when equilibrium is reached once more the amounts of free labour and of labour engaged in replacing capital resume their former proportion (at the same time the labourers lose part of, but not all, their recent increase in wages and the capital-goods regain part of, but not all, the value they have just lost). Employing this interesting result, we might regard the productivity function and the function for "extension of lifetime", which have been selected, i.e.

$$a = f(n) = kn^{\nu}$$
$$\text{and } P = F(x, y) = cx^{\alpha}y^{\beta} \quad (\alpha + \beta = 1),$$

as typically *normal* functions from which, taking them as the simplest elements in the problem, we must start in the analysis of the more complicated phenomena of the real world.

[1] In a stationary state these quantities will themselves be constant.

[2] The two coefficients k and c refer only to the value of units, and therefore leave no room for varying conditions in other respects.

With these constants, the values of M and A plainly become

$$M = \frac{\beta A}{\alpha(\nu + \phi(\nu)) + \beta} \text{ and } A - M = \frac{\alpha(\nu + \phi(\nu))A}{\alpha(\nu + \phi(\nu)) + \beta} \quad (20\ bis).$$

Let $\nu = \frac{1}{2}$: then $\phi(\nu) = 1{\cdot}27$. Further, let $\alpha = \beta = \frac{1}{2}$.[1] Then

$$M = \frac{A}{2{\cdot}77}, \quad A - M = \frac{1{\cdot}77}{2{\cdot}77} A.$$

Rather more than a third of the existing supply of labour should therefore be engaged in manufacturing axes, and the remainder —about two-thirds—in the *application* of the existing stock of axes for the delivery of saw-logs. This result we achieve without taking the amount of axe-capital into account, for, with a small supply of capital in the form of axes, as long as our assumptions hold, they must necessarily be manufactured so as to last for a correspondingly short period, and will therefore need renewal all the more often.

M being determined, the whole problem can be solved without any further difficulty. The remaining unknowns are (1) the amount of capital expressed in terms of the product or of money (for the price of the product is taken as fixed on the great staple markets), (2) the product per annum in terms of the same unit, (3) the duration or lifetime of the capital-goods (axes), (4) wages per annum, (5) the value of the yearly uses of an axe, and (6) the rate of interest prevailing in equilibrium and current throughout the economy. It does not matter which of these is taken as the independent variable, for in any case all the other quantities vary as certain powers of this parameter, each being multiplied by its own constant co-efficient. If we choose n as our independent variable, i.e. if we imagine an equilibrium situation where the total period for which the capital-good lasts is n years, and let C_1, C_2, etc., be the constant coefficients, we obtain

$$K = C_1 n^{1 + \beta(1-\nu)}, \qquad \pi = C_2 n^{\beta(1-\nu)},$$
$$l = C_3 n^{\beta(1-\nu)}, \qquad b = C_4 n^{-\alpha(1-\nu)},$$

and, as before,

$$\rho = \phi(\nu) n^{-1}.$$

[1] It follows from this second assumption that capital and labour are equally important in production, so that a percentage increase in one factor has the same *effect* as an equal increase in the other, which of course is only conceivable in a special situation.

It follows immediately that the exponentials are solely dependent on ν and $\beta (= 1 - \alpha)$. The coefficients depend on k and c, the meaning of which is well understood. In addition, C_1 and C_2, the first two coefficients, contain A as a factor; for by dividing by A we had obtained the capital and product per head (of labourers) of the population.

Thus with the simplifying assumptions we selected the problem is now solved. But we must of course be very careful in drawing general conclusions from the results obtained if only because of the above reservation (and quite apart from the fact that they are no longer applicable as soon as our quantities move in a negative direction, for what is not valid in a special case is still less so in the general). But a few observations may still be permissible.

As ν is < 1, the capital K clearly increases simultaneously with n, and conversely n with K. For the reason mentioned in our review, this interrelation must be general. Similarly, π grows when n (and K) increase, but much less than the latter, since the index is smaller by one whole unit.[1] The conclusion that an increase in fixed capital also produces an increase in the annual product should also be perfectly general, independently of our particular assumption, as we shall immediately attempt to show.

Similarly, l increases when n and K increase, but b diminishes when n and K increase. This conclusion ought also to be general in its validity, as we shall soon show.

Since the expressions for π and l have the same index, the ratio $\frac{\pi}{l}$ remains a constant, in other words, with increasing capital, wages remain an *unvarying* part of the increasing product, which is a necessary consequence of our assumptions. Given our particular productivity function, the *sum* of the wages of free labour in each firm and throughout industry constitutes an unvarying portion of the product, which follows from (18) and (18 *bis.*) And besides since, according to our function for the "extension of lifetime", $A - M$, the total number of free workers remains constant, every free labourer (and therefore

[1] If $\nu = \frac{1}{2}$ and $\beta = \frac{1}{2}$, K becomes proportional to $\sqrt[4]{n^5}$, but π only to $\sqrt[4]{n}$.

all labourers) receives a constant part of the national dividend when capital increases (though of course labour *now* invested is paid in consumption-goods which are ready *now*, and not in the consumption-goods which they themselves help to make). Naturally, this conclusion cannot be general.

If the proportionate share of the labourers in the total national dividend is constant, then the capitalists' share is also constant. But, as we have maintained, this result holds for the interest on all the capital at the moment of time in question, if the rate of interest is ρ.

Hence $\rho\,\dfrac{K\Delta t}{\pi\Delta t}$ must be a constant. This result is correct, for $\rho\dfrac{K}{\pi} = \phi(\nu)\dfrac{C_1}{C_2}$, since the powers of n cancel out.

It may be added that the number of capital-goods (axes) in use at the same time, which on the above analysis is

$$M\frac{n}{a} = M\frac{1}{k}n^{1-\nu}$$

necessarily increases with n and also with K, although in a smaller proportion than either, since $1 + \beta(1-\nu) = 1 - a(1-\nu) + 1 - \nu > 1 - \nu$. This result *is general* and holds as we shall soon show, even in the exceptional case when *M diminishes* with an increase in K.

Let us turn to the transition from one equilibrium to another. It is now possible to discover to what extent the closely-related proposition originally advanced by von Thünen that the rate of interest corresponds to the "marginal productivity" of capital is corroborated by our formulæ in the modified form put forward by Åkerman. By logarithmic differentiation we obtain directly

$$\frac{\Delta K}{K} = (1 + \beta(1-\nu))\frac{\Delta n}{n} \text{ and } \frac{\Delta\pi}{\pi} = \beta(1-\nu)\frac{\Delta n}{n}.$$

Therefore

$$\frac{\Delta\pi}{\Delta K} = \frac{\beta(1-\nu)}{1+\beta(1-\nu)}\,\frac{\pi}{K}$$

We can easily express the value of the ratio $\dfrac{\pi}{K}$ without needing to bother about the rather complicated constants C_1 and C_2. Since the share of capital in the product is equal to the interest

on all the capital $= \rho K$ (cancelling Δt out), it must clearly be $\pi - Al$, or, if we take (18 *bis*) and (20 *bis*) into account, it is

$$= \pi \frac{\beta(\nu + \phi(\nu) - 1)}{\nu + \phi(\nu)}.$$

Thus we obtain

$$\frac{\pi}{K} = \frac{\nu + \phi(\nu)}{\beta(\nu + \phi(\nu) - 1)}\rho,$$

and finally

$$\frac{\Delta \pi}{\Delta K} = \frac{1 - \nu}{1 + \beta(1 - \nu)} \frac{\nu + \phi(\nu)}{\nu + \phi(\nu) - 1}\rho. \tag{22}$$

Our ratio is therefore proportionate, but not equal, to ρ. If $\nu = \frac{1}{2}$, $\phi(\nu) = 1{\cdot}27$, and $\beta = a = \frac{1}{2}$, it becomes $\cdot 92\rho$ approximately, i.e. *rather* less than ρ. This discrepancy is only to be expected, when the increase in capital is partly absorbed by the resulting increase in wages and only part of it is effective in raising production. But since this explanation does not hold here, we may infer that the principle is *not* general. If β is quite small, i.e. if the capital-goods have only a minor significance for production as compared with free labour, then as long as $\nu = \frac{1}{2}$, the first fraction approaches $1 - \nu = \frac{1}{2}$ as closely as possible, whilst the other is always $\frac{1{\cdot}77}{\cdot 77}$, i.e. > 2. Strangely enough, this ratio is thus *greater* than ρ.

In these circumstances, it is already obvious *a priori* that von Thünen's thesis is no longer verified, even in the form in which Åkerman proposes to recast it. In his analysis on p. 152, Åkerman starts by replacing the divisor ΔK by $\Delta K - K\frac{\Delta l}{l}$, and thus subtracts that part of the increase in capital absorbed by the rise of wages. This method of approach is perfectly justifiable (cf. my review) for Böhm-Bawerk's thesis, as we can see from a simple inspection of the formulæ on p. 113 of my *Uber Wert*, etc.[1] But in this particular case, it does not hold good.

[1] If ΔK is replaced by $\Delta K - K\frac{\Delta l}{l}$ in the equation at the bottom of the page, p. 113, op. cit.,

$$\frac{Ap'dt}{dK} = \frac{2p'dt}{ldt + tdl} = \frac{2p'}{l - t^2p''},$$

t^2p'' disappears from the denominator in the fraction on the extreme right, which is reduced to $\frac{2p'}{l} = z$ (the rate of interest).

We obtain without any difficulty

$$\Delta K - K\frac{\Delta l}{l} = K\left(\frac{\Delta K}{K} - \frac{\Delta l}{l}\right) = K\frac{\Delta n}{n}$$

and if $\Delta\pi$ is divided by this expression, the new ratio can be written as

$$\left(\frac{\Delta\pi}{\pi} \div \frac{\Delta n}{n}\right)\frac{\pi}{K} = \beta(1-\nu)\frac{\pi}{K} = \frac{(1-\nu)(\nu+\phi(\nu))}{\nu+\phi(\nu)-1}\rho. \quad (23)$$

The new ratio differs from the old only in this respect, that the factor in the denominator depending on β drops out. Since this is always > 1 as also in this case, the new ratio $\frac{\Delta\pi}{\Delta K}$ is always greater than the old one, but it is not therefore equal to ρ. On the contrary, we should be in a position to show that it must always be *greater* than ρ, except in both the limiting cases, where either ν is very small and $n\rho = \phi(\nu)$ is therefore very large, or where ν approaches unity and $\phi(\nu)$ tends to zero. In *both* these cases the R.H.S. of the equation is reduced to the value of ρ; this is self-evident for the first case and can easily be proved for the second by the method of limits.[1] I cannot enter now on the explanation of this very puzzling formula; presumably it belongs to the sphere of "dynamic" theory, where we cannot confine ourselves to the comparison of two different equilibria, but must also study the transition from one to the other.

Finally, I shall tackle the question which really constitutes the starting point for the whole of this fragmentary essay. It is the validity of the principle that an increase in capital (*measured* in units of product, or the value of the product remaining unaltered) must, *as a general rule*, always produce an increase

[1] Let $\nu = 1 - \epsilon$ where ϵ is a small positive fraction. The value of $\phi(\nu)$ then approximates to 2ϵ, and the value of the denominator thus becomes $+\epsilon$. The denominator cannot change signs between the limits $\nu = 0$ and $\nu = 1$ since it would be at a minimum between these points, which can easily be proved to be impossible. Therefore it always remains positive. We can now also prove that this quantity $\nu + \phi(\nu) - 1$ always has a sign *opposite* both to the second derivative of l with respect to n, ρ remaining constant, and to the second derivative of ρ, l remaining constant, when their first derivative becomes $= 0$; whence the values of l and ρ respectively, obtained above, always describe a real maximum. This need not hold in the general case (*vide infra*).

in the volume of production. We have already seen that it is valid on our assumptions.

But even this conclusion now appears more complex to me than I had first believed. The proof I shall advance rests on the assumption that a rise in wages relatively to the use-value of the machine, that is to say an increase in $\frac{l}{b}$, always brings about an extension of lifetime whenever such an extension can be profitable (in other words if all the data are taken as continuously variable). According to (10) and (10 *bis*) n varies quite simply as a positive power of $\frac{l}{b}$ and *vice versa*, but this conclusion follows from $a = kn^{\nu}$, our function for 'extension of lifetime'. If instead we take a more general function, $a = f(n)$, of which it is only assumed that it becomes zero when n is zero, and increases more *slowly* than n, then the matter is no longer self-evident. For brevity, substitute x for $\frac{l}{b}$. We now obtain the corresponding changes in x and n by differentiating (4) and (6), which hold simultaneously for a given value of $x = \frac{l}{b}$, when ρ has reached its maximum. Thus—

$$\frac{1 - e^{-\rho n}}{\rho} = \frac{l}{b} f(n) = xf(n), \tag{4 bis}$$

and also

$$e^{-\rho n} = xf'(n), \tag{6 bis}$$

where $f'(n)$ is the first derivative of $f(n)$. This expression should now be differentiated with respect to n, x, and ρ, for it involves a shifting of the maximum points themselves. Let $f''(n)$ be the *second* derivative of $f(n)$ and let $\frac{f'(n)}{f(n)} = p$ and $\frac{f''(n)}{f'(n)} = q$. Then on eliminating $\Delta\rho$, we obtain

$$\frac{\Delta x}{\Delta n} = x\left(\frac{1}{n} - p\right)\frac{\rho + q}{\rho + p - \frac{1}{n}} \tag{24}$$

Clearly, on our assumption ($f(n) = 0$ when $n = 0$ and $f'(n)$ diminishing when n increases), p must be $< \frac{1}{n}$ and $q < 0$. The expression $\frac{1}{n} - p$ is therefore always positive, and in the numerator and the denominator of the next fraction q and $p - \frac{1}{n}$ are both *negative*. But we cannot presume without further analysis that they are simultaneously $<$ or simultaneously $> \rho$.[1] It is therefore not *a priori* impossible for Δx and Δn to have opposite signs. Let us return to our function $a = f(n) = kn^{\nu}$. Then clearly $p = \frac{\nu}{n}$ and $q = -\frac{(1-\nu)}{n}$. Consequently, the numerator and denominator are here identical (if multiplied by n they both become $\rho n + \nu - 1 = \nu + \phi(\nu) - 1$) and our equation is simplified thus—

$$\frac{\Delta x}{\Delta n} = (1 - \nu)\frac{x}{n},$$

which can be directly obtained by logarithmic differentiation of (10). Now since $f(n)$, whatever its actual form, has the same *general* form as our special function, we may infer even now that x and n vary approximately to the same degree. But it is not impossible that they might sometimes vary in *different*

[1] But it can easily be proved that the denominator $\rho + p - \frac{1}{n}$ is always > 0. From (6 *bis*) and (4 *bis*) we find that it must here always have the value

$$\frac{e^{-\rho n} + \rho n - 1}{n(1 - e^{-\rho n})}.$$

The denominator of this fraction is certainly > 0, and so is the numerator, since its value becomes $= 0$ for $\rho n = 0$, but later rises continuously, as $-e^{-\rho n} + 1$ its derivative (with respect to ρn) is always > 0.

It is impossible to get any further without knowing something about $\rho + q$. Still we can easily show that the inequality $\rho + q > 0$ (which for $f(n) = kn^{\nu}$ becomes $\phi(\nu) + \nu - 1 > 0$) constitutes the *second* condition necessary for the emergence of a maximum value for l or ρ in the general case. This condition, however, need not be satisfied throughout. As far as I can see, if $\frac{l}{b}$ is given and n is increasing, there is nothing to prevent a sequence in which there first emerges a maximum value for ρ, then a *minimum*, and then a maximum again, and so on. An interesting consequence of this phenomenon will soon be mentioned.

directions, from which it plainly follows that $x(=\frac{l}{b})$ and n are not uniquely determined by each other but that x may have two (or more) values for the same value of n or, conversely, n may have different values for the same value of x.

In actual fact this last possibility may often be reached, but it should not on that account give rise to any serious dilemma. The only practical significance it can have is that an increase of capital is sometimes distributed between two different investments—two types of machine of different durability (though otherwise identical), both yielding the *same* maximum return on capital. We have confined the number of different investments to *two*, because for technical reasons it often does not pay to manufacture capital-goods lasting for intermediate periods.[1]

It would have very much more serious effects on the following proof, if two different values of $x = \frac{l}{b}$ could hold for the same value of n. But fortunately *this can never happen*. If it could, the conditions of our equations (4 *bis*) and (6 *bis*) could simultaneously be satisfied for the same value of n with two different x-values, x_1 and x_2, and with concomitantly different values for ρ, ρ_1 and ρ_2 $(\rho_1 > \rho_2)$. In other words we should obtain at the same time first $\frac{1 - e^{-\rho_1 n}}{\rho_1} = x_1 f(n)$ and $e^{-\rho_1 n} = x_1 f'(n)$, and secondly $\frac{1 - e^{-\rho_2 n}}{\rho_2} = x_2 f(n)$ and $e^{-\rho_2 n} = x_2 f'(n)$, from which dividing we should obtain

[1] If ρ has two maxima (as distinguished from a minimum) for small values of $\frac{l}{b}$ the manufacturer of machines naturally chooses the *larger*, which we shall assume corresponds to the smaller value of n.

Were capital and $\frac{l}{b}$ to increase, the maximum corresponding to the *higher* value of n may become the greater. Now when ρ has two equal maxima (for different values of n), there must be a case in the transition period analogous to that described in my *Lectures*, p. 163. For a time the increase in capital is divided between two different investments, in which l and b and their ratio $\frac{l}{b}$ do not undergo any further change; for since an ever-growing part of the capital is successively transferred to the longer investment, M is diminished and A-M increased, so that the proportion between free labour and the available uses of the machines remains unchanged. But I have not been able to complete any research into this interesting question in detail.

$$\frac{e^{\rho_1 n} - 1}{\rho_1} = \frac{e^{\rho_2 n} - 1}{\rho_2}$$

or

$$\frac{\rho_1 - \rho_2}{2} + \frac{\rho_1^2 - \rho_2^2}{6} n + \frac{\rho_1^3 - \rho_2^3}{24} n^2 + \ldots\ldots = 0.$$

If the values of n and ρ are positive, all the terms in the series are also positive, and our assumption therefore involves something absurd.

We may, consequently, proceed on the assumption that an increase in $\frac{l}{b}$ always produces an extension of the lifetime of capital-goods, even if this extension does not always occur continuously; at times it may take place in jumps (or more correctly in such a way that capital is distributed among capital-goods of the same profitability but of different durations).

On this hypothesis the proof of the thesis we previously advanced takes more or less the following form.

When real capital increases it must always increase in "height", in so far as an extension of the durability of machines is technically possible. For were it only to increase in "breadth", so that the only effect would be an increase in the number of machines of the old type, the labour permanently engaged in maintaining it would clearly have increased, once equilibrium had been reached. Hence it follows that the amount of free labour would have diminished at the same time as the number of capital-goods had increased. This must clearly result in a shifting *upwards* of $\frac{l}{b}$, in which case we must infer from our conclusion, which we have just shown to possess general validity, that an *extension* of the lifetime of the capital-good becomes profitable. On the other hand there is no need for an inevitable increase in the "breadth" dimension of capital which follows from what we have said above. On our formula, with an increase in capital the amount of labour required for renewing capital should generally remain unaltered. We may therefore summarily assume that an increase in capital may very well occur with an accompanying *fall* in the breadth dimension. None the less even in this case *the number of capital-goods in existence at the same time* will have increased, for if it had diminished, since

the amount of free labour has now *increased,* $\frac{l}{b}$ would have shifted *downwards* and we cannot describe the position in which n has a new and higher value as an equilibrium one. It therefore emerges that there will be a larger number of machines simultaneously with a *larger* supply of free labour, which must obviously lead to an increase in the total product.[1]

Let us now take the commonest instance in which machine-capital increases in breadth as well as in height; then the amount of free labour will diminish. We can conceive of this change as occurring in *two* (or more) stages. Let capital grow in breadth to begin with and only later in height also—in other words, we first increase our M, n remaining constant, and afterwards n as well (with M constant).

The first part of this procedure is soon explained. For since the composition of machine-capital remains the same, the whole process can be regarded as though M units of labour invested in a certain way co-operated with $A - M$ free labour in each case. If M is increased, and $A - M$ diminished by one unit, then, ignoring infinitesimal quantities of higher orders, the total product is increased and there is a difference between the marginal productivies of invested and free labour. This difference must be positive, for as we have always regarded the Productivity Function as being homogeneous and linear (or that it has again become so after any change has taken place) the marginal productivity of each group necessarily coincides with its wages. These must clearly be *greater* for invested than for free labour, as the wages of the former also include *some* interest. Now let the lifetime of the same number of capital-goods increase, M remaining constant. Then it follows that the number of machines in existence must increase (for the number of machines per labourer working on machines is $\frac{n}{a} = \frac{n^{1-\nu}}{k}$) and, if the amount of free labour is constant, the total product must increase still more. If the increase in machine-capital is such that as far as the first part of our analysis is concerned the rate of

[1] Similarly, if we abstract from technical discoveries, which change $f(n)$ and $F(x, y)$ the basic functions themselves, wages must always rise with a relative increase in the amount of capital. The general character of the Productivity Function plainly involves the result that l and b always vary *inversely*; if l has increased n must also increase.

interest not only falls but is at the point of becoming zero, we simply stop at this point and allow n to grow until the rate of interest reaches its maximum (and becomes therefore > 0), and using this point as our starting-point we begin again with the same procedure.

Thus the net result is that a growth of capital, as long as it is such as to be profitable, is always accompanied by an increase in the total product. Consequently the paradox of a fall in the national dividend resulting from continued saving and capital accumulation does not apply to perfectly free *competition*, but the possibility of its holding for a situation in which capitalists combine cannot be excluded.

So far we have treated the lifetime of capital-goods as if it were altogether separated from their other property—their "Automatism", as Åkerman calls it. Actually, these properties are scarcely ever independent, greater durability is normally combined with greater efficiency in other respects. We ought to be able to express this mathematically so that the a-function does not actually have the simple form $f(n)$, but also contains a quantity g as a variable which objectively refers to the efficiency of the capital-good in question. Thus if, for example, g increases from g_1 to g_2, and $g_2 = 2g_1$, *ceteris paribus* we get a machine of a new type, which can replace two of the older machines in all respects. We need only substitute $f(n, g)$ for $f(n)$ in equation (4 *bis*), and partially differentiate with respect to n and g, to obtain a new equation corresponding to this variable. However, I shall not undertake it here, as I have already taken too much space.